A Study of Fuzzy Relation and Its Application in Medical Diagnosis

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Authors' contributions

This work was carried out in collaboration between both authors. Authors SK and CG designed the study, performed the analysis, wrote the protocol and wrote the first draft of the manuscript. Both authors read and approved the final manuscript.

Original Research Article

Abstract

Objective: Medical diagnosis process extends within the degree to which they plan to affect different complicating aspects of diagnosis. In this research work, the concept of fuzzy relation with medical diagnosis is studied and the application of fuzzy relations to such problems by extending the Sanchez's approach is introduced.

Method: An application of fuzzy relation with Sanchez's approach for medical diagnosis is presented. Based on the composition of the fuzzy relations, an algorithm for medical diagnosis as follows- first input the number of objects and attributes to obtain patient symptom matrix, symptom-disease matrix and the composition of fuzzy relations to get the patient-diagnosis matrix. Then find the maximum value to evaluate which patient is suffering from what disease.

Result: Using the algorithm for medical diagnosis, the disease for which the membership value is maximum gives the final decision. If almost equal values for different diagnosis in composition are obtained, the case for which non-membership is minimum and hesitation is least is considered. The output matched well with the doctor's diagnosis.

Conclusion: In the process of medical diagnosis, state of patient are given by the patient through linguistic terminology like as temperature, cough, stomach pain etc., consideration of fuzzy sets as grades for

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association instead of membership grades in [0,1] is more advantageous to model the state of the patient. Similarly fuzzy relation has been introduced representing the association between symptoms and diseases. Sanchez's approach has been extended for medical diagnosis in this reference. The approach used to form fuzzy matrix showing the association of symptoms and diseases is based on the sanchez's approach.

Keywords: Fuzzy logic; medical diagnosis; membership function; sanchez's approach.

Mathematics Subject Classification (MSC): 03E72, 03E02.

1 Introduction

Medical diagnosis process extend within the degree to which they plan to affect different complicating aspects of diagnosis like relative importance of symptoms, varied symptom pattern and therefore the relation between diseases themselves. Based on decision theory, within the past many mathematical models like crisp sets, fuzzy sets [1], intuitionistic fuzzy sets [2,3] were developed to affect complicating aspects of diagnosis.

Fuzzy Logic was proposed by Prof. L. Zadeh [1] in 1965 as a means of representing or manipulating data that is not precise but rather fuzzy. It is extremely useful for many people involved in medical diagnosis [4,5]. To diagnose the patient for diseases carries various stages which are certainly filled with uncertainties upto some extent. Physicians generally collect information by examining the patient physically and history of the patient. In physical examination, some symptoms may be overlooked and some important part of the history may not be revealed by the patient.

In addition physicians gather information from the laboratory tests which are often depend upon the precise interpretation of the results which are rare. However techniques are available to compute some symptoms. If almost equal values for various diagnosis in composition are obtained, the case that non-membership is minimum and hesitation is least is taken into account.

In this research work, the concept of fuzzy relation with medical examples is studied and the application of fuzzy relations to such problems by extending the Sanchez's approach [6,7] is introduced. A hypothetical example is discussed to illustrate the methodology.

2 Preliminaries

This section presents a review of some fundamental notions of Fuzzy set, Fuzzy relation, Reflexive relative, Symmetric relation, Intuitionistic fuzzy (IF) set etc.

2.1 Fuzzy set [1]

Definition 2.1.1 A **membership function** is a function which defines the degree of an element's membership in a fuzzy set. Thus a fuzzy set A of a set X is a function $\mu_A : X \rightarrow I([0,1])$.

Definition 2.1.2 A **fuzzy subset** A of the set X is a function from X to I and is denoted by a set of ordered pairs :$A = \{(x, \mu_A(x))$: $x \in A, \mu_A(x) \in [0, 1]\}$, where $\mu_A(x)$ is the grade or degree to which any element x in X belongs to the fuzzy set A. Thus the grade of non-belongingness of x into A is equal to $1 - \mu_A(x)$.

2.2 Fuzzy relation [8]

Definition   2.2.1   Let $X, Y \subseteq R$ be   universal   sets   then; $\{(x, y), \mu_r(x, y): (x, y) \in X \times Y \}$ is called fuzzy relation in $X \times Y \subseteq R$ or X and Y are two universal sets, the fuzzy relation $R(x, y)$ is given as $R(x, y) = \left\{ \frac{\mu_r(x,y)}{(x,y)} \middle| (x, y) \in X \times Y \right\}$

Fuzzy relations are often presented in the form of two dimensional tables. A $m \times n$ matrix represents a contented way of fuzzy relation R.

