The Entangled Informational Universe

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Author's contribution

The sole author designed, analyzed, interpreted and prepared the manuscript.

Original Research Article

ABSTRACT

From the perspective of quantum gravity research, since the archetypal quantum gravitational object, the black hole, was accidentally found function as a thermodynamic system, it is certainly natural to suggest that the secret of quantum gravity may lie in thermodynamic analysis. Until now, it was not possible to express the gravitational fine-grained entropy of a black hole using the rules of gravity. However, the black holes entropic information formula fills this gap by allowing a semi-classical gravitational approach to express the gravitational fine-grained entropy of black hole. The black holes entropic information formula calculates the entropy of Hawking radiation as the entangled information of the initial considered black hole, this down to the quantum level of the system, the degrees of freedom describing the black hole, and this independently of the Bekenstein-Hawking entropy area law, providing a sufficient microscopic description of how this entropy arises, showing that the process of black holes evaporation is consistent with the unitarity principle. Also, this approach avoids ultraviolet divergences. These perspectives must be understood like the fine-grained entropy formulas discovered by Ryu and Takayanagi. In fact, the black hole entropy turns out to be a special case of the Ryu-Takayanagi conjecture. The Ryu-Takayanagi formula being a general formula for the fine-grained entropy of gravity-coupled quantum systems. That put the accent on the emergence quantum gravity process through the fundamentality of the entangled quantum information.

Keywords: Information; gravitational fine-grained entropy; quantum gravity; black hole; Bekenstein bound; Bekenstein-hawking entropy; ryu-takayanagi; entropic information.

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1. INTRODUCTION

1.1 Information

We start this introduction with a very deep analysis of the bottom level of what we call the Reality. For that approach, we'll take the "it from bit" perspective route.

"It from bit" perspective symbolizes the idea that every item of the physical world has at bottom — at a very deep bottom, in most instances — an immaterial source and explanation; that what we call reality arises in the last analysis from the posing of yes-no questions and the registering of equipment-evoked responses; in short, that all things physical are information-theoretic in origin and this is a participatory universe" [1].

But "It's one thing to say that measurement requires information. It's another thing to say that the thing being measured is created by the observer doing the measuring" [2].

"Based on that view, the entropic information theory approach is founded on the bit of information such as the number of bits of the system, the number of bits necessary to specify the actual microscopic configuration among the total number of microstates allowed and thus characterize the macroscopic states of the system under consideration" [3].

In 1870, Boltzmann introduced the statistical entropy perspective who established a new field of physics based on the rigorous treatment of a large ensembles of microstates that constitute thermodynamic systems, the descriptive linkage between the macroscopic observation of nature and the microscopic view. In statistical mechanics, entropy is formulated as a statistical property using probability theory [4].

The number of arrangements counted by Boltzmann entropy reflects the amount of Shannon information that would be needed to implement any arrangement ... of matter and energy; indeed, the thermodynamic entropy and Shannon entropy are conceptually equivalent [5].

We can say that entropy, is information: indeed, in the micro-canonical language entropy is determined by the number of microstates compatible with a given macrostate [6].

Following Shannon in his celebrated 1948 work that started the development of information theory, the number of states in which something can be, is precisely the definition of "information" (more precisely, "lack of information") [7].

The macroscopic approach and the statistical one was the first approach of the concept of information. Indeed, the macrostate formula from Boltzmann and his statistical entropy perspective has been followed by the statistical ensemble of distribution on the microstates approach of Gibbs. After what, entropy has been a measure allowing to characterize a statistical distribution as the probability of event for a specific distribution, it is the Shannon perspective of information. After this statistical approach of the entropy, we arrive at the quantic perspective, we dive into quantum physics approach with the Planck-Einstein relation. With mass-energy equivalence relation from Einstein, we are at confluence of the principles of mechanics, principle of relativity and electromagnetic theory for this quantic way of description of the information concept [4].

But we can go further into the exploration of the notion of information by introducing the mass-energy-information equivalence principle [8] which following the Landauer's principle [9] propose that a bit of information is not just physical, but it has a finite and quantifiable mass while it stores information. This assumption can be testable with the experimental protocol for testing the mass–energy–information equivalence principle [10].

