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Laplace Transforms Method on a System of Differential Equations for Non-isothermal Chemically Reactive Flow

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Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

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Abstract

This study analyses Laplace transforms method on a system of partially coupled differential equations for non-isothermal chemically reactive flow through a cylindrical channel. The dimensionless governing equation for velocity, temperature and concentration was solved using Laplace Transform. Various parameters such as Temperature boundary parameter, Concentration boundary parameter, Cooling Parameter, Grashof number, pressure gradient and Magnetic field, as well as perturbation parameter had an effect on the velocity profile as well as temperature and concentration profile. The graphs were obtained with the results showing that an increase in the temperature boundary parameter resulted to an increase in the temperature of flow, an increase in perturbation parameter resulted to an increase in temperature profile of a body and an increase in Grashof temperature number results to an increase in the velocity of the body.

Keywords: Laplace transform; differential equations; temperature; concentration; velocity.

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Nomenclatures

$u^{'}$: *Dimensional velocity components*
$z^{'}$: *Dimensional axisymmetric flow of the channel*
$r^{'}$: *Dimensional radius*
$T^{'}$: *Dimensional fluid temperature*
ρ : *Fluid density*
k : *Thermal conductivity*
g : *Acceleration due to gravity*
μ : *Viscosity of the fluid*
C_p : *Heat capacity*
β : *Coefficient of volumetric expansion*
T_∞ : *Free stream temperature*
A : *Concentration injection parameter*
λ : *Cooling parameter*
θ_a : *Temperature boundary parameter*
φ_a : *Concentration boundary parameter*
Gr : *Grashof number*
Pr : *Pressure gradient*
M : *Magnetic Field*
$u_0(r)$: *Dimensionless velocity*
$u(r)$: *Velocity profile*
$\theta_0(r)$: *Dimensionless temperature*
$\theta(r)$: *Temperature profile*
$\varphi_0(r)$: *Dimensionless Concentration*
$\varphi(r)$: *Concentration profile*

1 Introduction

Laplace transform is one of the integral transforms that is widely used in many fields especially Mathematics, Physics, and Electrical Engineering, Control Engineering, Optics and signal processing. It was named after a French mathematician and astronomer Pierre-Simon Laplace (1749 – 1827). The Laplace transform operates on a function of time and yields another function with complex frequency as it converts a system of differential equations into a system of algebraic simultaneous equations. The Laplace transform has become an integral part of society, even if it is not common knowledge, especially considering how attached members of today’s society are to their cell phones. The reason for this being the Laplace transform is undoubtedly partially responsible for the device working, as it is in many other types of two-way receivers. The Laplace transform’s applications are numerous, ranging from heating, ventilation, and air conditioning systems modelling, to modelling radioactive decay in nuclear physics. [1] gives a clear introduction to Laplace transform. Vaithyasubramanian et al. [2] gave theory, problem worked on and application of Laplace transform in the research paper study on Applications of Laplace Transformation. A methodical review on applications of Laplace transform as a significant tool was used to respond diverse research problem modelled as differential or integral equation. The results of numerous studies, allow them to recommend the use of this technique to model their research problem mathematically and to find the solution to the same. [3] discussed the application of Laplace transforms in the area of mathematics and also in the area of physics. Also, [4] discussed the properties, master techniques used by researchers, formulation of Laplace transform of important functions (like periodic functions, unit impulse function) and application of Laplace transform in the various fields. [5] studied the properties and broad range of Applications of Laplace transform in various fields. Jadhav et al. [6] provided solid foundation of what Laplace transform is, its properties and its application in various fields which are useful in real life as well. [7] Discussed application of Laplace Transform in different engineering fields, how to use Laplace Transform in nuclear physics as well as Automation engineering, control engineering and signal processing. [8] Showcased some of the most important uses of the transform. [9] investigated solutions of first order constant coefficients complex equations with Laplace transforms using several areas of mathematics. Pavani et al. [10] provided a solid foundation in the fundamentals of Laplace transforms. Dhaigude & Dhaigude [11] studied the system of Linear Differential Equations of first order, second order and Third order in two variables as well as in three variables and obtained the set of Pre-functions and extended pre-functions are the closed form of solutions through Laplace Transform technique [12]. Investigates the usefulness of the Laplace Transforms in solving mixed problems associated to an equation in Partial Derivatives, which facilitates the calculations. Seeing the solution of the hyperbolic equation with the boundary conditions, in Partial Differential Equations the equations were solved using a variable separation method that requires knowledge about the Fourier series. Opanuga et al. [13] presented the solution of boundary value problems using finite difference scheme and Laplace Transform which gave a closed form solution while the extended interval enhanced the convergence of the solution in finite difference scheme.

The study analyses the method of Laplace Transform in solving a system of partially coupled differential equations in a Cylindrical Domain. The dimensionless governing equation for velocity, temperature and concentration was solved using Laplace Transform. Numerical Simulation using Wolfram Mathematica were performed while investigating the pertinent parameters on the temperature and velocity profiles. The effects of various parameters such as Temperature boundary parameter, Concentration boundary parameter, Cooling Parameter, Grashof number, pressure gradient and Magnetic field, as well as perturbation parameter are discussed in this paper.

This paper is presented as follows: The mathematical models of the problem are presented in section 2 and detailed solution is derived in Section 3. The results are presented and discussed in Section 4. Section 5 gives the conclusion of the paper.

This paper is aimed at extending the work of Nwaigwe and Amadi [14] by solving the system using the method of Laplace transform.