Example 1: Suppose, we have two sets A and B, where A is a set of diseases and B is a set of symptoms such that Set A = {Typhoid (T), Viral Fever (V), Cold(C), Dengue(D) }, Set B={ Fever(F), Cough(Q), Rashes(R), Shivering(S)}

If the membership values for A and B are {0.9,0.7,0.6,0.3} and {1,0.5,0.1,0.5} respectively, Then, fuzzy relation (R*) between set A and set B are :

R*	F	S	Q	R
T	0.9	0.5	0.5	0.1
V	0.7	0.5	0.5	0.1
C	0.6	0.5	0.5	0.1
D	0.3	0.3	0.3	0.1

2.3 Reflexive relation [8]

Let R be a fuzzy relation in $X \times X$, then R is called reflexive, if $\mu_r(x,x) = 1, \ \forall x \in X$

Example: Let X = {1,2,4,5,6}, then

	1	2	3	4	5
1	1.0	0.8	0.3	0.5	0.6
2	0.8	1.0	0.7	0.4	0.2
3	0.3	0.7	1.0	0.9	0.3
4	0.5	0.4	0.9	1.0	0.2
5	0.6	0.2	0.3	0.2	1.0

R = is a reflexive relation.

2.4 Anti-reflexive relation [8]

Fuzzy relation R ⊂ X×X is anti-reflexive if, $\mu_r(x,x) = 0, x \in X$

Example: $R = \begin{bmatrix} 0 & 0 & 0.6 \\ 0.3 & 0 & 0 \\ 0 & 0.3 & 0 \end{bmatrix}$

$here, \quad \mu_R(x,x) = 0, x \in X$

Therefore, this relation is an anti-reflexive relation.

2.5 Symmetric relation [8]

A fuzzy relation R is called symmetry if, $\mu_R(x,y) = \mu_R(y,x), \ \forall x,y \in X$

Example: Let $X = \{x_1, x_2, x_3\}$

$R = \begin{matrix} x_1 \\ x_2 \\ x_3 \end{matrix} \begin{bmatrix} 0.8 & 0.1 & 0.7 \\ 0.1 & 1 & 0.6 \\ 0.7 & 0.6 & 0.5 \end{bmatrix}$ is a symmetric relation.

2.6 Intuitionistic Fuzzy (IF) set [2]

Definition 2.6.1 An IF set A in a nonempty set X is an object having the form $A = \{(x, \mu_A(x), \nu_A(x)) : x \in X$ $\}$, where $\mu_A(x)$ and $\nu_A(x)$ are functions from X to $I = [0, 1]$ such that $0 \leq \mu_A(x) + \nu_A(x) \leq 1, \forall x \in X$. The numbers $\mu_A(x)$ and $\nu_A(x)$ represent the degree of membership and degree of non-membership for each element $x \in X$ to A respectively.

For each element $x \in X$, the quantity $\pi_A(x) = 1 - (\mu_A(x) + \nu_A(x))$ is called the degree of indeterminacy or hesitation or uncertainty of the element $x \in X$ to the IF set A [9], which may relate to either membership value or non-membership value or both. It is clear that $\pi_A(x) \in [0, 1]$. If A is a fuzzy set, then $\pi_A(x) = 0, \quad \forall x \in X$.

3 Results

3.1 Application of fuzzy relation with sanchez's approach

E. Sanchez [6] in 1976 introduced a concept of relationships between symptoms and diseases. In 1979, he introduced fuzzy relationship between symptoms and diagnoses by the concept of "medical knowledge".

In this section, an application of fuzzy set theory with Sanchez's approach for medical diagnosis is presented. Suppose S is a set of symptoms of disease, D is a set of diseases and P is a set of patients. By applying fuzzy set theory, a technique through Sanchez's method to diagnose which patient is suffering from what diseases.

Sanchez proposed a method for medical diagnosis for fuzzy sets. De et al. generalized this method for Fuzzy sets which is based on a max-min and min-max composition. They obtained medical knowledge from a set of symptoms to a set of diagnoses.

This involves three steps:

1. Determination of symptoms
2. Formulation of medical knowledge based on fuzzy relations
3. Diagnosis on the basis of composition of fuzzy relations.

Let P be a set of patients with symptoms from a set S and let D be a set of diseases. Then, in this method the diagnosis is a fuzzy relation T from P to D which is obtained by the following membership grades:

$$\mu_T(p_i,d) = \{ \underset{s \in S}{\vee} [\mu_Q(p_i,s) \wedge \mu_R(s,d)] \}$$

$$\nu_T(p_i,d) = \{ \underset{s \in S}{\wedge} [\nu_Q(p_i,s) \vee \nu_R(s,d)] \}$$

$$\forall p_i \in P \quad \text{and} \quad \forall d \in D.$$

3.2 Algorithm

Based on the composition of the fuzzy relations, an algorithm for medical diagnosis as follows:

Step I: Input the number of objects and attributes to obtain patient-symptom matrix Q.
Step II: Input the number of objects and attributes to obtain symptom-disease matrix R.
Step III: Perform the composition of fuzzy relations to get the patient-diagnosis matrix T = QoR

Step IV: Find $S_T = \max \quad (p_i, d)$. Then we conclude that the patient p_i is suffering from the disease d.

