Applied Mathematics, **2011, 2, 348-354**
doi:10.4236/am.2011.23041 Published Online March 2011 (http://www.SciRP.org/journal/am)

Bianchi-Type VI_0 Bulk Viscous Fluid Models with Variable Gravitational and Cosmological Constants

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Received December 16, 2010; *revised January* 19, 2011; *accepted January* 22, 2011

Abstract

This paper deals with Bianchi type VI_0 anisotropic cosmological models filled with a bulk viscous cosmic fluid in the presence of time-varying gravitational and cosmological constant. Physically realistic solutions of Einstein's field equations are obtained by assuming the conditions 1) the expansion scalar is proportional to shear scalar 2) the coefficient of the bulk viscosity is a power function of the energy density and 3) the cosmic fluid obeys the barotropic equation of state. We observe that the corresponding models retain the well established features of the standard cosmology and in addition, are in accordance with recent type Ia supernovae observations. Physical behaviours of the cosmological models are also discussed.

Keywords: Bianchi VI_0, Cosmology, Bulk viscosity, Variable G and Λ.

1. Introduction

The adequacy of spatially homogeneous and isotropic Friedman-Robertson-Walker (FRW) models for describing the present state of the universe is no basis for expecting that they are equally suitable for describing the early stages of evolution of the universe. Cosmological models which are spatially homogeneous but anisotropic have significant role in the description of the universe at it's early stages of evolution. Bianchi space I-IX are useful tools in constructing models of spatially homogeneous cosmologies [1]. Considerable work has been done for constructing various Bianchi type cosmological models and their inhomogeneous generalizations. Among these Bianchi type I spaces are simplest which subsequent generalizations of zero-curvature FRW models are. Bianchi type VI_0 spaces are of particular interest since they are sufficiently complex, while at the same time, they are simple generalizations of Bianchi type I spaces. Barrow [2] has pointed out that Bianchi type VI_0 models of the universe give a better explanation of some of the cosmological problems such as primordial helium abundance and they also isotropize in a special sense.

Ellis and MacCallum [3] obtained solutions of Einstein's field equations for a Bianchi type VI_0 space-time in the case of a stiff-fluid. Collins [4] and Ruban [5] presented some exact solutions of Bianchi type VI_0 for perfect fluid distributions satisfying specific equations of state. Dunn and Tupper [6] investigated a class of Bianchi type VI_0 perfect fluid cosmological models associated with electromagnetic field. Lorentz [7] has generalized the dust model of Ellis and MacCallum [3]. Roy and Singh [8] derived some exact solutions of Einstein-Maxwells equations representing a free gravitational field of magnetic type with perfect fluid and incident magnetic field. Shri Ram [9] presented an algorithm for generating exact per- fect fluid solutions of Einstein's field equations, not satis- fying the equation of state, for spatially homogeneous cosmological models of Bianchi type VI_0.

Bulk viscosity is supposed to play a very important role in the early evolution of the universe. There are many circumstances during the evolution of the universe in which bulk viscosity could arise. The bulk viscosity coefficient determines the magnitude of the viscous stress relative to the expansion. Ribeiro and Sanyal [10] studied Bianchi type VI_0 models containing a viscous fluid in the presence of an axial magnetic field. Patel and Koppar [11] presented some Bianchi type VI_0 viscous fluid cosmological models with expansion and shear. Bali *et al*. [12] studied a Bianchi type VI_0 magnetized barotropic bulk viscous fluid massive string universe. Bali *et al*. [13] obtained some exact solutions for a homogeneous Bianchi type VI_0 space-time filled with a magnetized bulk viscous fluid in the presence of a massive comic string. Bali *et al.* [14] have also discussed the properties of the free gravitational fields and their invariant characterizations

and imposing certain conditions over the free gravitational fields.