2 Methodology

Following the dimensional analytical Solution of Non-Isothermal flow, we present the equations in a Cylindrical Geometry as follows:

$$\frac{\partial u'}{\partial z'} = 0 \tag{1}$$

$$-\frac{\partial p'}{\partial z'} + \mu\left(\frac{\partial^2 u'}{\partial r^2} + \frac{1}{r}\frac{\partial u'}{dr'}\right) + \rho g \beta_T (T' - T_\infty) - \alpha {B'_0}^2 u = 0 \tag{2}$$

$$\frac{k}{\rho C_p}\left(\frac{\partial^2 T'}{\partial r'^2} + \frac{1}{r'}\frac{\partial T'}{\partial r'}\right) - \lambda^2 (T' - T_\infty) = 0 \tag{3}$$

$$\left(\frac{\partial^2 C'}{\partial r'^2} + \frac{1}{r'}\frac{\partial C'}{\partial r'}\right) + \alpha^2 (C' - C_\infty) = 0 \tag{4}$$

With the given boundary conditions

$$u' = 0, \theta' = \theta_a, \varphi' = \varphi_a \; on \; r' = a \tag{5}$$

$$u', \theta', \varphi' < \infty \; for \; all \; r' \tag{6}$$

Equation (1) to (4) with equation (5) and (6) as the boundary conditions, we present the non-dimensional form by Nwaigwe and Amadi [14] to obtain the governing equation given as:

$$-\frac{\partial p}{\partial z} + \frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - M^2 u + Gr\theta = 0 \tag{7}$$

$$\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r}\frac{\partial \theta}{\partial r} - \lambda^2 \theta = 0 \tag{8}$$

$$\frac{\partial^2 \varphi}{\partial r^2} + \frac{1}{r}\frac{\partial \varphi}{\partial r} + \alpha^2 \varphi = 0 \tag{9}$$

Subject to:

$$\left.\begin{array}{l} u = 0, \theta = \theta_a, \varphi = \varphi_a \ on \ r = 1 \\ u, \theta, \varphi < \infty \end{array}\right\} \tag{10}$$

3 Methods of Solution

Equations (7) – (9) are coupled nonlinear partial differential equations and are reduced to a set of ordinary differential equations with the expressions for velocity (u), temperature (θ) and concentration (φ) in dimensionless form as follows

$$\theta = \theta(r) = \ \theta_0 + \varepsilon\theta_1 + 0(\varepsilon^2) \tag{11}$$

$$\varphi = \varphi(r) = \varphi_0 + \varepsilon\varphi_1 + 0(\varepsilon^2) \tag{12}$$

$$u = u(r) = u_0 + \varepsilon u_1 + 0(\varepsilon^2) \tag{13}$$

$$\frac{\partial p}{\partial z} = P_0 + \varepsilon P_1 \tag{14}$$

Substituting equation (11) to (14) in the set of equations (7), (8) and (9) and equating non – harmonic and harmonic terms and neglecting the higher order terms of $0(\varepsilon^2)$, the following set of ordinary differential equations are obtained.

$$\frac{d^2u_0}{dr^2} + \varepsilon\frac{d^2u_1}{dr^2} + \frac{1}{r}\left(\frac{du_0}{dr} + \varepsilon\frac{du_1}{dr}\right) - M^2(u_0 + \varepsilon u_1) = -P_0 - \varepsilon P_1 - Gr(\theta_0 + \varepsilon\theta_1) \tag{15}$$

$$\frac{d^2\theta_0}{dr^2} + \varepsilon\frac{d^2\theta_1}{dr^2} + \frac{1}{r}\left(\frac{d\theta_0}{dr} + \varepsilon\frac{d\theta_1}{dr}\right) - \lambda^2(\theta_0 + \varepsilon\theta_1) = 0 \tag{16}$$

$$\frac{d^2\varphi_0}{dr^2} + \varepsilon\frac{d^2\varphi_1}{dr^2} + \frac{1}{r}\left(\frac{d\varphi_0}{dr} + \varepsilon\frac{d\varphi_1}{dr}\right) + \alpha^2(\varphi_1 + \varepsilon\varphi_1) = 0 \tag{17}$$

By Splitting, Equation (15) to (17) is reduced to Equations (18a) to (20b)

$$\frac{d^2u_o}{dr^2} + \frac{1}{r}\frac{du_0}{dr} - M^2u_o = -P_o - Gr\theta_0 \tag{18a}$$

$$\frac{d^2u_1}{dr^2} + \frac{1}{r}\left(\frac{du_1}{dr}\right) - M^2u_1 = -P_1 - Gr\theta_1 \tag{18b}$$

$$\frac{d^2\theta_0}{dr^2} + \frac{1}{r}\frac{d\theta_0}{dr} - \lambda^2\theta_0 = 0 \tag{19a}$$

$$\frac{d^2\theta_1}{dr^2} + \frac{1}{r}\frac{d\theta_0}{dr} - \lambda^2\theta_1 = 0 \tag{19b}$$

$$\frac{d^2\varphi_0}{dr^2} + \frac{1}{r}\frac{d\varphi_0}{dr} + \alpha^2\varphi_0 = 0 \tag{20a}$$

$$\frac{d^2\varphi_1}{dr^2} + \frac{1}{r}\frac{d\varphi_1}{dr} + \alpha^2\varphi_1 = 0 \tag{20b}$$

Hence, we apply Laplace transform method in solving. We first solve for the temperature and concentration model, and then substitute our result into the velocity model.

3.1 Solution to temperature model

Let $-(\lambda)^2 = (i\lambda)^2$, then equation (19a) becomes

$$\frac{rd^2\theta_0}{dr^2} + \frac{d\theta_0}{dr} + (i\lambda)^2 r\theta_0 = 0 \quad (21)$$

Applying the Laplace transform to equation (21)

$$\mathrm{L}\left\{\frac{rd^2\theta_0}{dr^2}\right\} + L\left\{\frac{d\theta_0}{dr}\right\} + (i\lambda)^2 L\{r\theta_0\} = 0 \quad (22)$$

We obtain

$$\frac{-d}{ds}\left(s^2\theta_0(s) - s\theta_0(0) - \theta'_0(0)\right) + s\theta_0(s) - \theta_0(0) - (i\lambda)^2\frac{d\theta_0}{ds} = 0 \quad (23)$$

Simplifying equation (23),

$$\frac{-d}{ds}\left(s^2\theta_0(s)\right) + \frac{d}{ds}\left(s\theta_0(0)\right) + \frac{d}{ds}\left(\theta'_0(0)\right) + s\theta_0(s) - s\theta_0(0) - (i\lambda)^2\frac{d\theta_0}{ds} = 0 \quad (24)$$

$$-s^2\frac{d\theta_0}{ds} - (i\lambda)^2\frac{d\theta_0}{ds} - 2s\theta_0(s) + s\theta_0(s) = 0 \quad (25)$$

$$(-s^2 - (i\lambda)^2)\frac{d\theta_0}{ds} - 2s\theta_0(s) + s\theta_0(s) = 0 \quad (26)$$

