Designing of Track Axis Alignment with the Use of Satellite Measurements and Particle Swarm Optimization

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Authors' contributions

This work was carried out in collaboration between all authors. Author WC developed general concept of studies and designed calculation algorithms. Author CS elaborated technology of mobile satellite measurements and performed the fieldwork. Author PC created the computer programs and performed numerical analysis of different options. Author KP applied multi-criteria optimization processes to selecting the best variant. All authors read and approved the final manuscript.

Original Research Article

ABSTRACT

Designing of the track's alignment is a key issue from the point of view of maintaining of proper geometries. The paper presents a design method for sections of railway line located in the horizontal arch. The method is adapted to the technique of mobile satellite measurements. The general principles of this measurement method have been described in the article. A project's solution has been presented using mathematical notation in a form of the set of universal equations describing the whole geometrical layout which was analyzed. The design procedure lets to varying both the types and lengths of transition curves. Calculation algorithm has been implemented in Scilab environment and the basic features and functions of the algorithm has been presented. The implemented algorithm allows for generating various variants until the acceptation of main parameters of designed layouts. Moreover, the geometrical parameters could be determined in the process of multi-criteria optimization. Considering the process of alignment design, it has been

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assumed to achieve the highest possible trains speed together with the minimizing of the track dislocations. The optimization process is supported by the Particle Swarm Optimization algorithm (PSO). The presented method enables for relatively fast satellite measurements data processing as well as helps to reconstruct the model of a real route location. Therefore, this can be a basis for taking an appropriate action in modernization problems. A new concept of the designing of the track axis adjustment has been illustrated by a particular example of the calculations.

1. INTRODUCTION

Commercial auxiliary computer software have been used for many years in designing geometrical layouts of overland transport routes (roads and railways). Such programs are, in principle, adjusted to designing new geometrical layouts. With regard to railways, there is marked dominance of the situations where the existing conditions are modernized; i.e. alignment of track axis is designed. Here the key issue is to define corrected parameters of the existing geometrical layout. On the one hand, they have to ensure the highest possible speed of trains, while on the other hand they have to guarantee minimization of lateral displacements of the track.

In application of commercial computer programs, defining parameters of the existing geometrical layout has in itself always constituted a significant problem (not to mention the correction of these parameters). Traditional geodetic (optical) measurements [1,2] did not allow for establishing coordinates of a given railway in an absolute reference system. Various approximate measures, based, for example, on horizontal arrow graphs, were used to identify the location of individual geometrical elements (circular arcs and transition curves). Even today, despite the fact that satellite geodesy is used, to a limited extent, the procedure discussed here is still, to a high degree, intuitive in its character.

For many years there are attempts to obtain an analytical form of reconstruction of the existing railway track shape, often using the spline functions [3,4,5,6,7]. The article presents the assumptions of a new, analytical method for designing geometrical layouts of the track, adjusted to the technology of mobile satellite measurements. This method creates a possibility to define horizontal ordinates sequentially, with the use of appropriate mathematical formulas. It may prove to be particularly useful in defining and, later, correcting the parameters of the existing geometrical layout, i.e. it may fill the gap in currently applied design procedures for track alignment. This method, together with appropriate software, allows for generating subsequent route options and subjecting them to optimization process in order to select the best alternative. The presented concept makes it necessary to assume suitable optimization criteria and define their weights. The problem which is being solved is of significant importance, both with regard to science and application.

2. TECHNOLOGY OF MOBILE SATELLITE MEASUREMENTS

Using satellite technology in land navigation and – in the future – the main direction of commercial GNSS development are undoubtedly connected, due to virtually unlimited consumer market, with equipping mass-produced cars in integrated navigation system, supported by direction and speed meters, a vector map and a GPS receiver. This kind of application does not require, however, high accuracy in establishing coordinates, which also concerns, inter alia, defining train location on a railway route with the help of satellite technology.