1.2 Entropy Definition

"Statistical entropy is a probabilistic measure of uncertainty or ignorance; information is a measure of a reduction in that uncertainty" [11].

We can now develop our proper definition of entropy which can be enounced as following:

"The entropy of a thermodynamic system in equilibrium measure of the uncertainty as to which all its internal configurations compatible with its macroscopic thermodynamic parameters (temperature, pressure, etc.) are actually realized" [3].

1.3 Local Thermodynamic Equilibrium

Here we must emphasize on terms used in this definition, indeed when one speaks of thermodynamic system in equilibrium we must think about local thermodynamic equilibrium.

Where equilibrium does not require either local or global stationarity.

Each small locality must change slowly enough to practically sustain its local Maxwell–Boltzmann distribution of molecular velocities. Local thermodynamic equilibrium (LTE) means that those intensive parameters are varying in space and time, but are varying so slowly that, for any point, one can assume thermodynamic equilibrium in some neighborhood about that point. In other words, it doesn't require that every small locality must have a constant temperature.

1.4 Black Hole Thermodynamics

Thus, let's look to the black hole as a thermodynamic point of view from Jacob Bekenstein saying in substance: "Therefore, the decrease in area and entropy is of a statistical nature, and is quite analogous to the decrease in entropy of a thermodynamic system due to statistical fluctuations" [12].

We see so that Black hole are subject to the LTE, in accordance with definition of the entropy concept from the theory of entropic information.

The four laws of black hole mechanics, which are analogous to those of thermodynamics, were originally derived from the classical Einstein equation [13]. With the discovery of the Hawking quantum radiation [14], it became clear that the analogy is in fact an identity [15].

1.5 Degree of Freedom

Following a thermodynamic analysis of black hole, as the archetypal quantum gravitational object, the black hole, accidentally turned out to just like a thermodynamic system. we can say that there are fundamental degrees of freedom, considered as the minimum number of coordinates required to specify a configuration, that give rise to horizon temperature and entropy [3].

In general, entropy is related to the number of possible microstates according to the Boltzmann principle:

Where S is the entropy of the system, k Boltzmann's constant, and Ω or W is the number of microstates.
W = number of states or microstates, characterized by the position and velocity of all particles so if we consider that the degree of freedom of a system can be viewed as the minimum number of coordinates required to specify a configuration. So YES ... W reflects the degree of freedom of a system [3].

Concerning the degree of freedom, the Newtonian side of black hole is dictated by the fact that the degree of freedom of the system reflects the number of states or microstates, characterized by the position and velocity of all particles; while the quantum relativist side of black hole is based on the thermal radiation of the black hole's event horizon which is a place of quantum effects.

1.6 Fine-grained and Coarse-grained Entropy

At this level, we can define a fine-grained and coarse-grained entropy. "Where fine grained entropy is the entropy of the density matrix calculated by the standard methods of quantum field theory in curved spacetime. In the literature, this is often simply called the von Neumann entropy" [16]. It is Shannon's entropy with distribution replaced by density matrix. It is invariant under unitary time evolution. While the coarse-grained entropy is defined as follows: we only measure simple observables A_i. And consider all possible density matrices which give the same result as our system $\text{Tr}[\tilde{\rho}A_i]= \text{Tr}[\rho A_i]$. We then choose the maximal von Neumann entropy over possible density matrices $S(\tilde{\rho})$. It increases under unitary time evolution, i.e. thermodynamics entropy [17].

2. METHODOLOGY

We start from the hidden thermodynamic of Louis De Broglie, where we replace the mass by the mass of bit of information [4]. After we used this mass of bit of information at the Hawking temperature in the framework of the black hole thermodynamics and in the perspective of the Hawking radiation, the process of evaporation of the black hole, we will compare some black hole data from all over the black holes scale to confirm that the black holes entropic information formula equal to the Bekenstein-Hawking entropy, but, also equal to the Bekenstein bound [3]. Results are put in relation to the work of Casini from 2008 [18] about Bekenstein and Von Neumann entropy, about the fine-grained entropy and the coarse grained entropy. After this we will put in relation the black holes entropic information to the Ryu-Takayanagi formula which

is a general formula for the fine-grained entropy of gravity-coupled quantum systems [17].

