A Comparative Study on the Exercises of "Linear Equation with One Unknown" in the Middle School Mathematics Textbooks between China and the United States Based on Core Literacy

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Authors' contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

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Abstract

With the introduction of the "Chinese Compulsory Education Mathematics Curriculum Standard" in 2022, the state attaches more importance to the cultivation of students' core literacy in mathematics, As an important tool for cultivating students' abilities, textbook exercises play a crucial role in cultivating students' core literacy. In the context of globalization, as a major national project, education should also strengthen international exchange and cooperation, and learn from the advanced experience of other countries. As a fundamental concept in mathematics, equations have a wide range of applications in daily life, it plays an

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important role in disciplines such as physics, economics, engineering, and biology. The linear equation with one unknown is the simplest equation in mathematics, and students' learning of linear equation with one unknown can cultivate their logical thinking and problem-solving abilities, laying a foundation for future algebra learning. Based on this, we selects the exercises in the chapter "Linear Equation with one unknown" of the Virginia edition of the U.S. junior high school mathematics textbook and the three Chinese mathematics textbooks editions, Zhejiang Education edition (ZJE), People's Education Press edition (PEP), and Beijing Normal University edition (BNU), as the objects of study, the types and levels of mathematical core literacy in the four editions of textbook exercises were compared. Through the comparative study, it is found that different editions of the textbooks focus on different types of mathematical core literacy, the exercises in the four editions of the textbooks pay more attention to mathematical arithmetic literacy and less attention to mathematical modeling literacy; the distribution proportion of the levels of the mathematical core literacy embodied in the exercises of each edition of the textbooks is different, but each edition focuses on different levels, and the levels of the core literacy embodied in each edition of the textbooks are in the relatively basic stage.

1 Introduction

To face the Industrial Revolution 4.0, citizens should have skills in information, learning and innovation, life, and communication. The content of 21st-century learning has to always adapt to any changes that might happen in the industrial era [1]. The Mathematics Curriculum Standards for Compulsory Education (2022 Edition) (hereinafter referred to as the New Curriculum Standards) points out that at the present stage of compulsory education, it is necessary to focus on inheriting the successful experience of China's curriculum construction, but also to draw on the international advanced education concepts, and further deepen the curriculum reform. Strengthening the comprehensiveness and practicability of the curriculum, promoting the reform of parenting methods, and focusing on the development of students' core qualities [2]. As the main carrier of curriculum objectives, the textbook is the main tool and resource for teachers and students to carry out teaching and learning [3]. As an important part of the textbook, exercises have the functions of digesting and consolidating new knowledge, expanding and extending new knowledge, penetrating ideas and methods, and educating people [4]. Through the literature review, it is found that in the comparative study of mathematics teaching materials, the comparison of core literacy focuses more on the comparison of old and new teaching materials and the comparison between the current editions in different regions of China, while there are fewer studies on the comparison of core literacy among different countries. At this stage, the process of globalization is accelerating, We should actively participate in international cooperation in the process of Chinese modernization, and we should learn from the advanced education concepts of other countries, seek common ground while reserving differences, and incorporate the best of both worlds. Based on this, this paper starts from the requirements of the curriculum standard on the content of quadratic equations, and selects BIG IDEAS MATH 8 (hereinafter referred to as the Virginia edition) and the textbooks of ZJE, PEP, and BNU as the objects of study, and compares the chapter of "Linear Equation with one unknown" in the four editions of the textbooks. We compare the types and levels of core literacy in the chapter "Linear Equation with one unknown" in the four editions of the textbook, analyze the focuses of the core literacy types and levels of the four editions, and explore the differences and similarities in the focuses of the core literacy in mathematics among different countries and regions of the same country.

2 Methodology

2.1 Subjects of study

In this paper, we select the chapter "Linear Equation with one unknown" in the middle school mathematics textbook of Virginia edition of the United States, and the middle school mathematics textbooks of ZJE, PEP, and BNU editions of China for a comparative study. Linear equation with one unknown are the first equations encountered in junior high school mathematics, and they are also the basis for the future study of systems of quadratic equations, systems of cubic equations, fractional equations, and quadratic equations. Whether it is multivariate equations or higher equations, the core of them is solving them by "eliminating" or "reducing" them

into linear equation with one unknown, which shows the importance of linear equation with one unknown in studying equations. Therefore, the content of quadratic equations is chosen as the object of study. This paper is mainly based on the contents of the chapter on "linear equation with one unknown" in the ZJE edition as a reference. It includes the concept of quadratic equations, the basic properties of equations, the solution of quadratic equations, and the application of quadratic equations.

2.2 Research framework

In this paper, through reading the requirements of the New Curriculum Standards of "linear equation with one unknown", it is determined that the content of the chapter on "linear equation with one unknown" mainly embodies abstract ability, arithmetic ability, reasoning ability, modeling concepts, and application consciousness, creativity consciousness. The level of core literacy is mainly divided by the level division framework of the New Curriculum and Yu Ping's "A Framework for Evaluating Mathematical Core Literacy" as a reference. Taking the model concept as an example of literacy, level one requires the ability to understand that a linear equation with one unknown is the abstract result of a linear equation with one unknown of a variety of single unknown quantities, and the ability to solve problems by using the existing model; level two requires the ability to choose the appropriate linear equation with one unknown and construct a concrete model to solve the problem according to the conditions of the actual problem. Level 2 requires the ability to select appropriate linear equation with one unknown according to the conditions of a real problem and construct a concrete model to solve the problem. Level 3 requires the ability to apply knowledge of linear equation with one unknown to mathematical modeling in interdisciplinary integrative and practical activities.

2.3 Classification of studies

Classify the six core mathematical literacies (abstraction ability, arithmetic ability, reasoning ability, modeling concepts, application consciousness, and creativity consciousness) reflected in the chapter "Linear Equation with one unknown". The types of problems include exercises, problems, review questions, post-course problems, and review questions. The following provisions were made: (1) for questions containing several sub-topics, they were counted as one topic; (2) if an exercise embodied multiple core literacies, the number of types of core literacies embodied was increased by one; (3) if a topic embodied the level of multiple core literacies, it was counted according to the highest level of each core literacy.

2.3.1 Classification of exercises for math core literacy types

Example of abstraction ability: A balance is balanced with 3 ping-pong balls on the left side of the balance and a 5.4 g weight and 1 ping-pong ball on the right side. If the mass of one ping-pong ball is x (g), write an equation for x [5]. This problem of listing equations about x requires abstracting the physical context of the balance into a mathematical problem, seeking a relationship between the masses on both sides of the balance, and being able to express this using known mathematical notation, which fits the connotative description of mathematical abstraction and therefore falls into the Abstracting Competency Literacy category.

Example of arithmetic ability: Solve the following equation: $4(4-y)=3(y-3)$ [5]. To get the correct answer, the first step is to understand the basic properties of the equation, and through the basic properties of the equation, design the solution process, and finally get the result, which is in line with the connotation of mathematical operations, and therefore belongs to the type of arithmetic literacy.