3.3 Case study

Suppose there are four patient's p_1, p_2, p_3, p_4 in a hospital with symptoms temperature, cough, stomach pain, chest pain, and vomiting. Let the possible diseases relating to the above symptoms be viral fever, typhoid, stomach problem, jaundice, and pneumonia.

Suppose the patient P_1 whose symptoms were recorded by routine case-taking practice. A pair of values was attached to each symptom: first value showing the strength of association (membership value) of that symptom with the patient and the other showing non-association (non-membership value) as perceived by the practitioner.

The symptoms, practitioner decided to include, are like this: Suppose patient P_1 had been suffering from stomach-pain (0.3, 0.9). Here the first value is membership value and the other one is non-membership value.

Table 1. Relation (Q) between patients and symptoms

Q	Temperature	Cough	Stomach pain	Chest pain	Vomiting
P1	(0.6,0.2)	(0.5,0.1)	(0.3,0.9)	(0.5,0.2)	(0.3,0.5)
P2	(0,0.7)	(0.5,0.5)	(0.7,0.1)	(0.2,0.6)	(0.1,0.6)
P3	(0.5,0.1)	(0.7,0.1)	(0,0.7)	(0.3,0.8)	(0,0.4)
P4	(0.8,0.1)	(0.5,0.4)	(0.3,0.2)	(0.8,0.2)	(0.4,0.3)

Table 2. Relation (R) between symptoms and diseases

R	Viral Fever	Typhoid	Stomach problem	Jaundice	Pneumonia
Temperature	(0.4,0.3)	(0.5,0.6)	(0.2,0.2)	(0.1,0.6)	(0.2,0.7)
Cough	(0.2,0.4)	(0.3,0.5)	(0.5,0.1)	(0.1,0.3)	(0,0.6)
Stomach pain	(0.1,0.3)	(0,0.7)	(0.2,0.7)	(0.8,0)	(0.1,0.5)
Chest pain	(0.3,0.4)	(0.6,0)	(0.1,0.5)	(0.1,0.6)	(0.1,0.3)
Vomiting	(0.3,0.2)	(0.1,0.8)	(0.2,0.8)	(0.2,0.7)	(0.7,0.2)

Therefore, the composition T is given in

Table 3. Max-min-max composition (T)

T	Viral Fever	Typhoid	Stomach problem	Jaundice	Pneumonia
P1	(0.4,0.1)	(0.5,0.2)	(0.5,0.1)	(0.3,0.3)	(0.3,0.3)
P2	(0.2,0.3)	(0.3,0.5)	(0.5,0.5)	(0.7,0.1)	(0.1,0.5)
P3	(0.4,0.3)	(0.5,0.5)	(0.5,0.1)	(0.1,0.3)	(0.2,0.4)
P4	(0.4,0.3)	(0.6,0.2)	(0.5,0.2)	(0.3,0.2)	(0.4,0.3)

As the max-min-max composition is used for T, "dominating" symptoms were in fact only taken into account.

Table 4. Data statistics

S_T	Viral fever	Typhoid	Stomach Pain	Jaundice	Pneumonia
P1	0.4	0.5	0.5	0.3	0.3

4 Discussion

Using the algorithm for medical diagnosis, the disease for which the membership value is maximum gives the final decision. If almost equal values for different diagnosis in composition are obtained, the case for which non-membership is minimum and hesitation is least is considered. The output matched well with the doctor's diagnosis.

Table 5. Decision table for medical diagnosis

Patients	P1, P3	P2	P4
S_T	Typhoid with stomach problem	Jaundice	Typhoid

From the above Table (5) it is clear that, if the doctor agrees then patient P1 and P3 are suffering from Typhoid with stomach problem, patient P2 from Jaundice where as P4 from Typhoid. The concept of fuzzy relations in medical diagnosis is very helpful for general practitioners, specialists and doctors etc.

5 Conclusion

In the process of medical diagnosis, state of patient are given by the patient through linguistic terminology like as temperature, cough, stomach pain etc., consideration of fuzzy sets as grades for association instead of membership grades in [0,1] is more advantageous to model the state of the patient. Similarly fuzzy relation has been introduced representing the association between symptoms and diseases. Sanchez's approach has been extended for medical diagnosis in this reference. The approach used to form fuzzy matrix showing the association of symptoms and diseases is based on the sanchez's approach.

Competing Interests

Authors have declared that no competing interests exist.

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