The situation looks completely different if we want to define geometrical shape of the axis of an operated track based on the trajectory of rail vehicle movement. The accuracy of measurements (i.e. the measurement error) required for design purposes should be less than 1 cm, while in diagnostics it would mean single millimeters. For a long time it has not been possible to achieve such accuracy in Poland. Entirely new perspectives with regard to defining real position of a railway track opened only in mid-2008, after the Active Geodetic Network ASG-EUPOS [8] had been launched. The first measurements performed by the scientific team of Gdansk University of Technology and the Naval Academy in Gdynia in February 2009 [9], with the use of GPS measurement technique in its mobile version, i.e. with receivers installed on board a rail vehicle in motion, showed that

application of this technique allows for very precise determination of basic data necessary to design modernization of a railway line. It makes it possible to simulate straight sections of the route (and to establish their lateral deformations), as well as the places where the route direction changes (turn angles of the route, circular arcs and transition curves). Thanks to satellite technology it has become possible to simulate, in a direct way, the real shape of the track in a horizontal plane [10,11,12,13,14,15].

Measurements connected with inventory and diagnostics of railway and tramway lines, which have been conducted for a number of years, made it possible to elaborate the applied research methodology and indicated the main issues which influence the possibilities offered by kinematic GNSS measurements. It has been decided that, at the present stage, the optimal solution, ensuring maximum accuracy in establishing GNSS coordinates during inventory examination of railway routes, is a two-system satellite geodetic network GPS/Glonass, which guarantees network coordinate solutions [16,17].

3. CALCULATION ALGORITHMS

Identification of the basic input data and the route ordinates design procedure(in the considered design case) has been presented in [10]. The measured ellipsoidal coordinates from the GPS measurements were transformed with the use of the Universal Mercator Projection to obtain conformal coordinates [16]. These coordinates form what is called in Poland, The National Space Reference System 2000, where a division of terrestrial sphere zones with the width equal to 3 degrees is used. The obtained coordinates X_i

and Y_i could be used to calculate the straight lines equations $X = A + BY$ (main directions of the route) using the method of least squares.

The equations of both straight lines enable to calculate the coordinates of the intersection point of the main directions of the route and to define the conditions determining the end zone coordinates of the main direction of the route on *Straight line 1*, as well as the initial zone of the second main direction of the route on *Straight line 2*. The determination of inclination angles φ_1 and φ_2 of both straight lines relative to *Y*-axis enables us to calculate the route inclination angle α.

In order to use the measured data to design the area of route direction change, the appropriate part of the data set should be extracted and undergo geometrical transformation (translation and rotation) of the coordinate system. The new coordinates of the route extended to the point $O(Y_O, X_O)$ and rotated by an angle β in the local coordinate system *x, y* are described by the following formulas [18]:

$$x = (Y - Y_O)\cos\beta + (X - X_O)\sin\beta \qquad (1)$$

$$y = -(Y - Y_O)\sin\beta + (X - X_O)\cos\beta \qquad (2)$$

The application of the transformations (1), (2) is presented in [19]. The author presented a solution to an elementary case: joining two straight lines with a circular arc and two transition curves of the same type and length (a symmetric case). The analytical formulas are used in the designing process to ensure the satisfaction of the tangency condition between the circular arc and the transition curve.
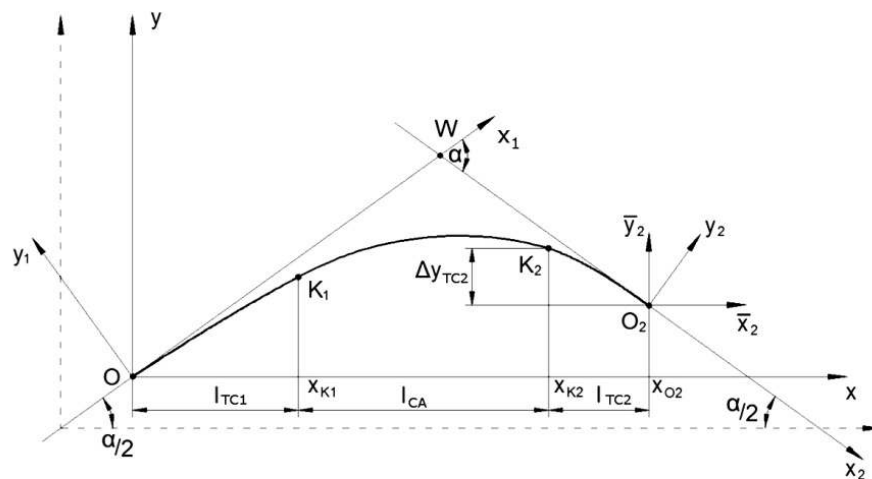

Fig. 1. The designed geometrical layout in a local coordinates system (Asymmetric case)

In [20] a solution to more complex case is presented: Joining two straight lines with a circular arc and two transition curves of any type and length (asymmetric case). The asymmetric case in a local coordinates system *Oxy* used in the design process is presented in Fig. 1. The design of the geometrical layout of the railway track is carried out in this local system and then – through appropriate transformation – the obtained solution is moved to the global system.