Example of reasoning ability: Take a 2×2 square from a month's calendar table, and know that the sum of the dates in the four squares enclosed by this square is 44, find the dates in these four squares [4]. To solve this problem, you first need to understand the pattern of arranging the dates on the calendar, and then use deductive reasoning to solve a specific calendar problem in conjunction with the content of the linear equation with one unknown, which is consistent with the description of the connotation of logical reasoning, and therefore belongs to the type of reasoning ability literacy.

Example of modeling concepts: A product is priced at a 30% increase in cost, and then sold at a 20% discount for ¥208. What is the cost of this product? [5] This question requires students to solve real-life problems based on mathematical perspectives, how to find the cost of goods, according to the question posed by the problem,

analyze the problem, build a model of one-variable equations to solve, and verify, and ultimately achieve the goal of solving the problem, in line with the connotation of mathematical modeling, and therefore belongs to the type of modeling concept literacy.

Example of application consciousness: On February 9, 2011, the state announced that the annual interest rate for a two-year lump sum savings deposit is 3.90%, and the interest is exempt from tax. Known that a depositor deposited two years after maturity to obtain the sum of the principal and interest for 3234 yuan, ask the depositor to deposit the principal how much? [5] This question requires students to apply the knowledge of one-variable equations to solve real-world problems, applying mathematical knowledge to real life, enhancing the ability of students to apply mathematical knowledge in life, in line with the connotation of application awareness, and therefore belongs to the application awareness literacy model.

Example of creativity consciousness: Make up a real-world application problem that requires the listed equation to be [5]. This problem requires students to understand the equation, create the situation of the problem, change from being the doer to the maker of the problem, and experience the use of linear equation with one unknown to solve real-world problems from different perspectives. The innovative mode of problem-solving is in line with the meaning of creativity and therefore belongs to the type of creativity literacy.

2.3.2 Classification of exercises at the math core literacy level

Level example: Figure, a cylindrical glass container with water inside the bottom radius of 10cm, the height of the container of water is 12cm, and a glass rod with a radius of 2cm is inserted vertically into the water, ask how many centimeters the container of water will rise (assuming that the water will not overflow)? [5] This problem requires the glass rod as a physical background abstracted into a mathematical problem, seeking the relationship between the volume of the glass rod and the height of the water changes, so belongs to the abstraction ability of level two; Secondly, students need to analyze the use of mathematical thinking, according to the knowledge of the learning of linear equation with one unknown, to find the equivalence in the problem, creatively build a model of linear equation with one unknown, solve the problem, so belongs to the modeling concept of level three.

2.4 Description of core literacy statistics

Notes on the statistics of the types of core literacy in mathematics: (1) Analyze the types of core literacy reflected in the problems of the chapter "Linear Equation with one unknown" in the four editions of the textbook, and calculate the proportion of each type of literacy in the total number of problems in each edition. (2) Summarize the total number of topics and the total number of core literacy topics in each edition of the textbook. Note on the statistics of core literacy levels in mathematics: Analyze the core literacy levels of the exercises in the chapter "Linear Equation with one unknown" in the four editions of the textbook, and calculate the proportion of each core literacy level in the total number of topics in each edition of the textbook.

3 Results

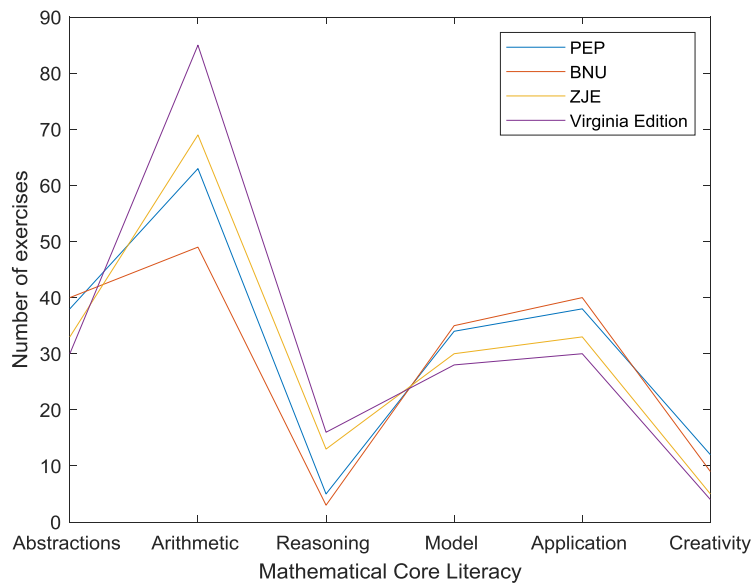## 3.1 Statistics of core literacy types

Table 1 shows the statistics of the exercises of "Linear equation with one unknown" in the four editions of the textbook on each type of core literacy.

Draw the corresponding line statistical chart according to Table 1 for comparative analysis, as shown in Fig. 1.

From the total number of exercises in the four editions, it can be seen that the Virginia edition ranks first, the PEP ranks second, the ZJE ranks third, and the BNU ranks last. The total number of exercises in the Virginia edition is more than that in the three editions of Chinese textbooks studied. The survey found that this is not an accident, the other editions commonly used in the United States are relatively large in the total number of exercises, and a large number of exercises is a common feature of all editions of American math textbooks [6]. According to the literature review, in the design of problem levels, the American textbooks emphasize the consolidation of knowledge through imitation and divergence, while the Chinese textbooks pay more attention to comprehensiveness and complexity.

Table 1. The total number and percentage of exercises of core literacy types in the four editions of textbooks

Textbook Editions	Abstractions	Arithmetic	Reasoning	Model concept	Application	Creativity	Total number
PEP	38	63	5	34	38	12	75
	50.7%	84.0%	6.7%	45.3%	50.7%	16.0%	
BNU	40	49	3	35	40	9	53
	75.5%	92.5%	5.6%	66.0%	75.5%	17.0%	
ZJE	33	69	13	30	33	5	74
	44.6%	92.0%	17.1%	39.0%	42.3%	6.3%	
Virginia Edition	30	85	16	28	30	4	87
	34.5%	97.7%	18.4%	32.2%	34.5%	4.6%	

Fig. 1. Statistics of the total number of exercises for core literacy types in the four editions of textbooks

In addition, in terms of the total number of core literacy questions, the Virginia edition has the most, 193 questions, while the BNU edition has the least, 176 questions. The number of core literacy questions in the four editions is similar, which is mainly affected by the total number of exercises.

This shows that American textbooks also pay attention to the cultivation of the core literacy of mathematics. In fact, the United States also has in-depth research on core literacy [7]. For example, the National Institute of Research states that mathematical literacy consists of five basic elements: first, conceptual understanding; Second, the process is smooth; Third, strategic ability; Fourth, value tendency; Fifth, appropriate reasoning.

3.2 Analysis of the types of core competencies

According to the requirements of the "New Curriculum Standard" on equations, equations pay attention to cultivating students' abstract ability, arithmetic ability, reasoning ability, model concept, application consciousness, and creativity consciousness. Based on this, the following analysis is carried out.

