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Certain Subclass of Bi-univalent Functions Defined by Sălăgean Differential Operator Related with Horadam Polynomials

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"Author's contribution

The sole author designed, analyzed, interpreted and prepared the manuscript."

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Abstract

The goal of this paper is to introduce and investigate a new subclass $\mathcal{F}_{\Sigma}(\sigma,\vartheta,k,r)$ of bi-univalent functions using the Horadam polynomials and Sălăgean differential operator. Furthermore, coefficient estimates are given for $|a_2|$, $|a_3|$ and Fekete-Szegő inequalities for this subclass are obtained.

Keywords: Bi-univalent function; Sălăgean differential operator; Horadam polynomials; "Fekete-Szegö inequalities".

"Subject Classification: 30C45, 30C50".

1 Introduction

Let $\mathcal{A}$ be the family of normalized functions of the form

$$f(z) = z + \sum_{j=2}^{\infty} a_j z^j \qquad (z \in U). \tag{1.1}$$

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Which are holomorphic in $\mathcal{U} = \{z \in \mathbb{C}: |z| < 1\}$ .Let S be the subclass of all univalent functions from $\mathcal{A}$ in $\mathcal{U}$. It is clear (see [1]) that every function f ∈ S has an inverse f^{-1} satisfying

$$"z = f^{-1}(f(z)), (z \in U) \text{ and } \quad w = f(f^{-1}(w)), \left(|w| < r_0(f), r_0(f) \geq \frac{1}{4}\right)"$$

Where

$$" f^{-1}(w) = g(w) = w - a_2w^2 + (2a_2^2 - a_3)w^3 - (5a_2^3 - 5a_2a_3 + a_4)w^4 " \quad (1.2)$$

If f and f^{-1} are univalent in $\mathcal{U}$, then function $f \in \mathcal{A}$ is called to be bi-univalent in $\mathcal{U}$ and denote this class by Σ defined in $\mathcal{U}$.
"In 2010, Srivastava et al. [2] revived the study of bi-univalent functions by their pioneering work on the study of coefficient problems. Several authors have introduced and investigated subclasses of bi-univalent functions and obtained bounds for the initial coefficients (see [3-8]). However, for the coefficient estimate problem for each of the following Taylor-Maclaurin coefficients is still an open problem".

If f(z) and g(z) be holomorphic in $\mathcal{U}$, then f(z) is said to be subordinate to g(z) if $\exists\ \Phi(z)$ which a Schwarz function, with $\Phi(0) = 0$ and $|\Phi(z)| < 1$ and denote by $f(z) \prec g(z), z \in \mathcal{U}$, Such that $f(z) = g(\Phi(z))$ $(z \in \mathcal{U})$ .Moreover, $f(z) \prec g(z)$ is equivalent to $f(0) = g(0)$ and $f(\mathcal{U}) \subset g(\mathcal{U})$

If g is univalent in $\mathcal{U}$.

From ([9], [10]) "The Horadam polynomials $h_n(r)$ are" defined as

$$" h_n(r) = prh_{n-1}(r) + qh_{n-2}(r), (r \in \mathbb{R}, n \in N - \{1,2\}, N = \{1,2,3,\dots\})" \quad (1.3)$$

With $h_1(r) = e$, $h_2(r) = br$, where $e, b, p, q \in \mathbb{R}$ .It is clear that $h_3(r) = pbr^2 + eq$.

The generating polynomials of $h_n(r)$ is

$$\psi(r,z) = \sum_{n=1}^{\infty} h_n(r)z^{n-1} = \frac{e + (b - ep)rz}{1 - prz - qz^2} \quad (1,4)$$

In1983 [11] Sălăgean defined differential operator $\mathcal{D}^k: \mathcal{A} \to \mathcal{A}$ as

$$"\mathcal{D}^0 f(z) = f(z)$$
$$\mathcal{D}^1 f(z) = \mathcal{D}f(z) = z f'(z)$$
$$\mathcal{D}^k f(z) = \mathcal{D}\left(\mathcal{D}^{k-1}f(z)\right) = z(\mathcal{D}^{k-1}f(z))' \quad k \in N = \{1,2,\dots\}"$$
$$\mathcal{D}^k f(z) = z + \sum_{n=2}^{\infty} n^k a_n z^n, \ k \in N_0 = N \cup \{0\} \quad (1.5)$$

"And further for functions g of the form (1.2) Vijay et al. [12]"

$$\mathcal{D}^k g(w) = w - 2^k c_2 w^2 + 3^k(2c_2^2 - c_3) w^3 - \dots \quad (1.6)$$