2.1 Contextualization

Here, the link between physical objects and entropy/information in the context of black hole physics is examined. The relationship between the initial information/entropy contained in the horizon of black hole and the final entropy carried by the outgoing and entangled quantum information of Hawking radiation is considered. In the context of the entropy of black holes and from a semi-classical approach.

2.2 Entanglement

Now, let's look to entropy and entanglement. Indeed, entropy is viewed as a measure of entanglement. Entropy provides one tool that can be used to quantify entanglement. If the overall system is pure, the entropy of one subsystem can be used to measure its degree of entanglement with the other subsystems.

It is a classical result that the Shannon entropy achieves its maximum at, and only at, the uniform probability distribution {1/n, ...,1/n}. Therefore, a bipartite pure state $\rho \in$ HA $\otimes$ HB is said to be a maximally entangled state if the reduced state of each subsystem of ρ is the diagonal matrix.

$$\begin{bmatrix} \frac{1}{n} & & \\ & \ddots & \\ & & \frac{1}{n} \end{bmatrix}$$

We must note that for mixed states, the reduced von Neumann entropy is not the only reasonable entanglement measure.

2.3 Bekenstein-Hawking Entropy

The Bekenstein–Hawking area law claims that the area of the black hole horizon is proportional to the black hole's entropy.

$$S_{BH} = \frac{K_B}{4}\left(\frac{c^3}{\hbar G}\right)A = k_B\ \frac{A}{4l_p^2} = S = k_B \ln$$

The black-hole entropy is proportional to the area of its event horizon A. This area relationship was generalized to arbitrary regions via the Ryu–

Takayanagi formula, which relates the entanglement entropy of a boundary conformal field theory to a specific surface in its dual gravitational theory [19].

The Bekenstein Hawking entropy formula, since it increases under time evolution, should be viewed as the coarse-grained entropy formula for the black hole [17].

The Bekenstein–Hawking entropy is a statement about the gravitational entropy of a system.

The Bekenstein–Hawking entropy is a measure of the information lost to external observers due to the presence of the horizon.

2.4 Mass of Bit of Information

About the hidden thermodynamics of isolated particles, it is an attempt to bring together the three furthest principles of physics: the principles of Fermat, Maupertuis, and Carnot, that De Broglie has had his final idea. Entropy becomes a sort of opposite to action with an equation that relates the only two universal dimensions of the form: [4]

$$\frac{action}{h} = \frac{-\text{entropy}}{k}$$

With action = E·time
And with entropy = k ln(w)

$$\ln(W) = \frac{action}{h} = \frac{mc^2t}{h}$$

In [4], the First equivalence from Entropic Information, we have:

$$\ln(W) = \frac{action}{h} = \frac{mc^2t}{h}$$

we replace m by the mass of bit information given as

$$m_{bit} = \frac{k\,T \ln{(2)}}{c^2}$$

where k = 1.38064 ×10-23 J/K is the Boltzmann constant
T is temperature at which the bit of information is stored.
t = time required to change the physical state of the information bit
we have:

$$\ln(W)=\frac{\frac{k\,T\ln(2)t\,c^2}{c^2}}{h}$$

We have by c^2 simplification:

$$\text{Ln}(W)=\frac{k\,T\ln(2)t}{h}$$

We obtain finally:

$$\ln(W)=\frac{action}{h}=\frac{mc^2t}{h}=\frac{k\,T\ln(2)t}{h}$$

mathematical proof given by Landauer principle as seen in [4]

$$\frac{mc^2t}{h}=\frac{k\,T\ln(2)\,t}{h}$$

We arrive by h and t simplification to:

$$k\,T\ln(2)=mc^2$$

There is a minimum possible amount of energy required to erase one bit of information, that is that Landauer's principle asserts and it's known as the Landauer limit.