The cosmological constant Λ and the gravitational constant G are two parameters present in Einstein's field equations. The Newtonian constant G plays the role of coupling constant between geometry and matter in Einstein's field equations. There have been numerous modifications of general relativity in which G varies with time in order to achieve possible unification of gravitation and elementary particle physics or to incorporate Mach's principle in general relativity. From the point of view of incorporating particle physics into Einstein's theory of gravitation, the simplest approach is to interpret the cosmological constant Λ in terms of quantum mechanics and the physics of the vacuum [15]. The Λ term has also been interpreted in terms of Higg's scalar field [16]. Linde [17] proposed that Λ term is a function of temperature and related it to the process of broken symmetries. The cosmological constant problem related to the existence of Λ has been extensively discussed in literature. A phenomena logical solution to this problem is suggested by considering Λ as a function of time, so that it was large in the early universe and got reduces with the expansion of the universe [18]. A number of authors e.g. Kalligas *et al*. [19], Arbab [20], Abdussattar and Vishwakarma [21] proposed linking of variations of G and Λ within the framework of general relativity. This approach is appealing as it leaves the form of Einstein equations formally unchanged by allowing a variation of G to be accompanied by change in Λ. Pradhan and Yadav [22] investigated bulk viscous anisotropic cosmological models with variable G and Λ. Pradhan *et al.* [23] derived FRW universe with verying G and Λ. Since Bianchi type I spaces are subsequent generalization of zero curvature FRW models, Singh *et al*. [24] obtained some Bianchi type I models with variable G and Λ. Singh *et al.* [25] obtained early viscous universe with variable G and Λ. Singh and Tiwari [26] presented Bianchi type I models in the presence of a perfect fluid with time varying G and Λ in general relativity. Singh and Kotambkar [27] discussed cosmological models with variable G and Λ in space-times of higher dimensions. Singh and Kale [28] dealt with Bianchi type I, Kantowski-Sachs and Bianchi type III anisotropic models of the universe filled with a bulk viscous cosmic fluid in the presence of variable G and Λ. Bali and Tinker [29] investigated Bianchi type III bulk viscous barotropic fluid cosmological model with variable G and Λ which leads to inflationary phase of the universe. Recently, Verma and Shri Ram [30] obtained Bianchi type III bulk viscous barotropic fluid cosmological model with variable G and Λ in simple and systematic way. Homogeneous cosmologies with Bianchi type VI_0 space filled with perfect fluids, satisfying specific equation of state linking the pressure and matter energy density are widely used to study different properties of solutions of Einstein's field equations. Pradhan and Bali [31] presented magnetized Bianchi type VI_0 barotropic massive string universe with decaying vacuum energy density. Recently, a new class of LRS Bianchi type VI_0 universe with free gravitational field and decaying vacuum energy is obtained by Pradhan *et al.* [32].

In this paper, we investigate Bianchi type VI_0 bulk viscous barotropic fluid cosmological models with time varying gravitational and cosmological constants. The paper is organized as follows, we present the metric and Einstein's field equation for a viscous fluid with time-dependent G andΛ. We deal with solutions of the field equations and we obtain solutions of the field equations under the assumptions that 1) the expansion scalar is proportional to the shear scalar 2) the bulk viscosity coefficient is a power function of the energy density and 3) the cosmic fluid obeys the barotropic equation of state. The corresponding models represent expanding, shearing and non-rotating universe which give essentially space for large time. We also discuss the physical and kinematical behaviours of Bianchi type VI_0 anisotropic cosmological models. Some concluding remarks have also been given.

2. Field Equations and General Expressions

We consider Bianchi type VI_o metric in the form

$$ds^2 = -dt^2 + A^2dx^2 + B^2e^{-2mx}dy^2 + C^2e^{-2mx}dz^2 \qquad (1)$$

where A, B and C are function of cosmic time t and m is a constant parameter.

The energy-momentum tensor for a bulk viscous fluid distribution is given by

$$T_i^j = (\rho + \bar{p})v_i v^j + \bar{p}g_i^j \qquad (2)$$

where

$$\bar{p} = p - \xi v^i_{;i} \qquad (3)$$

Here $\rho, p, \bar{p}$ and ξ are respectively, energy-density of matter, thermodynamic pressure, effective pressure and bulk viscosity coefficient. The four-velocity vector of the fluid satisfies.

$$v_i v^i = -1 \qquad (4)$$

A semicolon stands for covariant differentiation.