The identification of new track ordinates will be performed on the following input data:

- Inclination angle *α*,
- Circular arc radius *R*,
- Superelevation on the arc h_0,
- Transition curves lengths l_1 and l_2 of the assumed type.

The design process is carried out sequentially and comprises the following steps:

- Approximate assumption of the origin of the local coordinate system (point *O*) on one of the main direction of the route (*Straight line 1*); the assumed point *O* is, at the same time, the origin of the second auxiliary coordinates system Ox_1y_1, used for calculating the points of the transition curve *TC1*,
- Determination of the transition curve *TC1* ordinates in coordinate system Ox_1y_1 (Fig.1),
- Transformation of the transition curve *TC1* into an assumed local coordinate system using clockwise rotation of its reference system by the angle *α*/2; parametric equations *x(l)* and *y(l)* of the transition curve *TC1* are obtained,
- Determination of the transition curve *TC2* ordinates located in coordinate system Ox_2y_2 (Fig.1),
- Transformation of the transition curve *TC2* into the coordinate system $O_2x_3y_3$ using counterclockwise rotation of its reference system by the angle *α*/2; since the axes of that system are parallel to the axes of the local coordinate system *Oxy*, this transformation enables us to calculate the coordinates of the point K_2 (i.e. values of l_{TC2} and Δy_{TC2}),
- Determination of the equation of the circular arc *y(x)*, tangentially joined at the point K_1 with transition curve *TC1*,
- Calculation of the coordinates of the point $K_2(x_{K2}, y_{K2})$, satisfying the condition of

tangency of the circular arc and the transition curve *TC2*,
- Localization of the auxiliary coordinates system $O_2x_2y_2$ in the local coordinates system *Oxy* based on the coordinates of the point $O_2(x_{O2}, y_{O2})$,
- Determination of the parametric equations *x(l)* and *y(l)* of the transition curve *TC2* in the local coordinate system,
- Calculation of the origin of the local coordinate system i.e. the point *O* (on the appropriate main direction of the route) in the global coordinate system; this localization is determined by the coordinates of the vertex $W(x_W, y_W)$ in the system *Oxy*, since, based on the satellite measurements its coordinates Y_W and X_W in the System 2000 can be easily obtained,
- Transfer of the solution to the global system in accordance with the following formulas [18]:

$$Y = Y_O + x \cos\beta - y \sin\beta \qquad (3)$$

$$X = X_O + x \sin\beta + y \cos\beta \qquad (4)$$

A similar method can be applied to design more complex geometrical layouts. It may be useful during the reconstruction of the existing railway track shape. In paper [21] a case of compound curve consisting of two circular arcs with different radius and three transition curves is presented.

4. COMPUTER-GENERATED VARIANTS

A suitable computer program [22] for generating subsequent route options has been elaborated with the use of the calculation algorithm presented above. This program will then, by way of optimization, select the solution which best describes the shape of the existing track axis.

The calculation algorithm, implemented in the Scilab environment [23], allows the user to design a new option, which is not connected with any existing geometrical layout, as well as a modernization option for an existing layout, based on the measurement data relating to a given railway track.

The design process consists in generating subsequent route options and is, to a great extent, automated, which means that the role of the designer is mainly to make critical decisions. Therefore computer support transfers the effort

connected with performing calculations and making presentations to the decision-making process.

The user loads first the data array containing the coordinates of the points of the global system 2000. The next step is to determine of the route main directions (assuming that horizontal alignment process preserves the straight sections of the route).