2.5 Mass of bit of information at Hawking Temperature

Now, as seen in [3], at the black hole thermodynamics level, we insert into $\frac{k\,T\ln(2)t}{h}$ the Hawking Temperature, in relation to the storage of mass of bit of information; so, it is represented at Hawking Temperature in the frame of the Hawking radiation.

$$\text{Ln}(w)=\frac{k\,T\ln(2)t}{h}$$

With T = Hawking Temperature:

$$T=\frac{\hbar c^3}{8\pi GkM}$$

k =Boltzmann constant, c=speed of light, $\hbar$ = Reduced Planck's constant, G=Gravitational Constant, M= mass of the black hole, by replacing $\hbar$ by $h/2\pi$.

$$\frac{\frac{khc^3\ln(2)\,t}{16\pi^2GkM}}{h}$$

By simplification of k, h, we obtain:

$$\text{Ln}(W)=\frac{c^3\ln(2)t}{16\pi^2GM}$$

With $\ln(W)=\frac{action}{h}=\frac{mc^2t}{h}=\frac{k\,T\ln(2)t}{h}$, in the frame of Black hole evaporating by Hawking radiation at the Hawking temperature with a time required to change the physical state of all the bit of information of the system equal to the time required by the black hole to evaporate (t_{evap}).

$$\ln(W)=\frac{c^3\ln(2)t_{evap}}{16\pi^2GM}$$

We clearly see the relation between the degree of freedom and the process of evaporation of the black hole.

$$\text{Where } t_{evap}=\frac{64\pi^3M^3G^2}{\hbar\ln(2)c^4}$$

2.6 The black holes entropic information formula and coarse-grained entropy

We can see the black holes entropic information formula as new formulation of Bekenstein-Hawking entropy which is on the edge of thermodynamics, relativity, quantum, gravitation, but moreover, this new formulation takes in account the information theory.

See Fig.1

2.7 The Black holes entropic information formula and Bekenstein bound

The black holes entropic information formula given as follows:

$$S=k\ln(W)=\frac{kc^3\ln(2)t_{evap}}{16\pi^2GM}$$

The universal bound originally founded by Jacob Bekenstein in 1981 as the inequality [20,21].
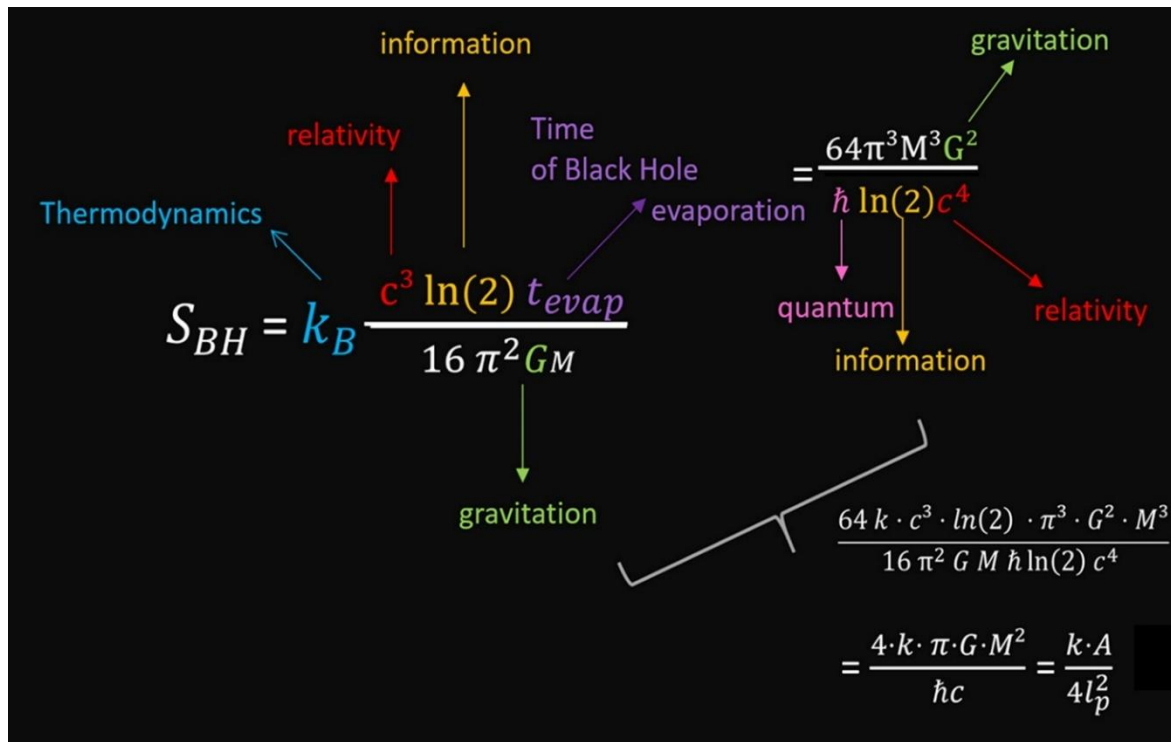
$$S\le\frac{2\pi kRE}{\hbar c}$$