Factorizing equation (26)

$$-(s^2 + (i\lambda)^2)\frac{d\theta_0}{ds} - (2s - s)\,\theta_0(s) = 0 \quad (27)$$

$$(s^2 + (i\lambda)^2)\frac{d\theta_0}{ds} + s\,\theta_0(s) = 0 \quad (28)$$

Solving equation (28), we obtain

$$(s^2 + (i\lambda)^2)\frac{d\theta_0}{ds} = -s\,\theta_0(s) \quad (29)$$

$$\frac{d\theta_0}{\theta_0} = \frac{-s}{(s^2 + (i\lambda)^2)}ds \quad (30)$$

$$\int\frac{d\theta_0}{\theta_0} = -\int\frac{s}{(s^2 + (i\lambda)^2)}\,\mathrm{ds} \quad (31)$$

$$\ln\theta_0 = -\frac{1}{2}\ln(s^2 + (i\lambda)^2) + \ln A \quad (32)$$

Simplifying equation (32),

$$\theta_0 = \frac{A}{\sqrt{(s^2 + (i\lambda)^2)}} \quad (33)$$

Note that if $L\{\theta_0(r)\} = \theta_0(s)\ then$

$$\theta_0(r) = L^{-1}\{\theta_0(s)\} \quad (34)$$

Applying the concept in equation (34) to solve equation (33),

$$\theta_0(r) = L^{-1}\left\{\frac{A}{\sqrt{(s^2 + (i\lambda)^2)}}\right\} = AL^{-1}\left\{\frac{1}{\sqrt{(s^2 + (i\lambda)^2)}}\right\} \quad (35)$$

Therefore, the inverse Laplace transform becomes

$$\theta_0(r) = AL^{-1}\left\{\frac{1}{\sqrt{(s^2+(i\lambda)^2)}}\right\} = AJ_0(i\lambda r) \tag{36}$$

The imaginary part of the equation (37) indicates that it is a modified Bessel function of order zero, thus we recall that

$$J_0(i\lambda r) = I_0(\lambda r) \tag{37}$$

Applying equation (37) into equation (36) we obtain

$$\theta_0(r) = AI_0(\lambda r) \tag{38}$$

The corresponding boundary conditions are

$$\left.\begin{aligned} &\theta_0 \to \infty,\ at\ \ r = 0 \\ &\theta_0 = \theta_a,\ at\ \ r = 1 \end{aligned}\right\} \tag{39}$$

Applying the boundary conditions in equation (39) in solving equation (38)

$$A = \frac{\theta_a}{I_0\lambda} \tag{40}$$

Substituting equation (40) into equation (38)

$$\theta_0(r) = \frac{\theta_a I_0(\lambda r)}{I_0(\lambda)} \tag{41}$$

Equation (41) is solution to equation (19a)

Applying same method in solving equation (19b) we obtain

$$\theta_0(r) = \varepsilon\left(\frac{\theta_a I_0(\lambda r)}{I_0(\lambda)}\right) \tag{42}$$

Hence equation (16) becomes

$$\theta_{(r,t)} = \frac{\theta_a I_0(\lambda r)}{I_0(\lambda)} + \varepsilon\left(\frac{\theta_a I_0(\lambda r)}{I_0(\lambda)}\right) \tag{43}$$

$$= \frac{\theta_a I_0(\lambda r)}{I_0(\lambda)}(1+\varepsilon) \tag{44}$$

Equation (44) is the final solution to equation (8)

3.2 Solution to the concentration model

Equation (20a) can be written as:

$$\frac{rd^2\varphi_0}{dr^2} + \frac{d\varphi_0}{dr} + \alpha^2 r\varphi_0 = 0 \tag{45}$$

Applying the Laplace transform to equation (45)

$$L\left\{\frac{rd^2\varphi_0}{dr^2}\right\} + L\left\{\frac{d\varphi_0}{dr}\right\} + \alpha^2 L\{r\varphi_0\} = 0 \tag{46}$$

We obtain

$$\frac{-d}{ds}(s^2\varphi_0(s) - s\varphi_0(0) - \varphi'_0(0)) + s\varphi_0(s) - \varphi_0(0) - \alpha^2\left\{\frac{d\varphi_0}{ds}\right\} = 0 \quad (47)$$

Simplifying equation (47) we obtain

$$\frac{-d}{ds}(s^2\varphi_0(s) + \frac{d}{ds}\left(s\varphi_0(0)\right) + \frac{d}{ds}\left(\varphi'_0(0)\right) + s\varphi_0(s) - \varphi_0(0) - \alpha^2\frac{d\varphi_0}{ds} = 0 \quad (48)$$

$$-s^2\frac{d\varphi_0}{ds} - \alpha^2\frac{d\varphi_0}{ds} - 2s\varphi_0(s) + s\varphi_0(s) = 0 \quad (49)$$

$$(-s^2 - \alpha^2)\frac{d\varphi_0}{ds} - 2\varphi_0(s) + s\varphi_0(s) = 0 \quad (50)$$

$$-(s^2 + \alpha^2)\frac{d\varphi_0}{ds} - (2s - s)\varphi_0(s) = 0 \quad (51)$$

$$(s^2 + \alpha^2)\frac{d\varphi_0}{ds} + s\varphi_0(s) = 0 \quad (52)$$

Solving equation (52), we obtain

$$(s^2 + \alpha^2)\frac{d\varphi_0}{ds} = -s\varphi_0(s) \quad (53)$$

$$\frac{d\varphi_0}{\varphi_0} = \frac{-s}{(s^2+\alpha^2)}ds \quad (54)$$

$$\int\frac{d\varphi_0}{\varphi_0} = -\int\frac{s}{(s^2+\alpha^2)}ds \quad (55)$$

$$\ln\varphi_0 = -\frac{1}{2}\ln(s^2 + \alpha^2) + \ln B \quad (56)$$

Simplifying equation (56) we have

$$\varphi_0 = \frac{B}{\sqrt{(s^2+\alpha^2)}} \quad (57)$$

Note that if $L\{\varphi_0(r)\} = \varphi_0(s),\ then$

$$\varphi_0(r) = L^{-1}\{\varphi_0(s)\} \quad (58)$$

Applying the concept in equation (58) to solve equation (57)

$$\varphi_0(r) = \left\{\frac{B}{\sqrt{(s^2+\alpha^2)}}\right\} = BL^{-1}\left\{\frac{1}{\sqrt{(s^2+\alpha^2)}}\right\} \quad (59)$$