The user selects the areas of straight line sections on both sides of the arc. During these operations, the user follows on the screen the coefficients *A* and *B* of the selected straight section, computed for the coordinates of global system 2000. Moreover the user sees R^2 and next R^2 coefficients for a current and the further selected areas. These operations can be performed for the moment, when the user decide about final straight line describing the chosen main direction of the route.

The designing process is held in a local coordinate system. The user enters input data and the computer program is expected to generate the variant of railway route between the specified earlier main directions.

After entering and confirming data the user sees on the screen the layout in a local coordinates system *x,y*. The designed transition curves and circular arcs are highlighted in the geometric layout of the route on the background of measured points.

Fig. 2 shows the solution obtained for the radius *R* = 965.48 m and the two transition curves (clothoids) of length l_1 = 95.94 m and l_2 = 129.08 m. To improve the readability of the layout, the beginnings and ends of the spirals are connected with each other by straight lines. The obtained differences of horizontal coordinates can be assessed with a help of additional chart which is displayed together with the generated variant. These differences *Δy* corresponding with the layout from Fig. 2 are presented in Fig. 3. In fact, the values of *Δy* do not correspond strictly to the values of potential lateral track displacements in a field phase but they are very helpful in decision phase during designing the final geometric layout. The variant of the route in the Fig. 2 and corresponding differences in the Fig. 3 have been obtained in the optimization process.

Finally, when the designer have completed the process of generating variants of the route, the program provides a result file of coordinates of

points along the designed geometric layout of the route. These points are the representation of the proposed route in the state spatial reference system 2000. In addition, it is possible to print a report containing the analytical record of the solution in the local coordinates x, y system, and the contained in the report data allow the user to restore the coordinates in the 2000 system.
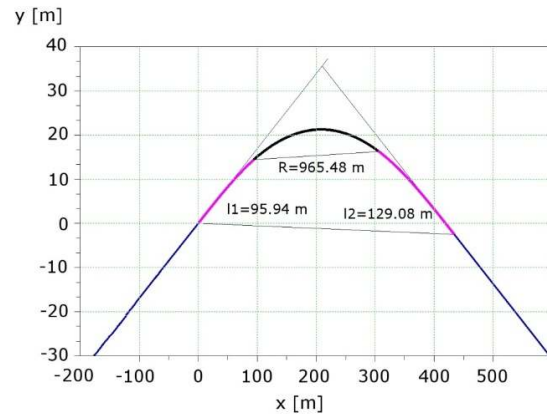

Fig. 2. Variant of the route in horizontal arc of radius *R* = 965.48 m together with two transition curves of length l_1 = 95.94 m and l_2 = 129.08 m (non-isometric scale)
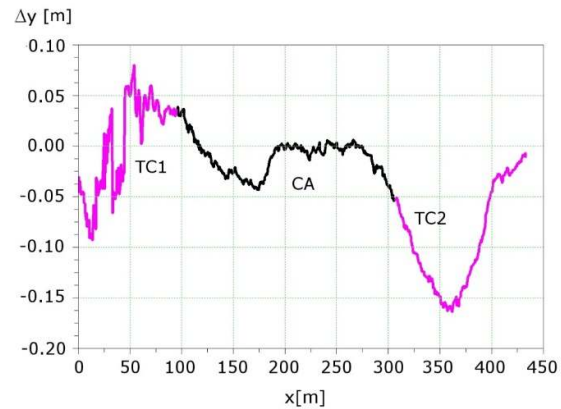

Fig. 3. The differences between the designed ordinates and the measured ones (Non-isometric scale), corresponding to the variant presented in Fig. 2

5. OPTIMIZATION OF THE PROCESS OF SELECTING A VARIANT DURING TRACK SHAPE RECONSTRUCTION

Generation and evaluation of the various variants may be controlled directly by the user or automatically in an optimization process. The optimization process is carried out using PSO

algorithm (*Particle Swarm Optimization*) [24]. A minimization of the existing track system transverse displacement is the main criterion in the process of reconstruction of the track layout axis shape.