The Einstein's field equations with time-dependent G and Λ are

$$R_{ij} - \frac{1}{2}Rg_{ij} = -8\pi G T_{ij} + \Lambda g_{ij}. \qquad (5)$$

For the line-element (1) with a bulk viscous fluid distribution, the field Equation (5), in commoving frame, give rise to the following equations:

$$\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{B}\dot{C}}{BC}+\frac{m^2}{A^2}=-8\pi G\,\bar{p}+\Lambda \tag{6}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{C}}{AC}-\frac{m^2}{A^2}=-8\pi G\,\bar{p}+\Lambda \tag{7}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}-\frac{m^2}{A^2}=-8\pi G\,\bar{p}+\Lambda \tag{8}$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{A}\dot{C}}{AC}+\frac{\dot{B}\dot{C}}{BC}-\frac{m^2}{A^2}=8\pi G\rho+\Lambda \tag{9}$$

$$\left(\frac{\dot{B}}{B}-\frac{\dot{C}}{C}\right)=0 \tag{10}$$

where a dot denotes differentiation with respect to t. An additional equation for time changes of G and Λ is obtained by the divergence of Einstein tensor, *i.e.* $\left(R_i^j-\frac{1}{2}Rg_i^j\right)_{;j}$, which leads to $\left(8\pi GT_i^j-\Lambda g_i^j\right)_{;j}=0$ yielding

$$8\pi\dot{G}\rho+\dot{\Lambda}+8\pi G\left[\dot{\rho}+\left(\rho+\bar{p}\right)\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)\right]=0 \tag{11}$$

The conservation of energy Equation (11), after using Equation (3), splits into two equation

$$\dot{\rho}+\left(\rho+p\right)\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)=0 \tag{12}$$

$$\dot{\Lambda}+8\pi\dot{G}\rho=8\pi G\xi\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)^2 \tag{13}$$

The average scale factor S for the metric (1) is defined by

$$S^3=ABC \tag{14}$$

The volume scale factor V is given by

$$V=S^3=ABC \tag{15}$$

The generalized mean Hubble parameter H is given by

$$H=\frac{1}{3}\left(H_1+H_2+H_3\right) \tag{16}$$

where $H_1=\frac{\dot{A}}{A},H_2=\frac{\dot{B}}{B},H_3=\frac{\dot{C}}{C}$ The expansion scalar θ. and shear scalar σ are given by

$$\theta=v^i_{;i}=\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right) \tag{17}$$

and

$$\sigma^2=\frac{1}{3}\left(\frac{\dot{A}^2}{A^2}+\frac{\dot{B}^2}{B^2}+\frac{\dot{C}^2}{C^2}-\frac{\dot{A}\dot{B}}{AB}-\frac{\dot{B}\dot{C}}{BC}-\frac{\dot{A}\dot{C}}{AC}\right) \tag{18}$$

An important observational quantity in cosmology is the deceleration parameter q which is defined as

$$q=-\frac{S\ddot{S}}{\dot{S}^2} \tag{19}$$

The sign of q indicates whether is model inflates or not. The positive sign corresponds to the standard decelerating model whereas the negative sign indicates inflation.

3. Solution of the Field Equations

We are at liberty to make certain assumptions as we have more unknown A, B, C, ρ, p, ξ, G and Λ with lesser number of field Equations (6)-(13). For complete determination of these field variables, we first assume that the expansion scalar θ is proportional to the shear scalar σ. This condition leads to

$$A=B^n \tag{20}$$

where n is a positive constant.