Therefore, the inverse Laplace transform becomes

$$\varphi_0(r) = BL^{-1}\left\{\frac{1}{\sqrt{(s^2+\alpha^2)}}\right\} = BJ_0(\alpha r) \quad (60)$$

The corresponding boundary conditions are

$$\left.\begin{array}{l}\varphi_0 \to \infty,\ at\ \ r = 0\\ \varphi_0 = \varphi_a,\ at\ \ r = 1\end{array}\right\} \quad (61)$$

Applying the boundary condition in equation (61) in solving equation (60)

$$B = \frac{\varphi_0}{J_0 \alpha} \quad (62)$$

Substituting equation (62) into equation (60)

$$\varphi_0(r) = \frac{\varphi_a J_0(\alpha r)}{J_0(\alpha)} \quad (63)$$

Equation (63) is solution to equation (20a)

Applying same method in soling (20b) we obtain

$$\varphi_1(r) = \varepsilon\left(\frac{\varphi_a J_0(\alpha r)}{J_0(\alpha)}\right) \quad (64)$$

Hence equation (17) becomes

$$\varphi_{(r,t)} = \frac{\varphi_a J_0(\alpha r)}{J_0(\alpha)} + \varepsilon \left(\frac{\varphi_a J_0(\alpha r)}{J_0(\alpha)}\right) \quad (65)$$

$$= \frac{\varphi_a J_0(\alpha r)}{J_0(\alpha)}(1 + \varepsilon) \quad (66)$$

Equation (66) is the final solution to equation (9)

3.3 Solution for velocity model

We shall start by solving homogenous parts of equation (18a)

Equation (18a) can be written as

$$\frac{rd^2u_o}{dr^2} + \frac{du_0}{dr} + (iM)^2 ru_o = 0 \quad (67)$$

Applying the Laplace transform

$$L\left\{\frac{rd^2u_o}{dr^2}\right\} + L\left\{\frac{du_0}{dr}\right\} + (iM)^2 L\{ru_0\} = 0 \quad (68)$$

We obtain

$$\frac{-d}{ds}\left(s^2u_0(s) - su_0(0) - u'_0(0)\right) + su_0(s) - u_0(0) - (iM)^2\frac{du_0}{ds} = 0 \quad (69)$$

Simplifying equation (69) we obtain

$$\frac{-d}{ds}\left(s^2u_0(s) - su_0(0) - u'_0(0)\right) + su_0(s) - u_0(0) - (iM)^2\frac{du_0}{ds} = 0 \quad (70)$$

Therefore

$$\frac{-d}{ds}\left(s^2u_0(s)\right) + \frac{d}{ds}(su_0(0)) + \frac{d}{ds}(u'_0(0)) + su(s) - u_0(0) - (iM)^2\frac{du_0}{ds} = 0 \quad (71)$$

$$-s^2\frac{du_0}{ds} - (iM)^2\frac{du_0}{ds} - 2su_0(s) + su_0(s) = 0 \quad (72)$$

$$(-s^2 - (iM)^2)\frac{du_0}{ds} - 2su_0(s) + su_0(s) = 0 \quad (73)$$

Factorizing equation (73)

$$-(s^2 + (iM)^2)\frac{du_0}{ds} - (2s - s)u_0(s) = 0 \quad (74)$$

$$(s^2 + (iM)^2)\frac{du_0}{ds} + su_0(s) = 0 \quad (75)$$

Solving equation (75), we obtain

$$(s^2 + (iM)^2)\frac{du_0}{ds} = -su_0(s) \quad (76)$$

$$\frac{du_0}{u_0} = \frac{-s}{(s^2+(iM)^2)}ds \quad (77)$$

$$\int \frac{du_0}{u_0} = -\int \frac{s}{(s^2+(iM)^2)}ds \quad (78)$$

$$\ln u_0 = -\frac{1}{2}\ln(s^2 + (iM)^2) + \ln D \quad (79)$$

Simplifying equation (79), we obtain

$$u_0 = \frac{D}{\sqrt{(s^2+(iM)^2)}} \quad (80)$$

Note that if $L\{u_0(r)\} = u_0(s), then$

$$u_0(r) = L^{-1}\{u_0(s)\} \quad (81)$$

Applying the concept in equation (81) to solve equation (80)

$$u_0(r) = L^{-1}\left\{\frac{D}{\sqrt{(s^2+(iM)^2)}}\right\} = DL^{-1}\left\{\frac{1}{\sqrt{(s^2+(iM)^2)}}\right\} \quad (82)$$

Therefore, the inverse Laplace transform becomes

$$u_0(r) = L^{-1}\left\{\frac{D}{\sqrt{(s^2+(iM)^2)}}\right\} = DJ_0(iMr) \quad (83)$$

The imaginary part in equation (83) indicates that it is a modified Bessel function of order zero, thus recall that

$$J_0(iMr) = I_0(Mr) \quad (84)$$

Applying equation (84) into equation (83), we have the homogenous solution of equation (18a) as

$$u_{0_h}(r) = DI_0(Mr) \quad (85)$$

Since the system is coupled with heat on momentum equation (18a), we substitute equation (85) into equation (18a) which is

$$\frac{d^2u_0}{dr^2} + \frac{1}{r}\frac{du_0}{dr} - M^2u_0 = -P_0 - \frac{\theta_a Gr}{I_0(\lambda)}I_0(\lambda r) \quad (86)$$

Since equation (86) is an inhomogeneous ordinary different equation of which the homogenous solution is known in equation (85), we let the particular integral to be

$$u_{0_p}(r) = D_1 + D_2I_0(\lambda r) \quad (87)$$

Differentiating equation (87) to second order and substitute the result into equation (86),

$$D_2\lambda^2 I_1^{'}(\lambda r) + \frac{\lambda}{r} D_2 I_1(\lambda r) - M^2(D_1 + D_2 I_0(\lambda r)) = -P_0 \frac{-\theta_a Gr}{I_0(\lambda)} I_0(\lambda r) \quad (88)$$

Thus equation (88) becomes

$$D_2\lambda^2 I_1^{'}(\lambda r) + \frac{\lambda}{r} D_2 I_1(\lambda r) - D_1 M^2 - D_2 M^2 I_0(\lambda r) = -P \frac{-\theta_a Gr}{I_0(\lambda)} I_0(\lambda r) \quad (89)$$