PSO algorithm deals with a population of the particles moving in the searching space. Each particle represents a potential solution to the problem i.e. circular arc radius R and transition curves lengths l_1 and l_2. In PSO algorithm each particle is characterized by its fitness value, current position, current velocity and a record of its past performance. Particles change their positions and velocities in the direction dependent on the best recorded own position, position of other particles and current velocity. Velocity of each particle in the swarm is updated in each iteration according to the following formula:

$$v_{id} = \omega \times v_{id} + C_1 \times \text{Rand}(\ ) \times (p_{id}^{best} - p_{id}) + C_2 \times \text{Rand}(\ ) \times (p_{gd}^{best} - p_{id}) \qquad (5)$$

where:

v_{id} – velocity of each particle in the swarm,
ω – inertia coefficient, changing during the execution of the algorithm as follows:

$$\omega = \omega_{max} - n \times \frac{\omega_{max} - \omega_{min}}{N} \qquad (6)$$

N – total number of iterations,
n – current iteration number,
C_1, C_2 – individual and swarm learning coefficients,
Rand() – random value from interval (0,1),
p_{id}^{best} – the best recorded so far particle position (individual best position),
p_{gd}^{best} – the best recorded so far position of other particles (global best position).

Position of each particle is updated in each iteration according to the equation:

$$p_{id} = p_{id} + v_{id} \qquad (7)$$

PSO algorithm flow chart is presented in Fig. 4. The optimization process is carried out using PSO-Toolbox v.0.7-1 for Scilab v. 5.4.0 [25]. Evaluation of each particle, representing potential solution to the problem, is carried out basing on differences between the ordinates of the reconstructed railway track and the measured ordinates according to the formula:

$$FF(R, l_1, l_2) = \frac{1}{n^2} \sum_{i=1}^{n} |y_i - py_i| \qquad (8)$$

where:

FF – fitness function value,
n – number of samples (i.e. measured railway track ordinates),
y_i – ordinate of the reconstructed circular arc with radius R, joining two straight lines in the geometrical layout with two transition curves of length l_1 and l_2,
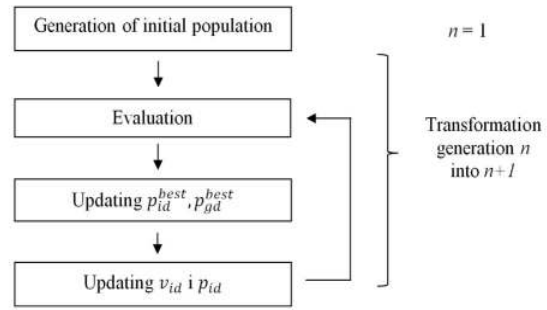py_i – measured ordinate.

Fig. 4. Particle Swarm Optimization algorithm flow chart

The parameters of the optimization processes are presented in Table 1.

Table 1. Parameters of the optimization processes

Parameter	Value
Initial inertia coefficient ω_{max}	0.9
Final inertia coefficient ω_{min}	0.4
Individual learning coefficient C_1	0.7
Swarm learning coefficient C_2	1.47
Iterations number	100

The aim of the optimization process is to obtain the circular arc radius R and transition curves lengths l_1 and l_2, ensuring minimal average transverse displacement of the circular arc. Applying of the inverse square of the sample number $\frac{1}{n^2}$ in the formula (8) served to suppress the tendency to over-extension of the transition curves length and minimize the displacements by shortening the length of the reconstructed arc.

6. OPTIMIZATION OF THE PROCESS OF SELECTING A VARIANT DURING THE TRACK AXIS ADJUSTING

The design of the track axis adjustment – employing methodology of the variant generation

similar to that used before, apart from minimizing transverse displacements of the track layout, should take into account at least one additional criterion, i.e. obtaining the highest possible train speed. The geometrical parameters of the designed track layout are obtained as a result of multi-criteria optimization [26].

6.1 Criteria of the Variant Selection

The variant is selected on the basis of two criteria:

- The differences between horizontal coordinates of the designed track layout and the existing track coordinates;
- Train speed limit.

In the optimization process different weights are assigned to these two criteria. The criterion importance depends on the relative value of its weight, which is assigned arbitrarily.

The values of the differences between horizontal coordinates of the designed track layout and the existing track coordinates can be used as preliminary assessment of the designed geometrical layout; the graph of the differences corresponding to the current variant is presented to the user (e.g. in the Fig. 3 the graph corresponding to the variant from Fig. 2 is presented). The presented differences Δy are a helpful indicator during selection of the particular design variant.