3.2.1 Exercises attach more importance to mathematical arithmetic ability accomplishment, but less importance to creativity consciousness accomplishment

As can be seen from Table 1, the four editions all attach importance to the cultivation of students' mathematical arithmetic ability literacy. The number of exercises containing arithmetic literacy is at the forefront of all types

of literacy questions, accounting for 84.0%, 92.5%, 92.0%, and 97.7% of the total number of exercises respectively, which indicates that all editions pay full attention to students' arithmetic literacy. Pay attention to excavating the depth of students' operational thinking, the plasticity of operational methods, and the creativity of operational ability in exercises. The exercises containing arithmetic literacy, also reflect the test of other literacy, indicating that the textbooks of each edition pay attention to the complementary relationship between mathematical arithmetic literacy and other core literacy, and also reflects that mathematical arithmetic literacy is the basis for the development of other literacy, and training arithmetic literacy can also provide guarantee for the development of other core literacy.

As can be seen from Table 1, the problems of the four editions reflect the creativity consciousness and literacy to a low degree, accounting for the least proportion of the core literacy problems of each edition, which are 16.0%,17.0%,6.3%, and 4.6% respectively. On the one hand, it is due to the content limitation of this chapter, and on the other hand, it also reflects that the emphasis of each edition on creativity consciousness and literacy needs to be strengthened.

The exercises in the four editions of textbooks also reflect the mathematical model concept is relatively small. It shows that the textbooks tend to regard this stage as the stage of consolidating the foundation, and do not make too many requirements on students' ability in various aspects.

3.2.2 Each edition has its own characteristics in the focus of core literacy types

As can be seen from Table 1, from the horizontal comparison, it can be seen that the BNU edition pays more attention to the balanced development of core literacy, the variance of the number of literacy questions is 292, among which the proportion of mathematical abstraction, model concept, and application awareness is not much different, while the other three editions show different performance in the proportion of literacy. The variance of the number of literacy questions in the Humanistic, Zhejiang, and Virginia editions is 361,408,645, respectively. From the vertical comparison, it can be seen that the Humanistic edition and Beijing Normal University edition pay attention to the cultivation of model concept literacy, while the Zhejiang edition and Virginia edition pay attention to the cultivation of reasoning ability literacy.

3.3 Level division of core literacy

3.3.1 Level division basis

By consulting a large number of data, we find that the existing division of mathematics core literacy level is more targeted at high school mathematics learning. Such a division framework lacks certain pertinence and direct guiding significance for the comparative study of junior high school mathematics textbooks. This article is based on Yu Ping's "Framework of Evaluation about Mathematics Key Competencies"[8] and Chandler K et al.'s "What should Common Core assessments measure"[9] as references, and takes the level division from the knowledge points as the starting point. This article divides knowledge learning into three levels: knowledge understanding, knowledge transfer, and knowledge creativity. Core literacy comes from knowledge, so the three levels of knowledge learning are used as the evaluation criteria of mathematics core literacy.

This article takes the knowledge of the chapter on linear equation with one unknown as an example, and analyzes it in combination with the level division framework of the "Compulsory Education Math Curriculum Standards"[10] and Yu Ping's "Framework of Evaluation about Mathematics Key Competencies". Six core literacies of Abstraction ability, Arithmetic ability, reasoning ability, Modeling concepts, Application consciousness, and Creativity consciousness highlighted in this chapter are divided into levels, and provide the basis for dividing each level. show as shown in Table 2.

3.3.2 Statistical results of core literacy levels

Table 3 shows the classification of core literacy levels for the four editions of the linear equation with one unknown exercises.

Based on the core literacy level statistics of the exercises, draw radar charts of each core literacy level, and visually display the level of different core literacy in four different editions of the textbooks, as shown in Figs. 2, 3, 4, 5, 6, 7.

Table 2. Core literacy level division table of the linear equation with one unknown

Core literacy	Level 1	Level 2	Level 3
Abstraction ability	Experience the process of equation generation and be able to understand the relevant concepts of linear equation with one unknown	Able to describe real objects using linear equation with one unknown and abstract the concept of linear equation with one unknown	Able to solve open-ended problems related to linear equation with one unknown, explore general laws of linear equation with one unknown, and establish connections between other mathematical knowledge
Arithmetic ability	Master the basic mathematical operations of linear equation with one unknown	Able to solve problems using linear equation with one unknown in real-life	Able to solve some open and comprehensive problems related to the operation of linear equation with one unknown and preliminary understanding of the significance of algorithmic ideas in mathematical operations
Reasoning ability	Understand the meaning of symbolic language in linear equation with one unknown, understand the meaning and structure of propositions in linear equation with one unknown, and understand the logic of mathematical language	Able to understand the deductive reasoning process of linear equation with one unknown, and infer methods for solving linear equation with one unknown through the properties of equations	Able to establish logical connections between linear equation with one unknown and other mathematical knowledge through reasoning
Modeling concepts	Understanding linear equation with one unknown is an abstract result of various single unknown first-order equations, and utilizing existing models to solve problems	Able to select appropriate linear equation with one unknown based on the actual conditions of the problem, and construct specific models to solve the problem	Able to apply knowledge of linear equation with one unknown for mathematical modeling in interdisciplinary synthesis and practical activities
Application consciousness	Realizing that mathematical knowledge of linear equation with one unknown can solve problems in mathematics itself, and being able to generalize and summarize them	Understand the real-life prototype of linear equation with one unknown and solve meaningful practical problems in reality	Recognizing the application of linear equation with one unknown in other disciplines, interdisciplinary synthesis and practical activities
Creativity consciousness	Willing to explore conjectures and problems of linear equation with one unknown	Able to think about the connection between real-life problems and linear equation with one unknown	Able to think independently and willing to try different methods and strategies to solve problems with linear equation with one unknown

Table 3. Classification results of core literacy level for linear equation with one unknown

Textbook edition	Abstraction ability			Arithmetic ability			Reasoning ability			Modeling concepts			Application consciousness			Creativity consciousness		
	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
ZJE	11	2	0	33	25	5	12	4	0	1	3	0	5	27	0	1	4	3
PEP	4	11	0	17	41	4	6	4	0	1	1	0	7	53	0	0	5	0
BNU	2	3	1	14	33	3	4	2	0	2	3	0	3	37	0	0	4	0
Virginia	5	1	4	38	30	2	11	2	1	8	1	0	6	31	1	0	1	1
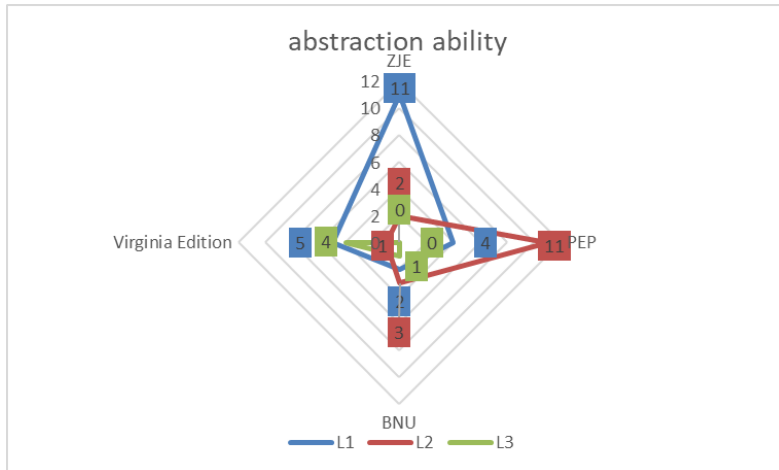
Fig. 2. Abstraction capability level

Through analysis, it can be found that in terms of abstraction ability, the four editions of textbooks have different emphases, and the proportion of each level is quite different.