Fig. 1. Black holes Entropic Information Formula with the time of evaporation of the black hole (t_{evap}) as new formulation of The Bekenstein–Hawking entropy additionally including the information notion; where k=Boltzmann constant, c=speed of light, ℏ = Reduced Planck's constant, G=Gravitational Constant, M= mass of the black hole and A is the two-dimensional area of the black hole's event horizon, l_p is the Planck length

As seen in [3], in informational terms, the relation between thermodynamic entropy S and Shannon entropy H is given by relation between S & *H*

S= k *H* ln (2)

Whence,

$$H = \frac{2\pi RE}{\hbar c \ln(2)}$$

where H is the Shannon entropy expressed in number of bits contained in the quantum states in the sphere. The ln 2 factor comes from defining the information as the logarithm to the base 2 of the number of quantum states

$$\frac{c^3 t_{evap}}{16\pi^2 GM} = \frac{2\pi RE}{\hbar c \ln(2)}$$

So, following S= k *H* ln(2)

$$S = k \ln(W) = k\frac{c^3 t_{evap}}{16\pi^2 GM}\ln(2) = k\frac{2\pi RE}{\hbar c}$$

where S is the entropy, k is Boltzmann's constant, R is the radius of a sphere that can

enclose the given system, E is the total mass–energy including any rest masses, ℏ is the reduced Planck constant, and c is the speed of light.

As a side note, it can also be shown that the Boltzmann entropy is an upper bound to the entropy that a system can have for a fixed number of microstates meaning:

S ≤ k lnW

With W reflecting the degree of freedom of a system as seen upper.

We have showed from [3] that the black holes entropic information formula $k\frac{c^3\ \ln(2)\ t_{evap}}{16\pi^2 GM}$ is equal to the universal bound, $\frac{2\pi kRE}{\hbar c}$ and that it is in relation with degree of freedom of black hole seen as a whole quantum system.

In [3], some calculations have been realized about $\frac{kc^3\ln(2)t_{evap}}{16\pi^2 GM}$ in regard to black hole data from along all the black hole size scale taking

extremes values to calculate the entropic information relation as proof of the validity of the black hole entropic information.

Rem: black hole saturates the bound.

Rem:

$$\text{Time\_ Bekenstein} = \frac{64\pi^3 M^3 G^2}{\hbar \ln(2) c^4}$$

With R=$\frac{2GM}{c^2}$

$$\text{Time\_Shannon} = \frac{32\pi^3 GRM^2}{\hbar \ln(2) c^2}$$

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Mass of Black Hole in solar mass                              :2.7


Classical method Results :
/////////////\\\\\\

Masse of the black Hole in kg                                 :    5.3703e+30
Schwarzschild radius of black hole in meter                   :    7976.022193357674
Entropy of the Black  Hole Classical Method   Dimensionless   :    7.650843355747418e+77
Entropy of the Black  Hole Classical Method                   :    1.0563129228269317e+55

Entropic Information Results :
/////////////\\\\\\

Entropic Information Time Shannon                             :    2.3186715453271243e+75
Entropic Information Entropy with time Shannon                :    1.0563129228269317e+55

Entropic Information Time Bekenstein                          :    2.3186715453271247e+75
Entropic Information Entropy with time Bekenstein             :    1.056312922826932e+55