More details associated with these polynomials see ([13-20])

2 Main Results

Definition 2.1: For $(0 \leq \vartheta \leq 1, \sigma \in \mathbb{C}/\{0\}, r \in \mathbb{R}, k \in N \cup \{0\})$ a function $f \in \Sigma$ of form (1.1) then $f \in \mathcal{F}\Sigma\ \sigma,\vartheta,k,r$ if the following subordination conditions are hold

$$1+\frac{1}{\sigma}\left[\frac{\vartheta z^3(\mathcal{D}^k f(z))''' + (2\vartheta+1)z^2(\mathcal{D}^k f(z))'' + z(\mathcal{D}^k f(z))'}{\vartheta z^2(\mathcal{D}^k f(z))'' + z(\mathcal{D}^k f(z))'} - 1\right] \prec \psi(r,z) + 1 - e$$

$$1+\frac{1}{\sigma}\left[\frac{\vartheta w^3(\mathcal{D}^k g(w))''' + (2\vartheta+1)w^2(\mathcal{D}^k g(w))'' + w(\mathcal{D}^k g(w))'}{\vartheta w^2(\mathcal{D}^k g(w))'' + w(\mathcal{D}^k g(w))'} - 1\right] \prec \psi(r,z) + 1 - e$$

Where $e \in \mathbb{R}$ and $g = f^{-1}$ is presented by (1.2)

Theorem 2.2: For $(0 \le \vartheta \le 1, \sigma \in \mathbb{C}/\{0\}, r \in \mathbb{R}, k \in N \cup \{0\})$ a function $f \in \mathcal{F}_\Sigma(\sigma,\vartheta,k,r)$ then

$$|a_2| \le \frac{|\sigma|\,|br|\sqrt{|br|}}{\sqrt{|[(2(3^{k+1})(2\vartheta+1) - 2^{2k+2}(\vartheta+1)^2)b - 2^{2k+2}(\vartheta+1)^2 p]br^2 - 2^{2k+2}(\vartheta+1)^2 eq|}} \quad (2.1)$$

$$|a_3| \le \frac{|\sigma|\,|br|}{3^{k+1}(2\vartheta+1)} + \frac{|\sigma|^2|br|^2}{2^{2k+2}(\vartheta+1)^2} \quad (2.2)$$

Proof: Let $f \in \mathcal{F}_\Sigma(\sigma,\vartheta,k,r)$"Then there are two holomorphic functions $s, t: \mathcal{U} \to \mathcal{U}$ presented by"

$$s(z) = s_1 z + s_2 z^2 + s_3 z^3 + \cdots \qquad (z \in \mathcal{U}) \quad (2.3)$$

$$t(z) = t_1 w + t_2 w^2 + t_3 w^3 + \cdots \qquad (w \in \mathcal{U}) \quad (2.4)$$

With

$s(0) = t(0) = 0$, max "$\{|s(z)|, |t(z)|\} < 1$; $z, w \in \mathcal{U}$", such that

$$\frac{1}{\sigma}\left[\frac{\vartheta z^3(\mathcal{D}^k f(z))''' + (2\vartheta+1)z^2(\mathcal{D}^k f(z))'' + z(\mathcal{D}^k f(z))'}{\vartheta z^2(\mathcal{D}^k f(z))'' + z(\mathcal{D}^k f(z))'} - 1\right] = \psi(r, s(z)) - e$$

$$\frac{1}{\sigma}\left[\frac{\vartheta w^3(\mathcal{D}^k g(w))''' + (2\vartheta+1)w^2(\mathcal{D}^k g(w))'' + w(\mathcal{D}^k g(w))'}{\vartheta w^2(\mathcal{D}^k g(w))'' + w(\mathcal{D}^k g(w))'} - 1\right] = \psi\big(r, t(z)\big) - e$$

Or equivalently

$$\frac{1}{\sigma}\left[\frac{\vartheta z^3(\mathcal{D}^k f(z))''' + (2\vartheta+1)z^2(\mathcal{D}^k f(z))'' + z(\mathcal{D}^k f(z))'}{\vartheta z^2(\mathcal{D}^k f(z))'' + z(\mathcal{D}^k f(z))'} - 1\right] = h_1(r) + h_2(r)s(z) + h_3(r)(s(z))^2 + \cdots - e \quad (2.5)$$

$$\frac{1}{\sigma}\left[\frac{\vartheta w^3(\mathcal{D}^k g(w))''' + (2\vartheta+1)w^2(\mathcal{D}^k g(w))'' + w(\mathcal{D}^k g(w))'}{\vartheta w^2(\mathcal{D}^k g(w))'' + w(\mathcal{D}^k g(w))'} - 1\right] = h_1(r) + h_2(r)t(z) + h_3(r)(t(z))^2 + \cdots - e \quad (2.6)$$