Equation (10), on integration, yields

$$B=lC \tag{21}$$

where l is an integration constant. Without loss of generality we can take l. From Equations (6) and (7), we obtain

$$\frac{\ddot{B}}{B}-\frac{\ddot{A}}{A}+\frac{\dot{B}}{B}\left(\frac{\dot{B}}{B}-\frac{\dot{A}}{A}\right)+\frac{2m^2}{A^2}=0 \tag{22}$$

Substitution of Equations (20) and (21) in Equation (22)

$$\frac{\ddot{B}}{B}+\left(n+1\right)\frac{\dot{B}^2}{B^2}+\frac{2m^2}{\left(1-n\right)}B^{-2n}=0 \tag{23}$$

which reduce to

$$2\ddot{B}+2\left(n+1\right)\frac{\dot{B}^2}{B}=\frac{4m^2}{\left(n-1\right)}B^{-2n+1}=0 \tag{24}$$

On assuming $\dot{B}=f(B)$, takes the form

$$\frac{d}{dB}\left(f^2\right)+\frac{2\left(n+1\right)}{B}f^2=M^2B^{-2n+1} \tag{25}$$

where $M^2=\frac{4m^2}{n-1},n\neq 1$. Equation (25) has the general solution

$$f^2=\dot{B}^2=\frac{M^2\left(B^4+a^2\right)}{4B^{2n+2}} \tag{26}$$

where a is the constant of integration. From Equation (26) we have

$$\frac{B^{n+1}dB}{\sqrt{B^4+a^2}}=\frac{M}{2}dt \tag{27}$$

The solution of Equation (27) is not valid for $n=1$. We can obtain physically realistic models by choosing the values of n for which Equation (27) is integrable.

3.1. Model I

When $n=2$ Equation (27) reduces to

$$\frac{B^3 dB}{\sqrt{B^4+a^2}} = \frac{M}{2}dt \tag{28}$$

which, after integration, leads to

$$B^2 = \left[\left(c_1 t + c_2\right)^2 - a^2\right]^{1/2} \tag{29}$$

where c_1 and c_2 are arbitrary constants.

Therefore, the metric (1) can be written in the form

$$ds^2 = -dt^2 + \left(T^2-a^2\right)dx^2 + \left(T^2-a^2\right)^{1/2} \times\left(e^{-2mx}dy^2 + e^{2mx}dz^2\right)$$

where

$$c_1 t + c_2 = T \tag{30}$$

It is clear that, given $\xi(t)$, we can find the physical and kinematical parameters associated with metric (30). The effect of bulk viscosity is to produce a change in the cosmic fluid and therefore exhibits essential change on character of the solution. In most of the investigations, the bulk viscosity is assumed to be a simple power function of the energy density [33, 34]

$$\xi(t) = \xi_0 \rho^{\alpha} \tag{31}$$

where ξ_0 and $\alpha(>0)$ are constant. For small density, α may even be equal to unity [35]. The case $\alpha=1$ corresponds to a radiative fluid [34]. Near a big-bank, $0\le\alpha\le 1/2$ is a more appropriate assumption to obtain realistic models [36].

For the specification of $\xi(t)$, we also assume that the fluid obeys the equation of state

$$p = \gamma\rho \tag{32}$$

where $\gamma(0\le\gamma\le 1)$ is a constant. From Equations (12) and (32), we obtain

$$\rho' + \frac{2(1+\gamma)T}{T^2-a^2}\rho = 0 \tag{33}$$

where a dash denotes differentiation with respect to T.

Integration of Equation (33) yields

$$\rho = k\left(T^2-a^2\right)^{-(\gamma+1)} \tag{34}$$

where *k* is integration constant. Differentiating Equation (34), we obtain

$$\rho' = -2k(\gamma+1)T\left(T^2-a^2\right)^{-(\gamma+2)} \tag{35}$$

Also, from Equation (9), we find that

$$8\pi G\rho + \Lambda = \frac{\left(5-4m^2\right)T^2 + 4m^2a^2}{4\left(T^2-a^2\right)^2} \tag{36}$$

which on differentiation leads to

$$8\pi G'\rho + 8\pi G\rho' + \Lambda' = \frac{\left(5-4m^2\right)T^3 + \left(8m^2-10\right)T^2}{2\left(T^2-a^2\right)^3} + \frac{\left(4m^2a^2-5a^2\right)T - 8m^2a^2}{2\left(T^2-a^2\right)^3} \tag{37}$$