From different levels, for abstraction ability literacy, at level 1, the PEP has the highest number of questions, followed by the BNU, ZJE, and Virginia Edition. The last three editions of textbooks all reflect very little level one - abstraction ability literacy; In terms of Level 2, the PEP also has the highest number of exercises, followed by the BNU, ZJE, and Virginia Edition. The last three editions of textbooks have very few exercises that reflect Level 2; In terms of Level 3, the Virginia Edition has the highest number of questions, followed by the BNU, while the PEP and ZJE hardly reflect Level 3.
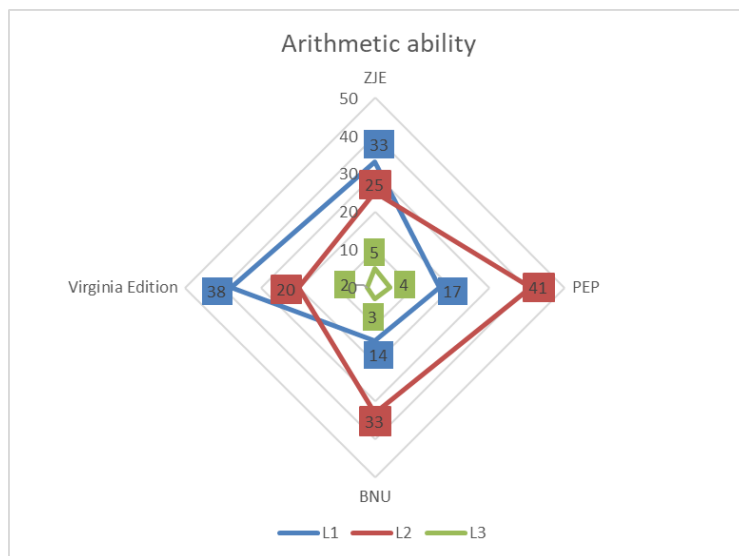

Fig. 3. Arithmetic ability level

In terms of the number of exercises reflecting the core literacy of Arithmetic ability, there is not much difference in the four editions of textbooks in terms of Arithmetic ability. Level three only accounts for a small part, while the number of exercises reflecting level one and the emphasis on level two differ.

From different levels, for level 1 of Arithmetic ability, the Virginia Edition of the textbook reflects the highest number of exercises, next is the ZJE, followed by the PEP, and BNU reflects the least; In Level 2, the PEP has the highest number of exercises, while the BNU has more exercises than the ZJE, and the Virginia Edition has the lowest number of exercises; In level 3, ZJE has the highest number of exercises, followed by the PEP, followed by the BNU, and the Virginia Edition has the lowest number of exercises.
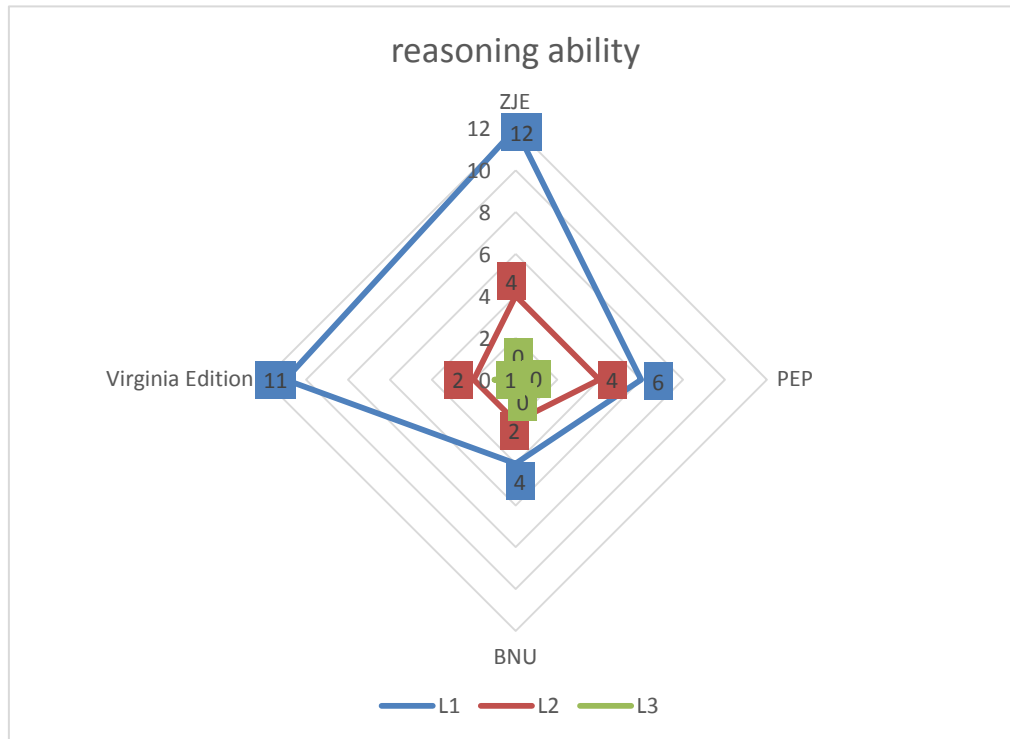
Fig. 4. Reasoning ability level

From the number of questions reflecting the core literacy of reasoning ability reflected in the exercises, it can be seen from Fig. 4 that in terms of reasoning ability, the proportion of different levels in the four editions varies. Among them, the proportion of levels in the Virginia edition and ZJE edition is relatively consistent, while the proportion of levels in the PEP and BNU is relatively consistent.

From different levels, for level 1 of reasoning ability, ZJE has the highest number of questions, followed by the Virginia Edition, followed by the PEP, and the BNU has the lowest number of questions; In Level 2, ZJE and PEP have the highest number of questions, while the Virginia Edition and BNU Press have the lowest number of questions; In level 3, only the Virginia Edition is reflected, while the other three editions of textbooks are not reflected.

From the number of exercises reflecting the core literacy of modeling concepts, it can be seen from Fig. 5 that in terms of modeling concepts literacy, except for the Virginia Edition, the other three editions of textbooks only reflect a small amount of modeling concepts literacy.