Where,

$$D_1 = \frac{P_0}{M^2}, D_2 = \frac{\theta_a Gr}{M^2 I_0(\lambda)}, D_2\lambda^2 \neq 0, I_1^{'}(\lambda r) = 0, \ \ \lambda D_2 \neq 0 \ \ I_1(\lambda r) = 0 \quad (90)$$

Substituting the value of the constant coefficient in equation (90) into equation (89) we have

$$u_{0_p}(r) = \frac{P_0}{M^2} + \frac{\theta_a Gr}{M^2 I_0(\lambda)} I_0(\lambda r) \quad (91)$$

Substituting equation (85) and (91) into equation (86),

$$u_0(r) = DI_0(Mr) + \frac{P_0}{M^2} + \frac{\theta_a Gr}{M^2 I_0(\lambda)} I_0(\lambda r) \quad (92)$$

Solving for the constant coefficient in equation (92) using the boundary condition in equation (10)

$$u_0\ (r = 1) = DI_0(M) + \frac{P_0}{M^2} + \frac{\theta_a Gr}{M^2 I_0(\lambda)} I_0(\lambda) = 0 \quad (93)$$

Simplifying equation (93) we obtain

$$D = \frac{P_0}{M^2 I_0(M)} - \frac{\theta_a Gr}{M^2 I_0(M)} \quad (94)$$

Substituting equation (89) into equation (92) we have

$$u_0(r) = \frac{P_0}{M^2}\left(1 - \frac{I_0(Mr)}{I_0(M)}\right) + \frac{\theta_a Gr}{M^2}\left(\frac{I_0(\lambda r)}{I_0(\lambda)} - \frac{I_0(Mr)}{I_0(M}\right) \quad (95)$$

Equation (95) is solution to equation (18a)

Applying same method in solving equation (18b) we obtain

$$u_1(r) = \left[\frac{P_0}{M^2}\left(1 - \frac{I_0(Mr)}{I_0(M)}\right) + \frac{\theta_a Gr}{M^2}\left(\frac{I_0(\lambda r)}{I_0(\lambda)} - \frac{I_0(Mr)}{I_0(M}\right)\right]\varepsilon \quad (96)$$

Let equation (95) = A and equation (96) = B

Therefore

$$u_{(r,t)} = A + B \quad (97)$$

$$u_{(r,t)} = \left[\frac{P_0}{M^2}\left(1 - \frac{I_0(Mr)}{I_0(M)}\right) + \frac{\theta_a Gr}{M^2}\left(\frac{I_0(\lambda r)}{I_0(\lambda)} - \frac{I_0(Mr)}{I_0(M)}\right)\right] 1 + \varepsilon \qquad (98)$$

4 Results and Discussion

To achieve the effect of various parameters, numerical evaluations of the analytical results and some important results are displayed graphically in Figs. 1 to 9. MATHEMATICA is used to obtain the numerical results and illustrations. All the obtained solutions are discussed graphically under the variations of various important parameters in the present section.

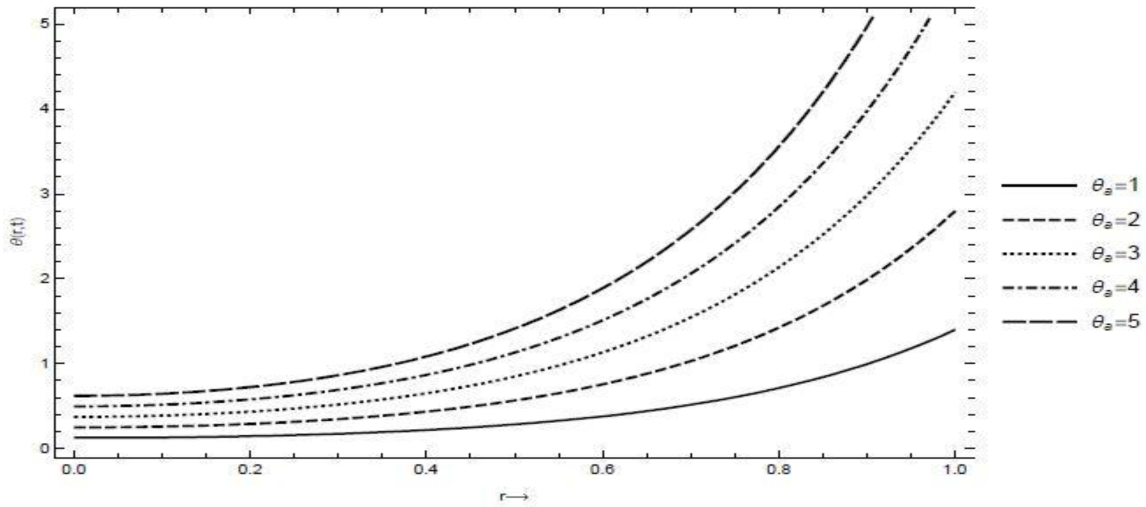

Fig. 1. The effect of wall temperature on temperature profile with other values
$\varepsilon = 0.4, \lambda = 4$

Fig. 1 shows the effect of temperature boundary parameter on the temperature of a fluid. It was observed that an increase in the temperature boundary parameter resulted in an increase in the temperature of flow. This is as a result of a reduced viscosity of the fluid due to the temperature increase at the wall of the cylindrical vessel. This is in line with Nwaigwe and Amadi [14] result on the graphical illustration of Temperature profile.