Using Equations (13), (31) and (35) in Equation (37), we get

$$G = \left[\frac{\left(5-4m^2\right)T^3 + \left(8m^2-10\right)T^2 + \left(4m^2a^2-5a^2\right)T - 8m^2a^2}{2\left(T^2-a^2\right)^3}\right] \times\left[\frac{32\pi\xi_0 k^{\alpha}T^2}{\left(T^2-a^2\right)^{\alpha(1+\gamma)+2}} - \frac{16\pi k(1+\alpha)T}{\left(T^2-a^2\right)^{(2+\gamma)}}\right]^{-1} \tag{38}$$

Using Equations (34) and (38) in Equation (36), we obtain

$$\Lambda(t) = \left[\frac{\left(5-4m^2\right)T^3 + \left(8m^2-10\right)T^2 + \left(4m^2a^2-5a^2\right)T - 8m^2a^2}{2\left(T^2-a^2\right)^{(4+\gamma)}}\right] \times\left[\frac{2k(1+\alpha)T}{\left(T^2-a^2\right)^{(\gamma+2)}} - \frac{4\xi_0 k^{\alpha}T^2}{\left(T^2-a^2\right)^{\alpha(1+\gamma)+2}}\right]^{-1} + \frac{\left(5-4m^2\right)T^2 + 4m^2a^2}{4\left(T^2-a^2\right)^2}. \tag{39}$$

The gravitational constant $G(t)$ is zero initially and gradually increases and tends to infinity at late time. We

also observe that the cosmological term Λ is initially infinite. It is decreasing function of time and approaches to zero at late time which is supported by recent result from the observations of type supernova explosion (SNIa). From Equations (31) and (34), we obtain

$$\xi(t) = \xi_0 k^{\alpha} \left(T^2 - a^2\right)^{-\alpha(1+\gamma)} \tag{40}$$

The physical and kinematical parameters of the model (30) are given by the following expressions.

$$V^3 = \left(T^2 - a^2\right) \tag{41}$$

$$H = 2T\Big/3\left(T^2 - a^2\right) \tag{42}$$

$$\theta = 2T\Big/\left(T^2 - a^2\right) \tag{43}$$

$$\sigma = T\Big/2\sqrt{3}\left(T^2 - a^2\right) \tag{44}$$

$$q = \left(T^2 + 3a^2\right)\Big/2T^2 \tag{45}$$

The value of the deceleration parameter is positive for all time which shows the decelerating behaviour of the cosmological model.

For model (30), we observe that the spatial volume increases with time T and it becomes infinite for large value of T. At $T = a$, the spatial volume is zero and ρ, p, θ, σ all are infinite but vanish for large T. Thus, the model has a big-bank singularity at the finite time $T = a$. The physical and kinematical parameters are all well behaved for $a < T < \infty.$. The bulk viscosity coefficient is infinite at $T = a$ and tends to zero for large time. Since $\dfrac{\sigma}{\theta}$ = constant, the anisotropy is maintained for all times. It can be seen that the model is irrotational. Therefore, the model describes a continuously expanding, shearing and non-rotating universe with a big-bang start at $T = a$.

3.2. Mode-II

From Equation (27), we get

$$dt = \frac{2B^{n+1}dB}{M\left(\sqrt{B^4 + a^2}\right)} \tag{46}$$

with the help of Eq. (46), the line-element (1) reduces to

$$ds^2 = -\frac{4B^{2n+2}dB}{M^2\left(B^4 + a^2\right)}dB^2 + B^{2n}dx^2 + B^2\left(e^{-2mx}dy^2 + e^{2mx}dz^2\right) \tag{47}$$

By a suitable transformation of coordinates, the line-element (47) reduces to

$$ds^2 = -\frac{4T^{2n+2}dT}{M^2\left(T^4 + a^2\right)}dT^2 + T^{2n}dx^2 + T^2\left(e^{-2mx}dy^2 + e^{2mx}dz^2\right) \tag{48}$$