Determination of the speed limit is more complex [27]. The speed limit on the whole track layout, consisting of circular arc and transition curves, depends on the relationship between the speed limit v_{max}^R resulting from the circular arc radius:

$$v_{max}^R = 3,6\sqrt{a_{per} + g\frac{h_{max}}{s}}\sqrt{R} \qquad (9)$$

and a lower speed limit on transition curve ($v_0^{l,\psi}$ or $v_0^{l,f}$):

$$v_0^{l,\psi} = 3,6\,\frac{\psi_{per}}{C a_{per}}\,l_k \qquad (10)$$

$$v_0^{l,f} = 3,6\,\frac{f_{per}}{C\,h_{max}}\,l_k \qquad (11)$$

where:

$v_{max}^R, v_0^{l,\psi}, v_0^{l,f}$ – speed limits [km/h],

a_{per} – limit of the unbalanced acceleration on a circular arc [m/s²],
h_{max} – limit of the superelevation on a circular arc [mm],
ψ_{per} – limit of the velocity of unbalanced acceleration changes [m/s³],
f_{per} – limit of the velocity of vertical wheels rising on gradient due to cant [mm/s],
R – radius of the circular arc [m],
l_k – length of the transition curve [m],
S – rail spacing [mm],
g – gravity constant [m/s²],
C – numerical coefficient dependent on a type of transition curve:

for clothoid	$C = 1$,
for Bloss curve	$C = 1,5$,
for cosin curve	$C = \pi/2$,
for sin curve	$C = 2$.

The speed limit on a whole system results from a speed limit on a circular arc v_{max}^R but in some cases can be additionally reduced due to an insufficient length of the transition curve.

In case $v_{max}^R \leq \min\left(v_0^{l,\psi}, v_0^{l,f}\right)$ the speed limit is determined as follows:

$$v_0^{R,l} = v_{max}^R \qquad (12)$$

In case $v_{max}^R > \min\left(v_0^{l,\psi}, v_0^{l,f}\right)$ the speed limit on a whole system is lower than the speed limit on a circular arc due to assignment $v_0^{R,l} < v_{max}^R$. In order to determine the speed limit $v_0^{R,l}$ the speed limits on a transition curve $v_0^{l,\psi}$ and $v_0^{l,f}$ should be increased by applying reduced values a_{per} and h_{max} in formulas (10) and (11).

Reduced values a_{per} and h_{max} are obtained by applying reduction factors $k_i \leq 1$; we obtain $a_0 = k_i a_{per}$ and $h_0 = k_i h_{max}$. The reduction factors used in formula (9) reduce the speed limit on a circular arc to the speed limit v_0^R.

The factors k_i should be adjusted to fulfill the following conditions:

- condition for factor k_1

$$v_0^{l,\psi} = v_0^R \qquad (13)$$

- condition for factor k_2

$$v_0^{l,f} = v_0^R \qquad (14)$$

The condition (13) leads to formula:

$$3{,}6 \frac{\psi_{per}}{Ck_1 a_{per}} l_k = 3{,}6 \sqrt{k_1 a_{per} + g \frac{k_1\, h_{max}}{s}} \sqrt{R}$$

from which we obtain the formula for k_1.

$$k_1 = \sqrt[3]{\frac{(\psi_{per})^2}{(Ca_{per})^2 \left(a_{per} + g\frac{h_{max}}{s}\right)} \frac{l_k^2}{R}} \qquad (15)$$

The condition (14) leads to formula:

$$3{,}6 \frac{f_{per}}{Ck_2 h_{max}} l_k = 3{,}6 \sqrt{k_2 a_{per} + g \frac{k_2\, h_{max}}{s}} \sqrt{R}$$

from which we obtain the formula for k_2.

$$k_2 = \sqrt[3]{\frac{(f_{per})^2}{(C\, h_{max})^2 \left(a_{per} + g\frac{h_{max}}{s}\right)} \frac{l_k^2}{R}} \qquad (16)$$

It can be easily shown that for limits a_{per}, h_{max}, ψ_{per} and f_{per} factor k_1 is significantly (more than twice) higher than k_2. Therefore, in the presented case more important is the reduction factor k_2 (16) and the speed limit on a whole system results from formula:

$$v_0^{R,l} = 3{,}6 \sqrt{k_2 a_{per} + g \frac{k_2\, h_{max}}{s}} \sqrt{R} \qquad (17)$$

Assuming the following values of the kinetic parameters: $a_{per} = 0{,}8$ m/s^2, $\psi_{per} = 0{,}5$ m/s^3, $f_{per} = 28$ mm/s, $s = 1{,}5$ m and $g = 9{,}81$ m/s^2, the computed speed limit for the layout in Fig. 2 is $v_{max} = 127.07$ km/h.