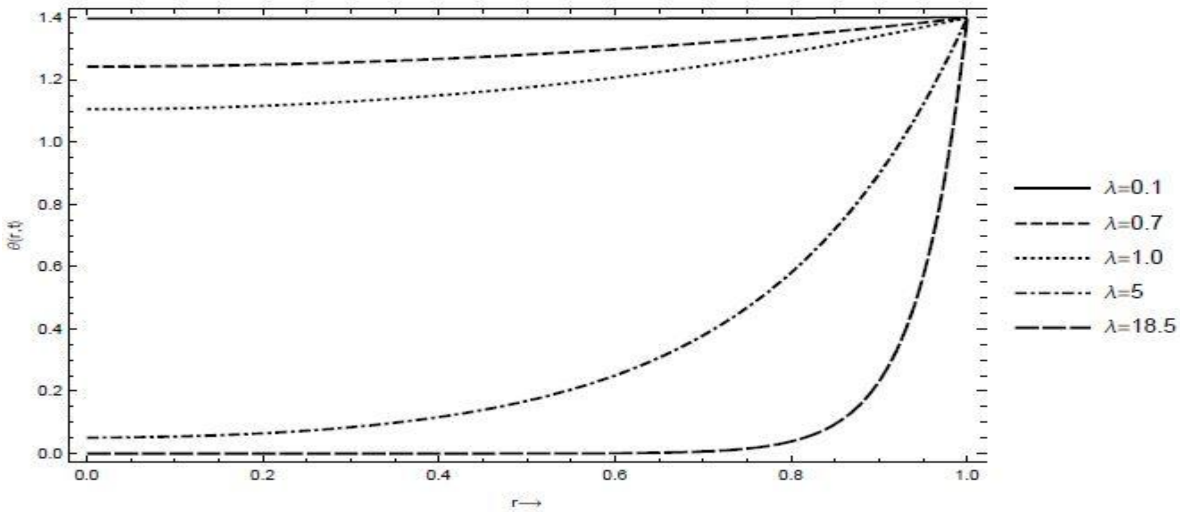

Fig. 2. The effect of cooling parameter on temperature profile with other values
$\varepsilon = 0.4, \theta_a = 1$

Fig. 2 shows the effect of cooling parameter on temperature profile. The illustration agrees with Nwaigwe and Amadi's result as an increase in the cooling parameter results in a decrease in the temperature of the body. This is as a result of a decrease in the surface tension of the fluid. For example, if hot water is placed in refrigerator, the temperature continues to drop as it cools.

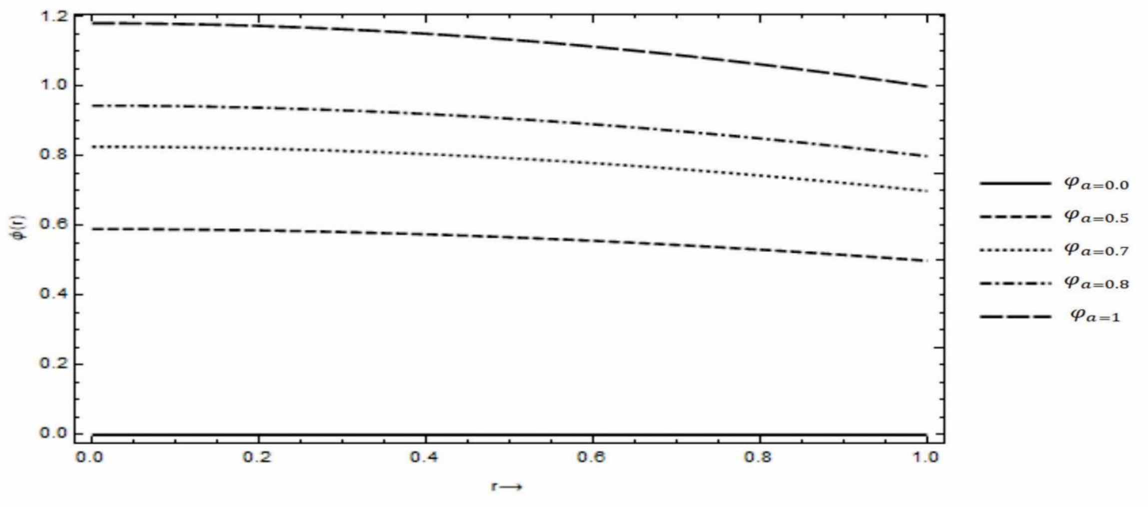

Fig. 3. The effect of boundary parameter on concentration profile with other values
$\varepsilon = 0.4\ \alpha = 0.2$

Fig. 3 shows the effect of concentration boundary parameter on concentration profile. It was observed that an increase in boundary parameter resulted to a decrease in concentration profile.

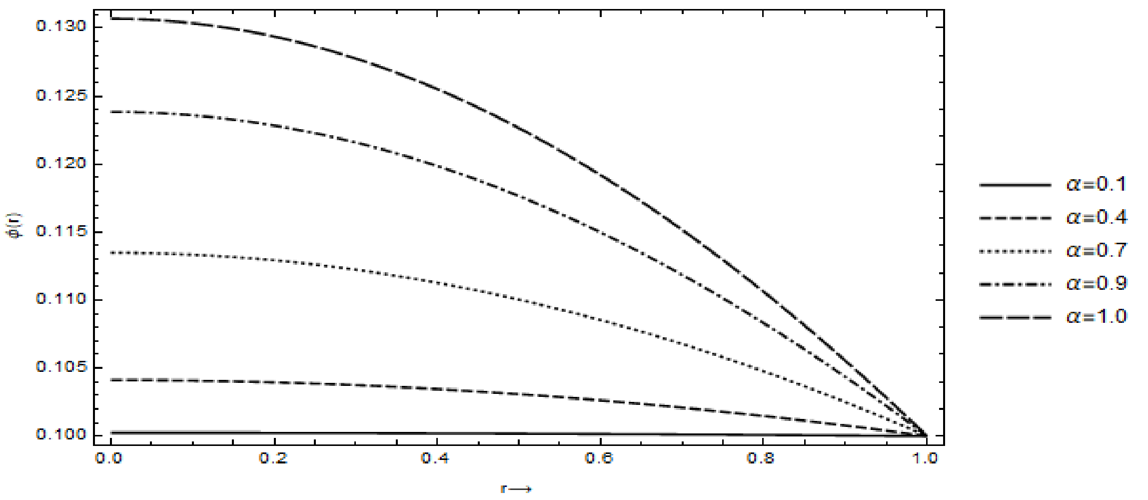

Fig. 4. The effect of injection parameter on concentration profile with other values
$\theta_0 = 0.1,\ \varepsilon = 0.4,\ \alpha = 0.2$

Fig. 4 shows the effect injection parameter on concentration profile. It was observed that an increase in the concentration injection parameter resulted to an increase in concentration profile.