For the model (48), the physical and kinematical parameters are given

$$V = T^{(n+2)/3} \tag{49}$$

$$\theta = \frac{(n+2)}{T} \tag{50}$$

$$H = \frac{(n+2)}{3T} \tag{51}$$

$$\sigma = \frac{(n-1)}{T} \tag{52}$$

$$q = \frac{(1-n)}{(n+2)} \tag{53}$$

Following the procedure as in Section (3.1), we obtain the expression for energy density ρ, G, Λ and ξ as under

$$\rho = k_1 T^{-(1+\gamma)(n+2)} \tag{54}$$

$$\begin{aligned} G = \left(T^4 + a^2\right)^{1/2} &\left[\frac{(2n+1)M^3}{2T^{3n+2}} - \frac{(2n+1)(2n+4)M^3\left(T^4 + a^2\right)}{8T^{3n+6}} + \frac{m^2 nM}{T^{3n}}\right] \\ &\times \left[\frac{8\pi\xi_0 k_1^{\alpha} M^2 (n+2)^2 \left(T^4 + a^2\right)}{4T^{(n+2)(2+\alpha+\alpha\gamma)}} - \frac{8\pi k_1 (n+2)(1+\gamma)}{T^{(2+\gamma)(n+2)}}\right]^{-1} \end{aligned} \tag{55}$$

$$\begin{aligned} \Lambda = \frac{k_1\left(T^4 + a^2\right)^{1/2}}{T^{(1+\gamma)(n+2)}} &\left[\frac{k_1(n+2)(1+\gamma)}{T^{(2+\gamma)(n+2)}} - \frac{\xi_0 k_1^{\alpha} M^2 (n+2)^2 \left(T^4 + a^2\right)}{4T^{(n+2)(2+\alpha+\alpha\gamma)}}\right]^{-1} \\ &\times \left[\frac{(2n+1)M^3}{2T^{3n+2}} - \frac{(2n+1)(2n+4)M^3\left(T^4 + a^2\right)}{8T^{3n+6}} + \frac{m^2 nM}{T^{3n}}\right] + \frac{(2n+1)M^2\left(T^4 + a^2\right)}{2T^{2n+4}} - \frac{m^2}{T^{2n}} \end{aligned} \tag{56}$$

$$\xi = \xi_0 k_1 T^{-\alpha(1+\gamma)(n+2)} \quad (57)$$

We observe that the spatial volume is zero at $T = 0$. At this epoch the energy density ρ, expansion θ, the shear scalar σ and the bulk viscosity coefficient are all infinite. Therefore the model (48) starts evolving with a big-bang at $T = 0$. The spatial volume tends to infinite and $\rho, \theta, \sigma, \xi$ become zero as for large time *T*. The gravitational constant G is zero initially and tends to infinity for large time *T*. The cosmological term Λ is infinite at the beginning of the model and tends decreases gradually to become zero at late time. The deceleration parameter *q* is positive for $n < 1$ and is negative $n > 1$. Therefore Equation (48) represents a model of a decelerating universe for $n < 1$ and a model of an accelerating universe for $n > 1$. The present day observations and literature favour accelerating model of the universe. The anisotropy in the models is maintained throughout.

4. Conclusions

In this paper we have studied Bianchi type VI_0 space-time models with bulk viscosity in the presence of time-dependent gravitational and cosmological constants. We have presented two physically viable anisotropic models of the universe. For $n = 2$, the model I evolves with a big-bang start at the finite time $T = a$ and does not approach isotropy as $T \to \infty$. For large T, energy density becomes zero. The rate of expansion in the model slows down tending to zero as $T \to \infty$. Since the deceleration parameter is positive for all time T, this model corresponds to an expanding, shearing, non-rotating and decelerating universe. For $o < n < 1$, the model II represents a decelerating universe whereas it represents an accelerating universe for $n > 1$. Model II starts evolving with a big-bang singularity at $T = 0$ and expands uniformly. The model with negative deceleration parameter is compatible with the recent supernovae Ia observations that the universe is undergoing a late time acceleration. The anisotropy is maintained in both the models. The gravitational constant $G(t)$ is zero initially and gradually increases and tends to infinity at late time. The cosmological term is infinite initially and approaches to zero at late time. These are supported by recent results from the observations of the type Ia supernova explosion (SN Ia).

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