Combining (2.3)(2.4)(2.5)and (2.6)

$$\frac{1}{\sigma}\left[\frac{\vartheta z^3(\mathcal{D}^k f(z))''' + (2\vartheta+1)z^2(\mathcal{D}^k f(z))'' + z(\mathcal{D}^k f(z))'}{\vartheta z^2(\mathcal{D}^k f(z))'' + z(\mathcal{D}^k f(z))'} - 1\right] = h_2(r)\, s_1 z + [h_2(r)\, s_2 + h_3(r)\, {s_1}^2]z^2 + \cdots \tag{2.7}$$

$$\frac{1}{\sigma}\left[\frac{\vartheta w^3(\mathcal{D}^k g(w))''' + (2\vartheta+1)w^2(\mathcal{D}^k g(w))'' + w(\mathcal{D}^k g(w))'}{\vartheta w^2(\mathcal{D}^k g(w))'' + w(\mathcal{D}^k g(w))'} - 1\right] = h_2(r) t_1 z + [h_2(r)\, t_2 + h_3(r)\, {t_1}^2]w^2 + \cdots \tag{2.8}$$

If $\max\{|s(z)|, |t(z)|\} < 1$;"$z, w \in \mathcal{U}$ then

$$|s_i| < 1 \; and |t_i| < 1 (\forall\, i \in N)" \tag{2.9}$$

From (2.7)and (2.8) it follows that

$$\frac{2^{k+1}(\vartheta+1)a_2}{\sigma} = h_2(r)\, s_1 \tag{2.10}$$

$$\frac{2(3^{k+1})(2\vartheta+1)a_3 - 2^{2k+2}(\vartheta+1)^2 a_2^2}{\sigma} = h_2(r)\, s_2 + h_3(r) s_1^2 \tag{2.11}$$

$$-\frac{2^{k+1}(\vartheta+1)a_2}{\sigma} = h_2(r)\, t_1 \tag{2.12}$$

$$\frac{a_2^2[2^2(3^{k+1})(2\vartheta+1) - 2^{2k+2}(\vartheta+1)^2] - 2(3^{k+1})(2\vartheta+1)a_3}{\sigma} = h_2(r)\, t_2 + h_3(r) t_1^2 \tag{2.13}$$

From (2.10)and(2.12) we get

$$s_1 = -t_1 \tag{2.14}$$

$$2\frac{2^{2k+2}(\vartheta+1)^2 a_2^2}{\sigma^2} = h_2^2(r)\,(s_1^2 + t_1^2) \tag{2.15}$$

If we add (2.11)and (2.13) we get

$$\frac{a_2^2[2^2(3^{k+1})(2\vartheta+1) - 2^{2k+3}(\vartheta+1)^2]}{\sigma} = h_2(r)\,(s_2 + t_2) + h_3(r)(s_1^2 + t_1^2) \tag{2.16}$$

Substituting the value of $(s_1^2 + t_1^2)$ from (2.15) in to (2.16) we deduce that

$$a_2^2 = \frac{\sigma^2 h_2^3(r)(s_2+t_2)}{h_2^2(r)\,[2^2(3^{k+1})(2\vartheta+1) - 2^{2k+3}(\vartheta+1)^2] + h_3(r)[2^{2k+3}(\vartheta+1)^2]} \tag{2.17}$$

"By further computations using (1.3), (2.9) and(2.17) we obtain"

$$|a_2| \le \frac{|\sigma|\,|br|\sqrt{|br|}}{\sqrt{|[(2(3^{k+1})(2\vartheta+1) - 2^{2k+2}(\vartheta+1)^2)b - 2^{2k+2}(\vartheta+1)^2 p]br^2 - 2^{2k+2}(\vartheta+1)^2 eq|}} \tag{2.18}$$

Next, by subtracting (2.13) from (2.11) we obtain

$$\frac{2^2(3^{k+1})(2\vartheta+1)a_3-a_2^2[2^2(3^{k+1})(2\vartheta+1)]}{\sigma} = h_2(r)(s_2-t_2)+h_3(r)(s_1^2-t_1^2) \tag{2.19}$$