From different levels, at level one, the Virginia edition has the highest number of questions, followed by the Beijing Normal University edition, while the People's Education Press and Zhejiang Education Press have the lowest number of questions; In Level 2, Zhejiang Education Press and Beijing Normal University Press have the highest number of questions, while Virginia Press and People's Education Press have the lowest number of questions; For level three and four editions of textbooks, they are not reflected.
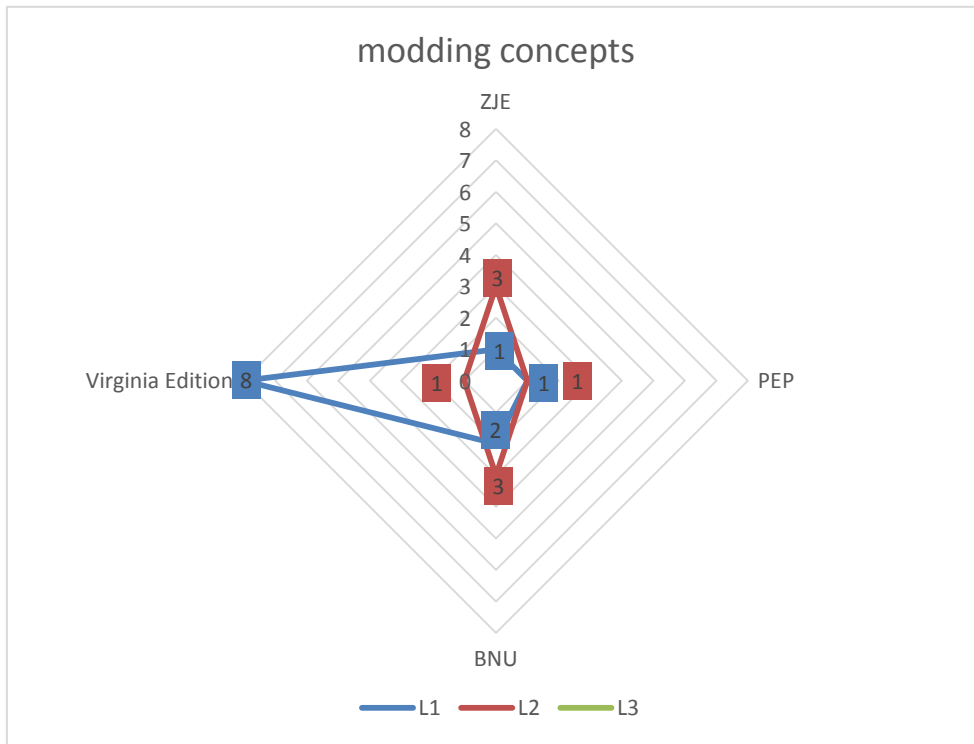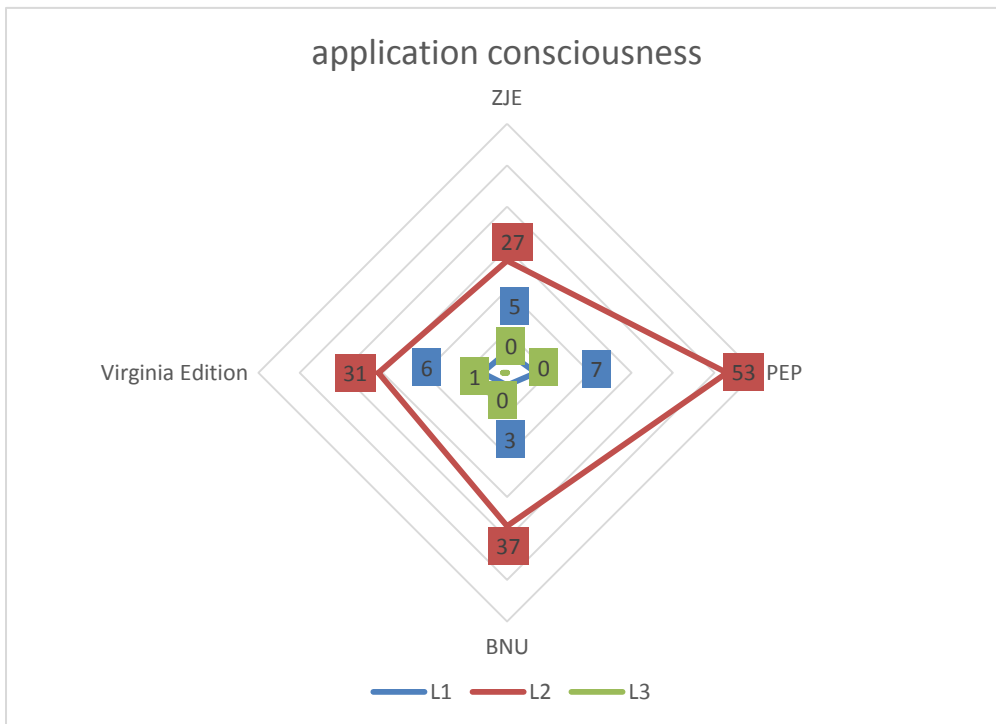
Fig. 5. Modeling concepts level

Fig. 6. Application consciousness level

In terms of the number of questions reflecting the core literacy of application consciousness in the exercises, the exercises in the four editions of the textbook all reflect a considerable level of application consciousness literacy, and the number of questions reflecting Level 2 is greater than Level 1.

Overall, the PEP has the highest number of questions reflecting the core literacy of application consciousness, followed by the BNU, followed by the Virginia Edition, and the ZJE has the lowest number of questions.

From different levels, at level 1, the PEP and ZJE have the highest number of questions, followed by the Virginia Edition and ZJE, while the BNU has the lowest number of questions; In Level 2, the PEP has the highest number of questions, followed by the BNU, followed by the Virginia Edition, while the ZJE has slightly fewer questions than the Virginia Edition; At level 3, only the Virginia Edition is reflected, and the other three editions of textbooks are not reflected.
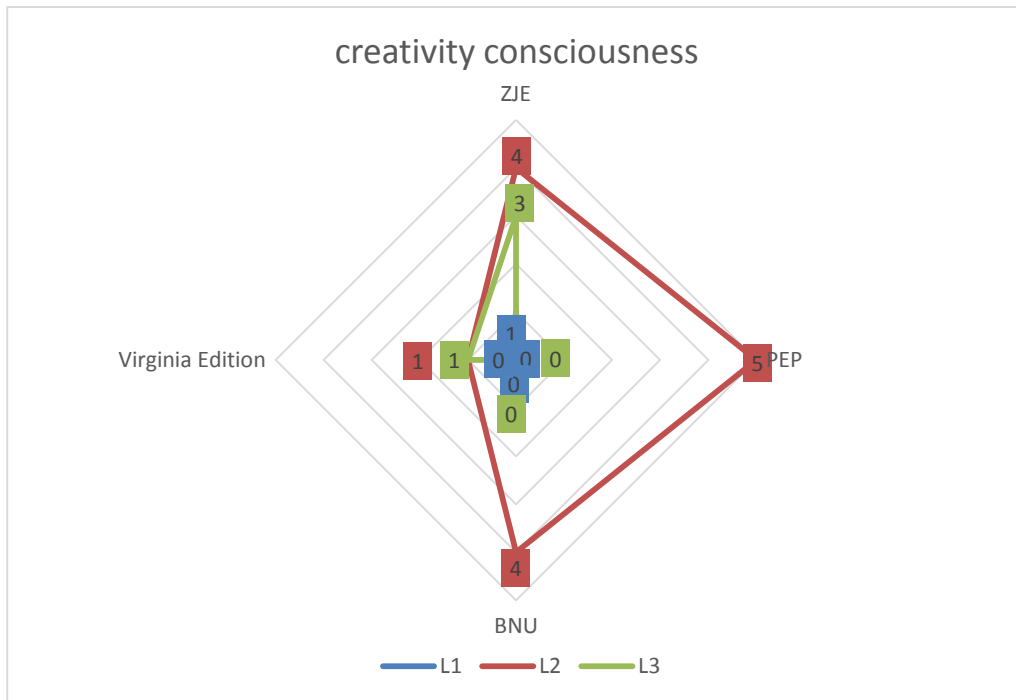