Comparison Results :
/////////////\\\\\\

Rapport between Classical Method and Entropic Information with Shannon  time    :    1.0
Rapport between Classical Method and Entropic Information with bekenstein time  :    0.9999999999999998
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Fig. 2. GW170817 with a given mass of 2.7 solar mass; from [1]

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Mass of Black Hole in solar mass                              :8.7


Classical method Results :
/////////////\\\\\\

Masse of the black Hole in kg                                 :    1.73043e+31
Schwarzschild radius of black hole in meter                   :    25700.515956374722
Entropy of the Black  Hole Classical Method   Dimensionless   :    7.943653410103184e+78
Entropy of the Black  Hole Classical Method                   :    1.096739713700555e+56

Entropic Information Results :
/////////////\\\\\\

Entropic Information Time Shannon                             :    7.757212663783706e+76
Entropic Information Entropy with time Shannon                :    1.0967397137005548e+56

Entropic Information Time Bekenstein                          :    7.757212663783709e+76
Entropic Information Entropy with time Bekenstein             :    1.0967397137005552e+56



Comparison Results :
/////////////\\\\\\

Rapport between Classical Method and Entropic Information with Shannon  time    :    1.0000000000000002
Rapport between Classical Method and Entropic Information with bekenstein time  :    0.9999999999999998
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Fig. 3. Cygnus X1 Black hole with a given mass of 8.7 solar mass; from [1]

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Mass of Black Hole in solar mass                                    :6500000000


Classical method Results :
///////////////\\\\\\\

Masse of the black Hole in kg                                   :   1.2928500000000002e+40
Schwarzschild radius of black hole in meter                     :   19201534909935.145
Entropy of the Black  Hole Classical Method   Dimensionless     :   4.434130751444835e+96
Entropy of the Black  Hole Classical Method                     :   6.12197818785156e+73

Entropic Information Results :
///////////////\\\\\\\

Entropic Information Time Shannon                               :   3.2351022361198057e+103
Entropic Information Entropy with time Shannon                  :   6.121978187851557e+73

Entropic Information Time Bekenstein                            :   3.2351022361198073e+103
Entropic Information Entropy with time Bekenstein               :   6.121978187851561e+73



Comparison Results :
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Rapport between Classical Method and Entropic Information with Shannon  time     :   1.0000000000000004
Rapport between Classical Method and Entropic Information with bekenstein time   :   0.9999999999999998
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Fig. 4. Messier87 Black Hole with a given mass of 6.5 x 10^9 solar mass; from [1]

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Mass of Black Hole in solar mass                                    :66000000000


Classical method Results :
///////////////\\\\\\\

Masse of the black Hole in kg                                   :   1.31274e+41
Schwarzschild radius of black hole in meter                     :   194969431393187.6
Entropy of the Black  Hole Classical Method   Dimensionless     :   4.571615042199692e+98
Entropy of the Black  Hole Classical Method                     :   6.311795736397963e+75

Entropic Information Results :
///////////////\\\\\\\

Entropic Information Time Shannon                               :   3.3867235411033223e+106
Entropic Information Entropy with time Shannon                  :   6.311795736397963e+75

Entropic Information Time Bekenstein                            :   3.386723541103322e+106
Entropic Information Entropy with time Bekenstein               :   6.3117957363979615e+75



Comparison Results :
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Rapport between Classical Method and Entropic Information with Shannon  time     :   1.0
Rapport between Classical Method and Entropic Information with bekenstein time   :   1.0000000000000002
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Fig. 5. Black hole TON618 with a given mass of 66 billion solar mass; from [1]