In order to take into account a speed limit criterion apart from a criterion based on the

differences between the designed track layout vertical coordinates and the measured track coordinates, fitness function FF (8) has been expanded as follows:

$$FF(R, l_1, l_2) = w_1 \cdot \frac{1}{n^2} \sum_{i=1}^{n} |y_i - py_i| - w_2 \cdot v_{max} \qquad (18)$$

where:

w_i – criterion weight,
v_{max} – speed limit on the whole layout.

The main goal of application of two criteria is to ensure a higher speed limit on the whole designed layout.

6.2 Multi-criteria Optimization Processes

In Table 2 the results of five optimization processes are presented. Case 1 takes into account only one criterion - minimization of lateral dislocations of the track, therefore the weight $w_2 = 0$, This case describes the situation depicted in Fig. 2. In cases 2÷5, while keeping the weight $w_1 = 1$, there is already the second criterion - for obtaining the maximum speed of trains. Here, there is a gradual increase of the weight w_2. Figs. 5 and 6 shows the designed layouts on the base of optimization from cases 2 and 5.

The conducted calculations show that increasing the weight w_2 causes the increase of the realizable v_{max}. This is done by shortening the circular arc and lengthening the transition curves. For weight $w_2 = 5*10^{-6}$ and $v_{max} = 130.4$ km/h (case 2 – Fig. 5) the respective lengths are: $l_{CA} = 183.96$ m, $l_1 = 116.45$ m, $l_2 = 151.69$ m. But for weight $w_2 = 1*10^{-4}$ and $v_{max} = 139$ km/h (case 5 – Fig. 6) there are: $l_{CA} = 111.44$ m, $l_1 = 186.90$ m, $l_2 = 186.83$ m. The observed tendency to extend the transition curves results shorter circular arc with decreased radius R (Table 2).

Table 2. The results of the optimization processes

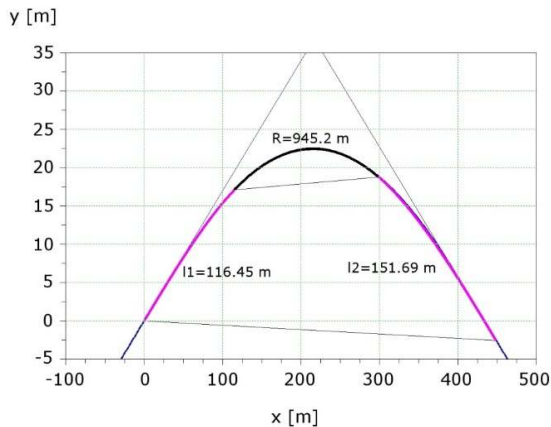| Case	Weight w_1	Weight w_2	Speed v [km/h]	Arc radius R [m]	Super-elevation h_0 [mm]	Arc length l_{CA} [m]	Length l_1 [m]	Length l_2 [m]
1	1	0	127.07	965.48	75	212.31	95.94	129.08
2	1	$5*10^{-6}$	130,42	945.20	90	183.96	116.45	151.69
3	1	$2*10^{-5}$	134.00	933.20	105	166.60	139.70	154.80
4	1	$8*10^{-5}$	134.73	942.19	105	175.43	141.02	142.09
5	1	$1*10^{-4}$	139.00	886.86	135	111.44	186.90	186.83