Fig. 7. Creativity consciousness level

From the number of exercises reflecting the core literacy of creativity consciousness, it can be seen from Fig. 7 that there are not many exercises in the four editions of textbooks that can reflect the core literacy of creativity consciousness.

Overall, ZJE has the highest number of questions reflecting creativity consciousness and core literacy, followed by the PEP, followed by the BNU, and the Virginia Edition has the lowest number of questions.

From different levels, at level 1, only the exercises in ZHE are reflected, while the other three editions of textbooks do not; In Level 2, the PEP has the highest number of questions, followed by ZJE and BNU, while the Virginia Edition has the lowest number of questions; At level three, only the ZJE and Virginia Edition have been reflected, with the ZJE reflecting a larger number of topics, while neither of the other two textbooks has been reflected.

Based on Level 1, compare the differences in the six core literacies of the four editions of the textbook, as shown in Fig. 8.

According to Fig. 8, it can be found that arithmetic ability is the core literacy that the fourth edition of the textbook mainly cultivates for students. The number of exercises in the Virginia Edition in the United States that reflect the modeling concepts is much higher than that in the third edition of the textbook in China. In terms of the total number of exercises, the Virginia Edition and ZJE edition reflect a higher number of exercises at level 1.
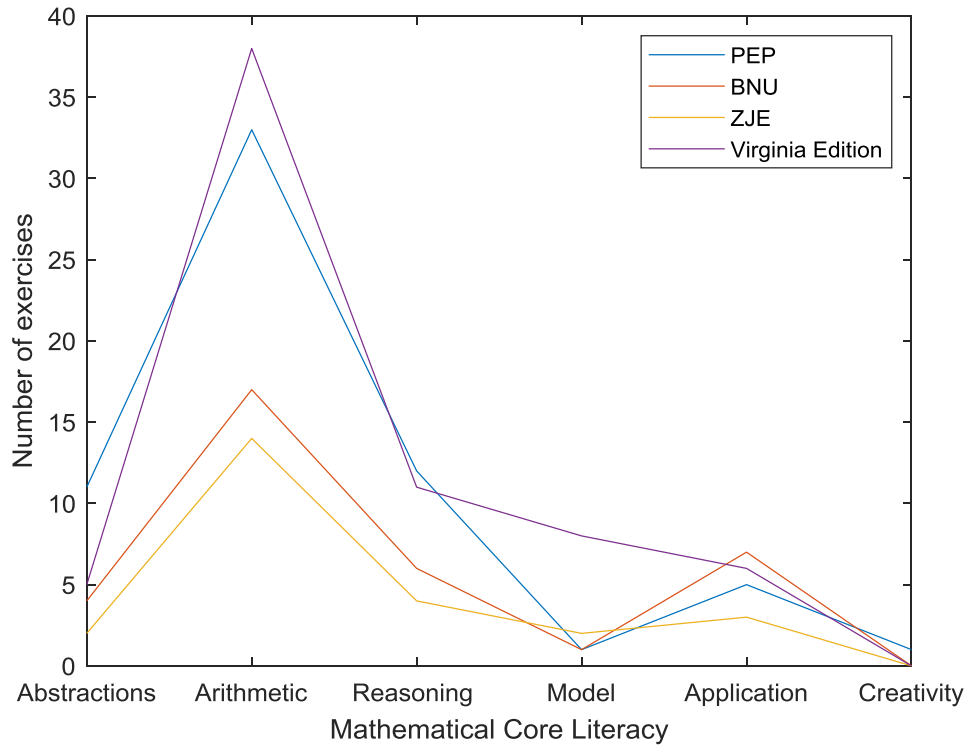
Fig. 8. Statistical chart of core literacy level 1 exercises in the fourth edition textbook

Based on Level 2, compare the differences in the six core literacies of the four editions of the textbook, as shown in Fig. 9.
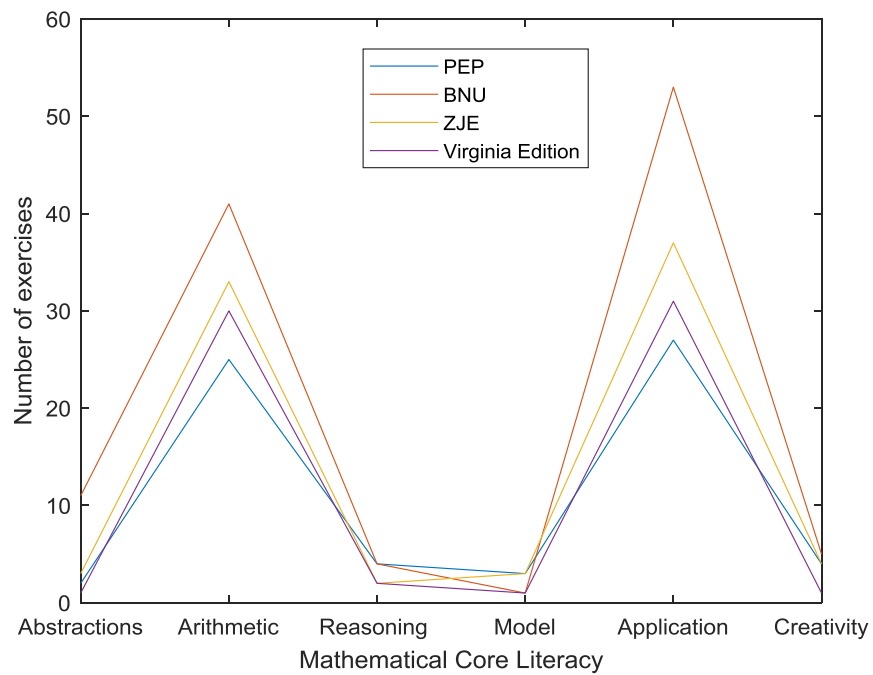
Fig. 9. Statistical chart of core literacy level 2 exercises in the fourth edition textbook

According to Fig. 9, it can be observed that the proportion of exercises for Core Literacy Level 2 in each textbook is relatively consistent, with a focus on cultivating students' arithmetic ability and application consciousness. However, for the other four Core Literacy Level 2 exercises, there are fewer exercises set. In terms of the total number of exercises, the PEP exercises reflect Core Literacy Level 2 the most, followed by BNU, As for ZJE and Virginia Edition, which is completely different from the situation of Level 1.