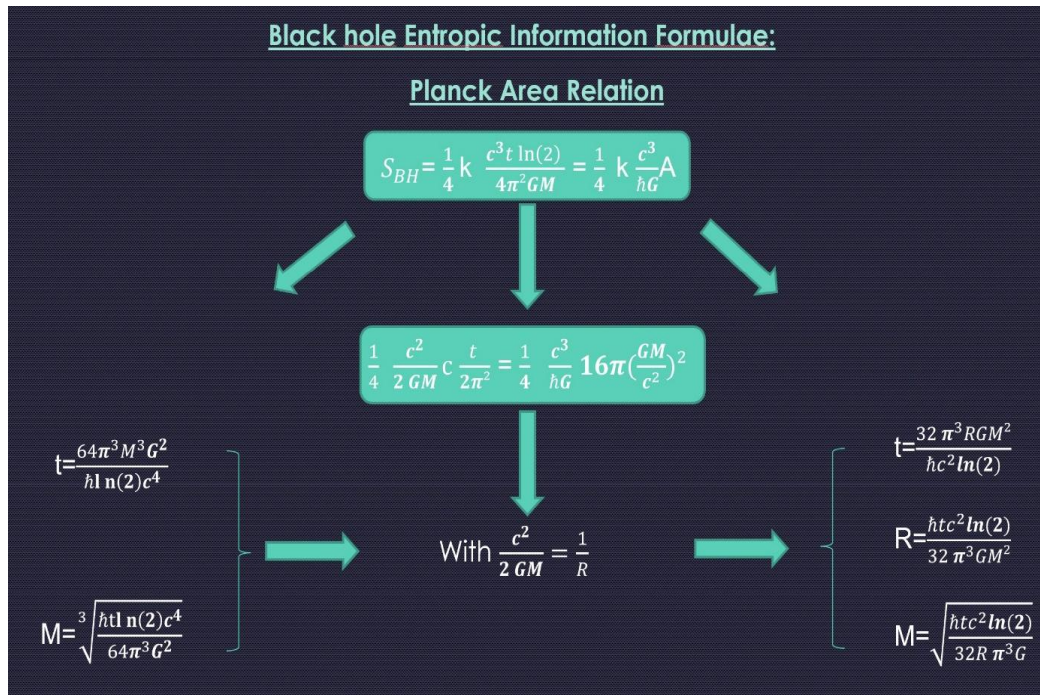- **The Black holes entropic information formula and Planck area relation**

Fig. 6. The black holes entropic information formula with the time of evaporation of the black hole (t_{evap}): Planck area relation

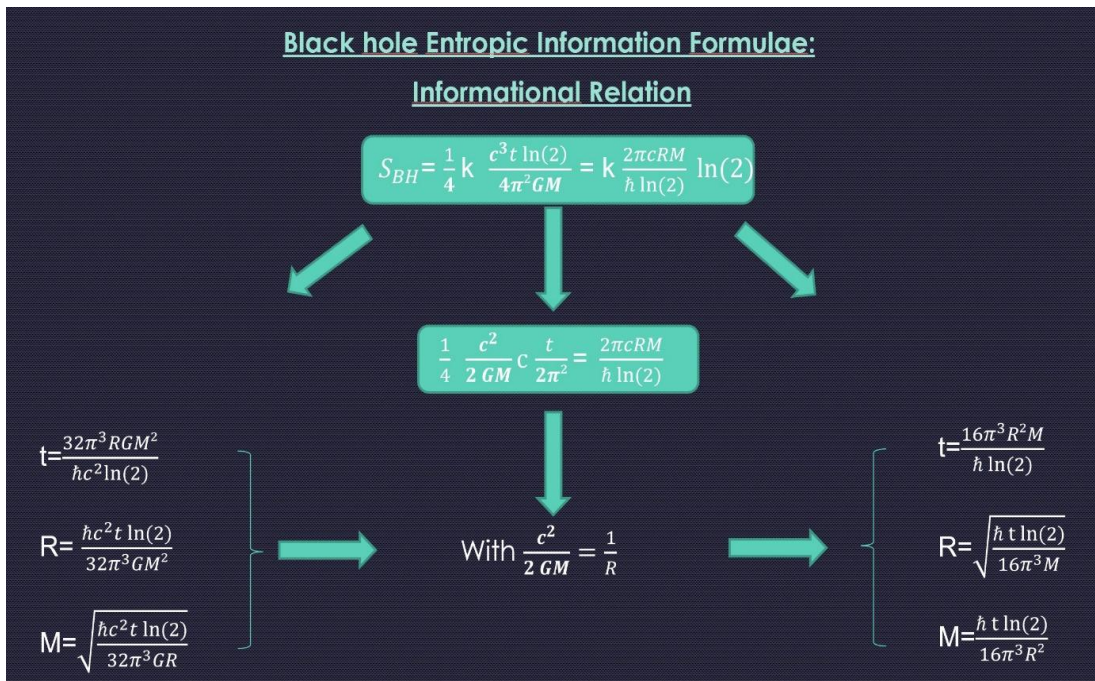- **The Black holes entropic information formula and informational relation**