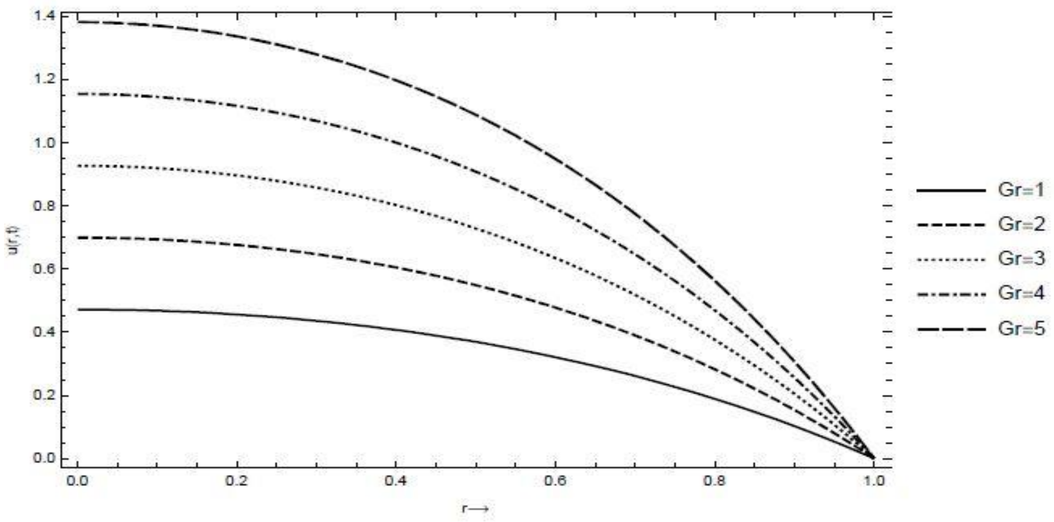

Fig. 5. The effect of Grashof Number on velocity profile with other values
$Gr = 1, \theta_a = 2, P_0 = 1, M = 1.5, \varepsilon = 0.4$

Fig. 5 shows the effect of Grashof number on velocity profile. It was observed that an increase in Grashof temperature number results in an increase in the velocity of the body. According to Amos and Omamoke [15], as the plate is cooled with convection currents, there is an increase in velocity.

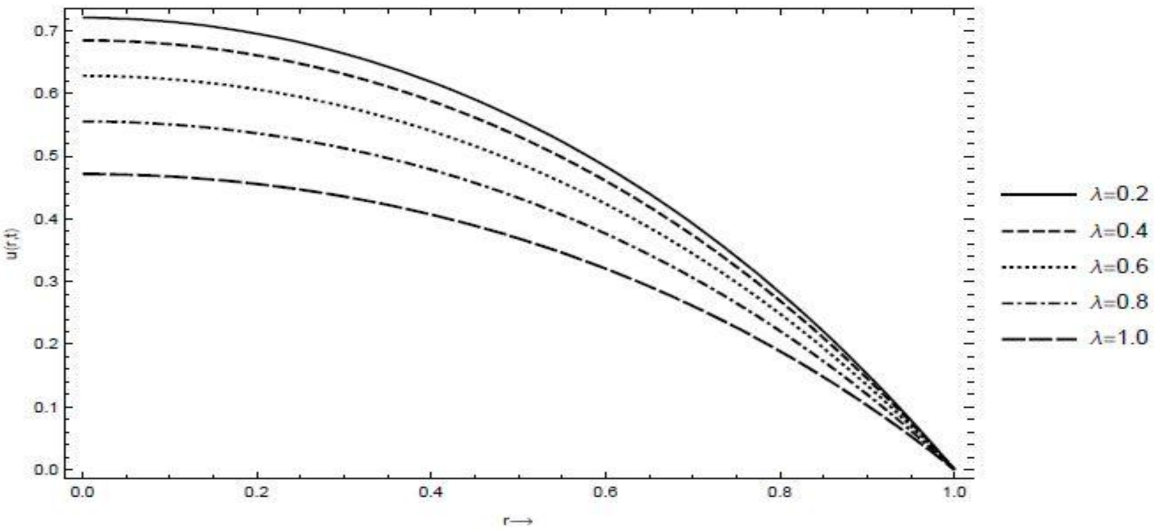

Fig. 6. The effect of cooling parameter on velocity profile with other values
$Gr = 1, \theta_a = 2, P_0 = 1, M = 1.5, \varepsilon = 0.4$

Fig. 6 shows the effect of cooling parameter on velocity profile. It was observed that as the cooling parameter increases, it results in a decrease in the velocity profile of a body. This is because an increase in viscosity of the fluid has been caused by an increased internal frictional force of that fluid.

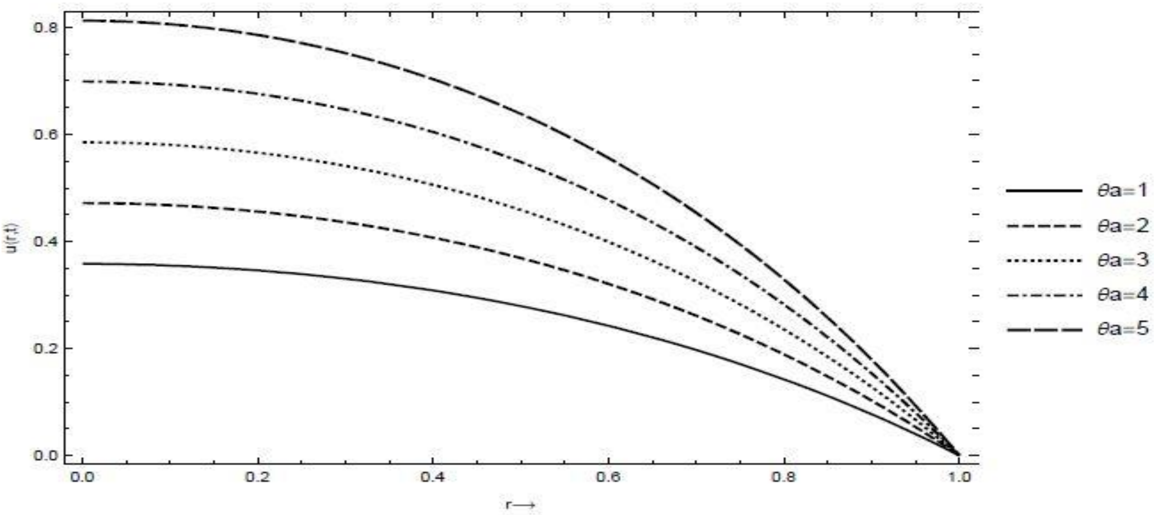

Fig. 7. The effect of wall temperature on velocity profile with other values
$$Gr = 1, \lambda = 4, P_0 = 1, M = 1.5, \varepsilon = 0.4$$

Fig. 7 shows the effect of temperature boundary parameter on velocity profile. It was observed that an increase in temperature boundary parameter of a cylindrical body causes an increase in the velocity profile of a fluid. This is because heating is applied to constricted tissue to cause a temperature increase in the area and increase flow. This is in line with Bunonyo and Amos [16].