In view of (2.14), (2.15) into (2.19)

$$a_3 = \frac{\sigma h_2(r)(s_2-t_2)}{2^2(3^{k+1})(2\vartheta+1)} + \frac{\sigma^2 h_2^2(r)(s_1^2+t_1^2)}{2^{2k+3}(\vartheta+1)^2}$$

Thus by applying (1.4) we get

$$|a_3| \le \frac{|\sigma|\,|br|}{2(3^{k+1})(2\vartheta+1)} + \frac{|\sigma|^2|br|^2}{2^{2k+2}(\vartheta+1)^2} \quad \blacksquare \tag{2.20}$$

Remark 2.3: If taking $k=0$ in above Theorem then $f \in \mathcal{F}_\Sigma(\sigma,\vartheta,0,r)$ and

$$|a_2| \le \frac{|\sigma|\,|br|\sqrt{|br|}}{\sqrt{|[(6(2\vartheta+1)-4(\vartheta+1)^2)b-4(\vartheta+1)^2p]br^2-4(\vartheta+1)^2eq|}}$$
$$|a_3| \le \frac{|\sigma|\,|br|}{3(2\vartheta+1)} + \frac{|\sigma|^2|br|^2}{4(\vartheta+1)^2}$$

Remark 2.4: If taking $\vartheta=1$ in above Theorem then $f \in \mathcal{F}_\Sigma(\sigma,1,k,r)$ and

$$|a_2| \le \frac{|\sigma|\,|br|\sqrt{|br|}}{\sqrt{|[(2(3^{k+2})-2^{2k+4})b-2^{2k+4}p]br^2-2^{2k+4}eq|}}$$
$$|a_3| \le \frac{|\sigma|\,|br|}{2(3^{k+2})} + \frac{|\sigma|^2|br|^2}{2^{2k+4}}$$

Remark 2.5: If taking $\vartheta=0$ in above Theorem then $f \in \mathcal{F}_\Sigma(\sigma,0,k,r)$ and

$$|a_2| \le \frac{|\sigma|\,|br|\sqrt{|br|}}{\sqrt{|[(2(3^{k+1})-2^{2k+2})b-2^{2k+2}p]br^2-2^{2k+2}eq|}}$$
$$|a_3| \le \frac{|\sigma|\,|br|}{2(3^{k+1})} + \frac{|\sigma|^2|br|^2}{2^{2k+2}}$$

"Now we present the Fekete-Szegö inequality for $f \in \mathcal{F}_\Sigma(\sigma,\vartheta,k,r)$"

Theorem 2.6: For $(0 \le \vartheta \le 1, \sigma \in \mathbb{C}/\{0\}, r \in \mathbb{R}, k \in N \cup \{0\})$ a function $f \in \mathcal{F}_\Sigma(\sigma,\vartheta,k,r)$ then

$$|a_3-\eta a_2^2| \le \begin{cases} \dfrac{|\sigma|\,|br|}{3^{k+1}(2\vartheta+1)} \text{ if } \\ |\eta-1| \le \dfrac{|[(2(3^{k+1})(2\vartheta+1)-2^{2k+2}(\vartheta+1)^2)b-2^{2k+2}(\vartheta+1)^2p]br^2-2^{2k+2}(\vartheta+1)^2eq|}{3^{k+1}(2\vartheta+1)|\sigma|\,b^2r^2} \\ \dfrac{|br|^3|\sigma|^2|\eta-1|}{|[(2(3^{k+1})(2\vartheta+1)-2^{2k+2}(\vartheta+1)^2)b-2^{2k+2}(\vartheta+1)^2p]br^2-2^{2k+2}(\vartheta+1)^2eq|} \text{ if } \\ : |\eta-1| \ge \dfrac{|[(2(3^{k+1})(2\vartheta+1)-2^{2k+2}(\vartheta+1)^2)b-2^{2k+2}(\vartheta+1)^2p]br^2-2^{2k+2}(\vartheta+1)^2eq|}{3^{k+1}(2\vartheta+1)|\sigma|\,b^2r^2} \end{cases}$$

Proof: Using (2.17)and (2.19) for some $\eta \in \mathbb{R}$,we get

$$a_3-\eta a_2^2 = \frac{\sigma h_2(r)(s_2-t_2)}{2^2(3^{k+1})(2\vartheta+1)} + \frac{(1-\eta)\sigma^2 h_2^3(r)(s_2+t_2)}{h_2^2(r)[2^2(3^{k+1})(2\vartheta+1)-2^{2k+3}(\vartheta+1)^2]+h_3(r)[2^{2k+3}(\vartheta+1)^2]}$$

$$"= \frac{h_2(r)}{2}\left[\left(\varphi(\eta,r)+\frac{\sigma}{2(3^{k+1})(2\vartheta+1)}\right)s_2 + \left(\varphi(\eta,r)+\frac{\sigma}{2(3^{k+1})(2\vartheta+1)}\right)t_2\right]"$$