Based on Level 3, compare the differences in the six core literacies among the four editions of the textbook, as shown in Fig. 10.
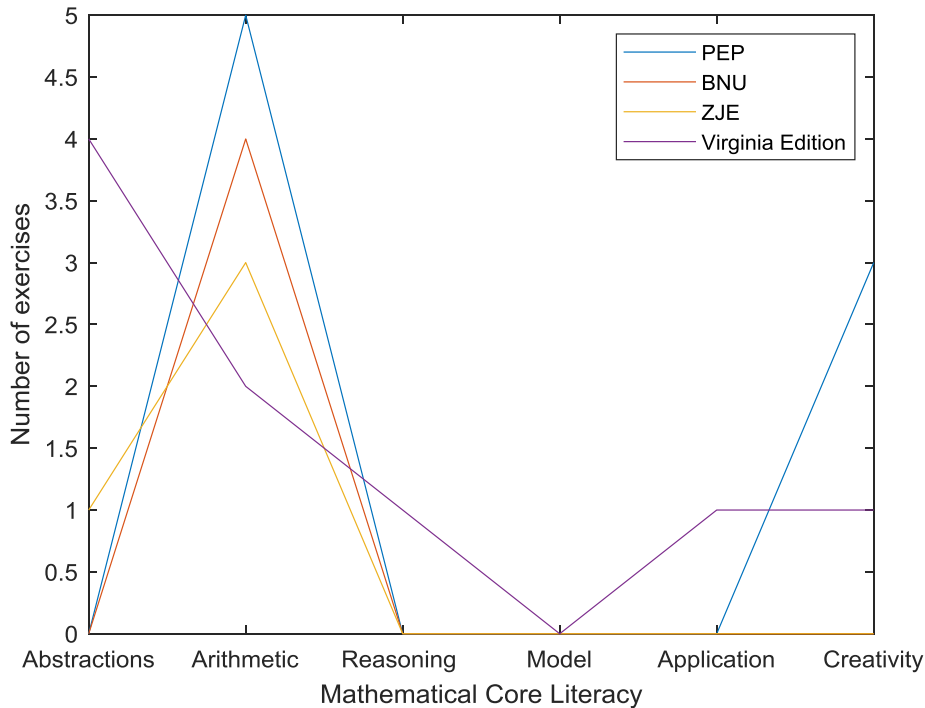
Fig. 10. Statistical chart of core literacy level 3 exercises in the fourth edition textbook

According to Fig. 10, it can be observed that the proportion of exercises set in the three editions of domestic textbooks that reflect the core literacy level 3 is relatively similar, with a focus on cultivating students' high level of computing ability. For the other core literacy levels three, there are not too many exercises set, while the ZJE sets some questions in the core literacy of activity consciousness, which is different from other textbooks, The Virginia Edition of foreign textbooks focuses more on the cultivation of abstraction ability level 3, while there are relatively few questions set for the other five core competencies. In terms of the total number of exercises, the number of exercises reflecting core literacy level 3 is far less than that of level 1 and level 2, indicating that exercises reflecting core literacy level 3 are generally difficult for students, so there are fewer questions set in each edition of textbooks.

4 Discussion

Based on the above statistical results, we analyze and evaluate the level of abstraction ability, arithmetic ability, reasoning ability, modeling concepts, application consciousness, and creativity consciousness reflected in the six core literacies of middle school mathematics in the four editions of textbooks.

4.1 Each edition of the textbook mainly reflects the basic level of core literacy

Through analysis, the exercises in the fourth edition of the textbook are generally more reflected in Level 1 and Level 2, with less or no involvement in Level 3.

For example, taking the modeling concepts as an example, the exercises in the fourth edition of the textbook reflect both Level 1 and Level 2, but none of the modeling concepts literacy is reflected in Level 3. For example, in the exercises in the fourth edition of the textbook, there is a type of question that provides a known model formula and requires students to use existing model formulas to solve mathematical problems, which reflects the requirements of Level 1. At the same time, the four editions of the textbook also have an activity exploration, Students are required to collect data on their own, choose appropriate methods, and use knowledge related to one-dimensional equations to establish appropriate mathematical models to solve problems, which reflects the requirements of Level 2.

Compared to this, level 3 of modeling concepts is not reflected in the exercises of the four editions of textbooks. According to the requirements of "Compulsory Education Math Curriculum Standards", the specific expression of modeling concepts in middle school includes four aspects. The fourth aspect requires the ability to apply mathematical knowledge and methods in interdisciplinary comprehensive and practical activities, construct simple models with disciplinary backgrounds, and solve interdisciplinary practical problems. For example, $h = \frac{1}{2}gt^2$ is a quadratic function in mathematics, while in physics it is a free-falling motion model. This is also the evaluation basis for level 3 of modeling concepts in this article: it can apply the knowledge of linear equation with one unknown to mathematical modeling in interdisciplinary synthesis and practical activities.

The level 3 of modeling concepts requires high standards for students. Applying mathematical knowledge to mathematical modeling in interdisciplinary comprehensive practical activities requires students' ability to design and establish mathematical models. Specifically, mathematical modeling activities require students' comprehensive mathematical abilities. It is closely related to their abilities in deduction and induction, convergence and divergence, intuition and logic, It is the result of the comprehensive effect of various mathematical abilities. The mathematical knowledge system of junior high school students has not yet been established, and their logical reasoning ability is also lacking in exercise. Their ability to conduct interdisciplinary modeling is weak, so it is difficult for each edition of textbook exercises to reflect level 3 of modeling concepts.

The other levels of literacy are also reflected in the same way. Level 3 is the highest requirement for a certain core literacy of students, which has significant difficulty. Textbooks are teaching materials for all students, and the difficulty of textbook exercises must ensure that most students can understand and answer them. Therefore, the core literacy reflected in textbook exercises is more inclined toward the basic level.

4.2 ZJE focuses on cultivating students' self-directed learning and creativity abilities

From the perspective of the core literacy level reflected in the exercises of ZJE, it is reasonable for ZJE to reflect the six core literacy levels. In terms of abstraction ability, arithmetic ability, and reasoning ability, which are the three basic mathematical abilities of students, The exercises of ZJE pay more attention to the embodiment of core literacy level 1 and the cultivation of students' basic knowledge and logical reasoning ability. However, in terms of modeling concepts and application consciousness, it more reflects level 2, ZJE also focuses on cultivating students' ability to apply mathematical knowledge, methods, and strategies to solve practical problems. Compared with the exercises in the other three editions of textbooks, the biggest feature of ZJE is that the number of exercises reflecting creativity consciousness literacy is the highest, and 37.5% of the exercises reflect level 3 of creativity consciousness literacy. This puts forward high requirements for students' autonomy and requires them to be able to think independently, Attempting different methods and strategies to solve the problem of linear equation with one unknown has honed students' creativity ability. However, the number of exercises reflecting modeling concepts literacy is slightly insufficient, and the penetration of modeling concepts ideas in textbook exercises needs to be improved.