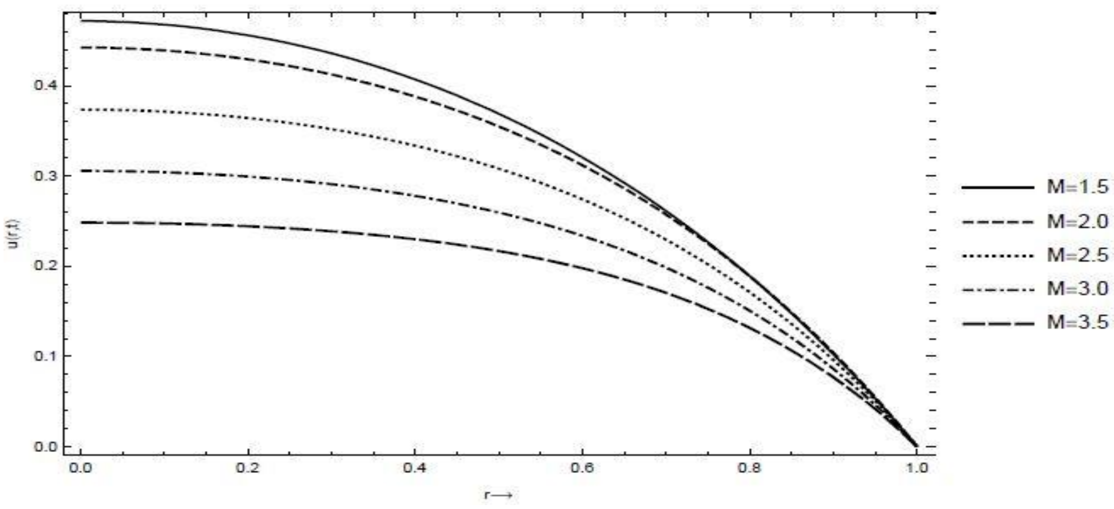

Fig. 8. The effect of Magnetic Field on velocity profile with other values
$$Gr = 1, \lambda = 1, P_0 = 1, \theta_a = 1, \varepsilon = 0.4$$

Fig. 8 shows the effect of magnetic field on velocity profile. It was observed that an increase in magnetic field results in decrease in the velocity of the fluid. This is because as the magnetic field increases, a Lorentz force is introduced and this inhibits the flow of the fluid. This is in line with Nwaigwe and Amadi [14], Omamoke and Amos [17], Bunonyo and Amos [16], Baoku et al. [18].

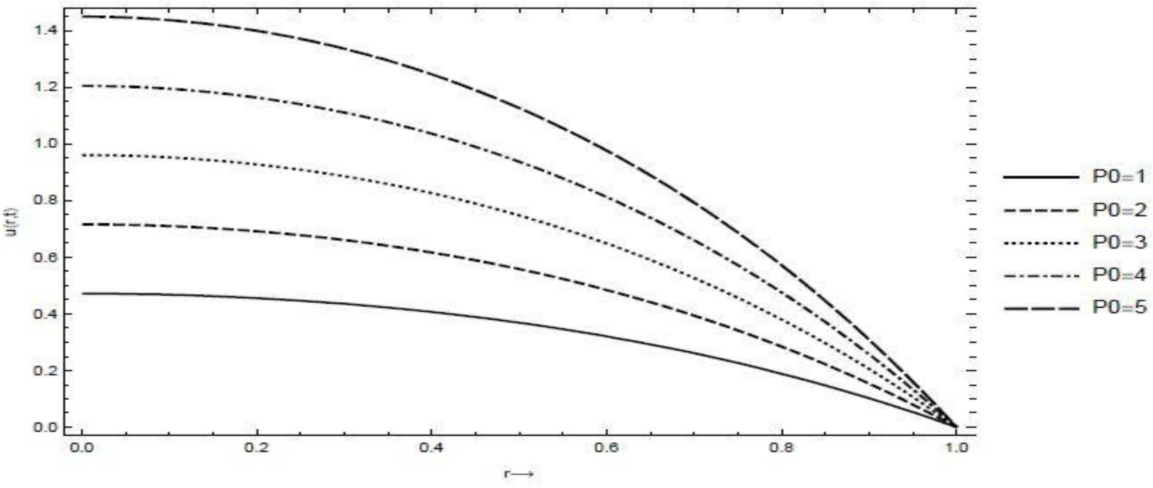

Fig. 9. The effect of pressure parameter on velocity profile with other values
$Gr = 1, \lambda = 1, M = 1.5, \theta_a = 1, \varepsilon = 0.4$

Fig. 9 shows the effect of pressure parameter on velocity profile. It was observed that an increase in pressure gradient (steady or pulsatile) results in an increase in velocity profile of the fluid in a cylindrical channel. This is in agreement with Nwaigwe and Amadi [14].

5 Conclusion

This study has shown that Laplace transform can be used to solve a system of partially coupled differential equations in a Cylindrical Domain. The Non-linear coupled partial differential equations of velocity, temperature and concentration in its dimensionless form was first reduced to a set of Ordinary Differential Equations by applying regular perturbation technique and solve using Laplace transform. The results obtained with illustrations and graph revealed that:

i. An increase in the temperature boundary parameter increases the velocity of the fluid flowing through the cylindrical channel.
ii. An Increase in the cooling parameter decreases the velocity of the fluid flowing through the cylindrical channel but the reverse was the case in increases of Grashof temperature number.
iii. An increase in magnetic field reduces the velocity of the fluid flowing through the cylindrical channel as a result of increases in the fluid viscosity.
iv. An increase in concentration injection parameter increases the concentration profile of the fluid
v. An Increase in pressure gradient (steady or pulsatile) the velocity of the fluid flowing through the cylindrical channel.

Competing Interests

Authors have declared that no competing interests exist.

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