Where

$$\varphi(\eta, r) = \frac{(1-\eta)\sigma^2 h_2^2(r)(s_2+t_2)}{h_2^2(r)\,[2(3^{k+1})\,(2\,\vartheta+1) - 2^{2k+2}\,(\,\vartheta+1)^2] + h_3(r)[2^{2k+2}\,(\,\vartheta+1)^2]}$$

According to (1.3) we have

$$|\,a_3 - \eta a_2^2| \le \begin{cases} \dfrac{|\sigma||br|}{2(3^{k+1})(2\,\vartheta+1)} & \text{if } 0 \le |\varphi(\eta, r)| \le \dfrac{|\sigma|}{2(3^{k+1})(2\vartheta+1)} \\ |br||\varphi(\eta, r)| & \text{if } |\varphi(\eta, r)| \ge \dfrac{|\sigma|}{2(3^{k+1})(2\vartheta+1)} \end{cases}$$

After simple computation we get

$$|a_3 - \eta a_2^2| \le \begin{cases} \dfrac{|\sigma|\,|br|}{3^{k+1}(2\vartheta+1)} \text{ if } \\ |\eta-1| \le \dfrac{\left|\left[\left(2(3^{k+1})(2\vartheta+1) - 2^{2k+2}(\vartheta+1)^2\right)b - 2^{2k+2}(\vartheta+1)^2 p\right]br^2 - 2^{2k+2}(\vartheta+1)^2 eq\right|}{3^{k+1}(2\vartheta+1)|\sigma|\,b^2r^2} \\ \dfrac{|br|^3|\sigma|^2|\eta-1|}{|[(2(3^{k+1})(2\vartheta+1) - 2^{2k+2}(\vartheta+1)^2)b - 2^{2k+2}(\vartheta+1)^2 p]br^2 - 2^{2k+2}(\vartheta+1)^2 eq|} \text{ if } \\ : |\eta-1| \ge \dfrac{\left|\left[\left(2(3^{k+1})(2\vartheta+1) - 2^{2k+2}(\vartheta+1)^2\right)b - 2^{2k+2}(\vartheta+1)^2 p\right]br^2 - 2^{2k+2}(\vartheta+1)^2 eq\right|}{3^{k+1}(2\vartheta+1)|\sigma|\,b^2r^2} \end{cases} \blacksquare$$

Remark 2.7: If taking $k = 0$ in above Theorem then $f \in \mathcal{F}_\Sigma(\sigma, \vartheta, 0, r)$ and

$$|a_3 - \eta a_2^2| \le \begin{cases} \dfrac{|\sigma|\,|br|}{3(2\vartheta+1)} \text{ if } \\ |\eta-1| \le \dfrac{|[(6)(2\vartheta+1) - 4(\vartheta+1)^2)b - 4(\vartheta+1)^2 p]br^2 - 4(\vartheta+1)^2 eq|}{3(2\vartheta+1)|\sigma|\,b^2r^2} \\ \dfrac{|br|^3|\sigma|^2|\eta-1|}{|[(6(2\vartheta+1) - 4(\vartheta+1)^2)b - 4(\vartheta+1)^2 p]br^2 - 4(\vartheta+1)^2 eq|} \text{ if } \\ : |\eta-1| \ge \dfrac{|[(6(2\vartheta+1) - 4(\vartheta+1)^2)b - 4(\vartheta+1)^2 p]br^2 - 4(\vartheta+1)^2 eq|}{3(2\vartheta+1)|\sigma|\,b^2r^2} \end{cases}$$

Remark 2.8: If taking $\eta = 1$ in above Theorem then $f \in \mathcal{F}_\Sigma(\sigma, \vartheta, k, r)$ and

$$|a_3 - a_2^2| \le \frac{|\sigma|\,|br|}{3^{k+1}(2\vartheta+1)}$$

2 Conclusion

The main results of this paper refer to introducing a new subclass of bi-univalent functions in Definition 2.1. using properties of Horadam polynomials and Sălăgean differential operator. Estimates on the first two Taylor–Maclaurin coefficients for the functions in this subclass are given in Theorem 2.2. and the corollaries that follow it. Fekete-Szegö inequalities are given in Theorem 2.6. for this newly introduced subclass of functions.

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"Competing Interests

Author has declared that no competing interests exist."

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