Fig. 5. Design variant calculated on the base of the optimization process - Table 2, case 2 (non-isometric scale)
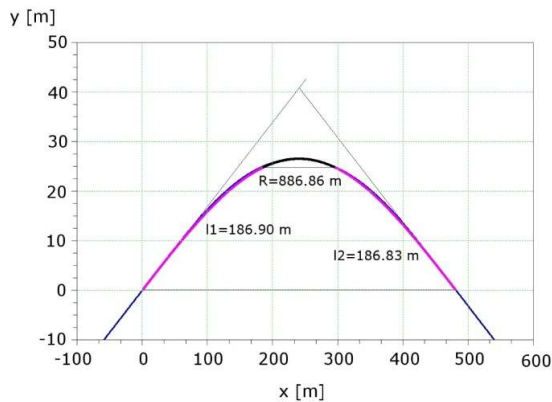

Fig. 6. Design variant calculated on the base of the optimization process - Table 2, case 5 (non-isometric scale)

Moreover, it was found, that for this geometrical scenario, the transverse displacements are within the similar limits. This is illustrated in Fig. 7, which shows graphs of horizontal differences over the length of the arc and transition curves.

It is typical for all cases that the largest differences are always in the part of connection the arc with the transition curve. So, there is a clear tendency for lengthening of the transition curves, shortening the length of circular arc l_{CA} together with a slightly decreasing radius of arc R. Moreover, while adjusting the weight w_2 the layout becomes to almost symmetrical arrangement.

Considered this way the geometrical layout improves its properties from the point of view of

the maximum possible speed. Interestingly, the level of lateral track displacements in the central part remain at the similar value.

Although, the analysis refers to one specific example, i.e. to the obtained by the mobile satellite measurements ordinates of a particular part of existing track, the method is general in its essence. Exactly the same procedure can be used for any of the measured track layouts. As a result, there are possibilities for designing axis adjustment of the track in a rational and very precise, virtually unattainable in the methods so far used in the field.
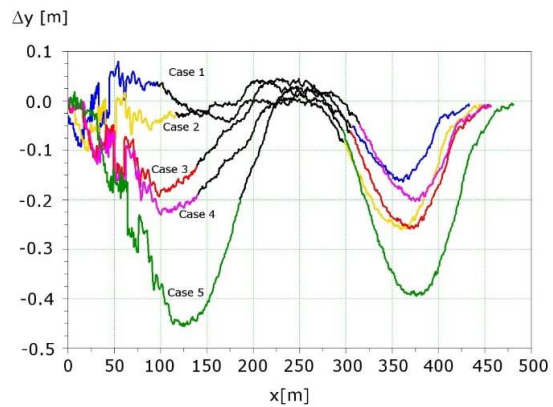

Fig. 7. Comparison of the differences between designed and measured horizontal ordinates of the analyzed arc - cases 1 ÷ 5 (Table 2)

7. CONCLUSIONS

Designing of the track alignment's adjustment is a key issue from the point of view of maintaining of proper geometries. Currently methods using in the process of track adjustment have a number of limitations. Those problems are simply not exist in the method of mobile satellite surveying techniques. Satellite methods allow for determination of the coordinates of an existing track location in more precise and incomparably faster way than traditional methods. On this basis, it is possible to evaluate the layout and modify the geometrical shape of the track in a reasonable manner, compatible with universal design principles. Similarly, the costs of the measurements by this method are much lower. Wide development of these techniques could eliminate in the future the need to allocate substantial financial outlays for creating railway control network which is currently used in adjustment of track alignment.

With the appropriate calculation algorithm, including a new and original way of designing of the route in a horizontal plane, a suitable computer program for generating variations has been created. The role of the designer is primarily on making the critical decisions regarding the selection of a particular variant.

The geometrical parameters of the layout are determined in the process of multi-criteria optimization. As the basic optimization criteria was established to achieve the greatest speed trains and minimize lateral displacements of the existing track alignment. The optimization process uses particle swarm algorithm (*Particle Swarm Optimization*).

The presented method enables fast processing of data obtained from satellite measurements and lets for determination of the route's analytical model. This can be a basis for taking appropriate actions in modernization problems.

The new design concept for a track axis adjustment has been illustrated by a particular calculation example. It has been shown how by adopting appropriate weights used in optimization criteria considered geometrical arrangement improves their properties from the point of view of the maximum speed. Although, the analysis refers to one arc section measured by the mobile satellite surveying, the same procedure can be used for any of the measured track layouts. As a result, there are possibilities to design the track axis adjustment in a rational manner.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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