ZJE is closer to cultivating students' autonomy, which originates from textbooks but is higher than textbooks. Zhejiang has good economic development, early reform and opening up, developed coastal trade, high social attention to education, and higher social requirements for talent positions, especially creativity ability. Since its establishment in 1983, Zhejiang Education Press has always been committed to "serving education, thriving academia, and accumulating culture" [11], Compared to other domestic textbooks, ZJE not only focuses on mastering basic mathematical knowledge but also pays more attention to mathematical creativity and open-ended Q&A. ZJE fully reflect this.

4.3 PEP focuses on cultivating students' ability to solve practical problems

From the perspective of the core literacy level reflected in the exercises of the PEP, among the six core literacy levels reflected in the PEP, the proportion of Level 2 is higher than Level 1. Only in terms of reasoning ability literacy, the number of Level 2 is slightly lower than Level 1, in terms of the three core literacies of abstraction ability, arithmetic ability, and application consciousness, the number of Level 2 is much higher than Level 1. It reflects the importance attached by the PEP to the application of knowledge in mathematics textbooks and emphasizes the characteristic of combining theory with practice. At the same time, the number of core literacy at level 1 is not small, which reflects the importance that the PEP attaches to basic mathematical knowledge in mathematics textbooks. The biggest feature of the PEP is that it reflects a lot of application consciousness as the core literacy. Each chapter has a practical problem analysis, which leads to corresponding mathematical knowledge points. At the same time, there are also many application problems in after-school exercises, and their scenarios are tailored to real life, It reflects a lot of level 2 application consciousness core literacy, but it is less reflected in the core literacy of modeling concepts, and there is a lack of questions reflecting modeling concepts in textbooks, which is not conducive to cultivating students' modeling concepts.

The PEP focuses on cultivating students' basic knowledge, while also emphasizing their ability to apply knowledge to solve practical problems. The PEP pays more attention to cultivating social application-oriented talents and emphasizing students' practical abilities. Most regions in China that adopt the PEP have average or relatively backward economic development levels, insufficient or uneven distribution of educational resources, and weak basic abilities among student groups, The differentiation is severe. Therefore, when compiling the PEP, it is necessary to consider the differences in learning foundations and educational resources among students in different regions, striving to make most students understand the textbook exercises. Therefore, the writing approach will focus on basic knowledge, setting the difficulty level of knowledge in mathematics textbooks to be very small, allowing students to gradually learn and then integrate practical application problems into practical situations to train their application abilities. However, such textbooks have low teaching efficiency, and spending too much time is not conducive to cultivating students' exploratory spirit and creativity ability

4.4 BNU focuses on training students' comprehensive mathematical abilities

From the perspective of the core literacy level reflected in the exercises of the BNU, the number of exercises reflecting core literacy is relatively small, but the distribution of different levels of core literacy is reasonable and comprehensive. The exercises of the BNU can reflect all the mathematical core literacy and effectively exercise students' comprehensive mathematical abilities. In terms of abstraction ability and arithmetic ability, the exercises of the BNU can reflect three levels, The difficulty level of exercises varies significantly from easy to difficult, reflecting the distinct differences in difficulty levels in the BNU. It can not only provide normal teaching for students with poor foundations but also provide a space for students with good foundations to improve and develop. At the same time, the proportion of exercises reflecting arithmetic ability and application consciousness literacy is very high, with the number of Level 2 accounting for a high proportion of the three levels, This reflects that the BNU places great emphasis on the cultivation of students' basic mathematical knowledge and abilities, emphasizing the concept of integrating knowledge theory with practice. The exercises in the BNU also reflect the core literacy of modeling concepts and creativity consciousness. This reflects that BNU does not forget to pay attention to the cultivation of students' mathematical modeling ideas and creativity abilities. However, the number of exercises in the BNU is relatively small, this places higher demands on teachers and the school's teaching and research team, requiring appropriate supplementary exercises for students to practice.

The regions used in the BNU are mostly urban areas with excellent basic education. The content arrangement of the BNU is relatively jumping, and the connection between knowledge points is weak. Moreover, the explanations of knowledge points in the textbooks are relatively simple, requiring students to have a certain degree of self-learning understanding and knowledge organization ability. There are many difficult, open, and comprehensive exercises in the BNU, Testing students' comprehensive mathematical abilities can effectively exercise their comprehensive development of mathematical core literacy.

4.5 The Virginia Edition focuses on cultivating basic mathematical abilities

From the perspective of the core literacy level reflected in the Virginia Edition of exercises, the Virginia Edition has the highest number of exercises reflecting core literacy among the four editions of textbooks. In addition to

application consciousness, the number of exercises reflecting level 1 of the four core competencies of abstraction ability, arithmetic ability, reasoning ability, and modeling concepts far exceeds level 2. The Virginia Edition will focus more on students' understanding of the essence of mathematics in exercises, Emphasizing the use of knowledge to solve problems in mathematics itself, and also emphasizing the combination of students' knowledge, theory, and practice. There are also quite a few core literacies of application consciousness reflected in exercises, including level 3 of application consciousness that is not reflected in the other three editions of textbooks. The exercises in the Virginia Edition truly reflect the concept of interdisciplinary and comprehensive practical activities using mathematical linear equation with one unknown. This is missing from the three editions of textbooks in China. However, the Virginia edition lacks sufficient exercises to reflect creativity consciousness literacy, making it difficult to cultivate students' ability to draw inferences and solve multiple problems, which is not conducive to the cultivation of students' creativity consciousness.

The Virginia edition of the junior high school mathematics textbook "BIG IDEAS MATH 8" reflects the author's emphasis on students' basic subject abilities. It can be seen that cultivating academic talents is the writing approach of the Virginia edition of the textbook, which is closely related to the concept of the United States' technological power and rejuvenation. Compared to the Virginia Edition, the exercises require students to fundamentally understand mathematical logic and knowledge, while the domestic three-edition textbooks focus more on applying knowledge and solving practical problems in social production, and more on cultivating applied talents.

5 Conclusion

The exercises in the four versions of the textbook have different focuses on the types of mathematical core literacies, but all four versions of the textbook attach great importance to arithmetic ability literacy, while the emphasis on modeling concepts literacy is relatively low.

The distribution proportion of mathematical core literacy levels reflected in each edition of textbook exercises varies but has commonalities. The main level of core literacy reflected in each edition of textbooks is at a relatively basic stage, and relatively few high-level core literacy levels are reflected.

Competing Interests

Authors have declared that no competing interests exist.

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