Fig. 7. The black holes entropic information formula with the time of evaporation of the black hole (t_{evap}): Informational relation

2.8 Hawking Radiation

Quantum field theory in curved spacetime (QFTCS) is an extension of quantum field theory from Minkowski spacetime to a general curved spacetime. The spacetime is treated by this theory as a fixed, classical background, while for the matter and energy propagating through that spacetime this theory gives a quantum-mechanical description.

QFTCS is expected to be a viable approximation to the theory of quantum gravity when spacetime curvature is not significant on the Planck scale. The spacetime in which the fields propagate is classical but dynamical. The curvature of the spacetime is given by the semiclassical Einstein equations.

In QFTCS, a single emission of Hawking radiation involves two mutually entangled particles. The thermal aspect of Hawking radiation comes from separating entangled outgoing Hawking quanta and interior Hawking quanta. A quantum of Hawking radiation is emitted by the outgoing particle escaping; the black hole swallows the infalling particle. Assume that in a finite time in the past a black hole has been formed and in the future; will be fully evaporated away in some finite time. The black hole will only emit a finite amount of information encoded within its Hawking radiation. The Hawking radiation is to put in relation with the (finite) entropy and thus the (non-zero) temperature of the black hole: the Hawking temperature. Hence the absence of the Hawking radiation would lead to violations of thermodynamical laws.

Hawking radiation occurs in an inertial frame where spatial distance doesn't come into it, and, where the horizon of a black hole is compact.

Hawking radiation is thermal radiation following Boltzmann distribution.

An observer at infinity will see a thermal bath of particles coming from the horizon, even though the quantum fields are in the local vacuum state near the horizon.

2.9 Black Hole System

Throw in a black hole a quantum system in a pure state and waits for some amount of time until the hole has evaporated, we end up with is a thermal state. A pure quantum state is a state which can be described by a single ket vector and of which the entropy is zero because there is no uncertainty in this state. A thermal state is a mixed state (described quantum mechanically by a density matrix rather than a wave function). So, we are face to a system which converts a pure state into a mixed state. During the transformation between a mixed state and a pure state, some information is lost. But prime directive of quantum mechanics indicate that quantum evolution should be unitary and, thus, information and entropy should be conserved.

In fact, if we had a very complex quantum system which starts in a pure state, it will appear to thermalize and will emit radiation that is very close to thermal. In particular, in the early stages, if we computed the von Neumann entropy of the emitted radiation it would be almost exactly thermal because the radiation is entangled with the quantum system [17].

As a side note, we must take in account that the black Hole information paradox can be established using the standard Copenhagen method of keeping the system separate from the measuring device. The black hole information paradox is independent of the quantum measurement problem. As such we can discuss the solutions to the information paradox without committing to any particular interpretation of quantum mechanics.

2.10 Casini and the Bekenstein Bound Proof

Casini proves the thermodynamics interpretation in the form of Bekenstein bound as valid. Indeed, we know following the work of Casini in 2008 [18] about the Von Neumann entropy and the Bekenstein bound, that the proof of the Bekenstein bound is valid using quantum field theory [22-30].

For example, given a spatial region V, Casini defines the entropy on the left-hand side of the Bekenstein bound as:

$$S_V = S(\rho_V) - S(\rho_V^0) = -tr(\rho_V \log \rho_V) + tr(\rho_V^0 \log \rho_V^0)$$

Casini defines the right-hand side of the Bekenstein bound as the difference between the expectation value of the modular Hamiltonian in the excited state and the vacuum state,

$$K_V = tr(K \rho_V) - tr(K \rho_V^0)$$

With these definitions, the bound reads $S_V \leq K_V$, which can be rearranged to give:

$$\text{tr} (\rho_V \log \rho_V) - \text{tr} (\rho_V \log \rho_V^0) \geq 0.$$

This is simply the statement of positivity of quantum relative entropy, which proves the Bekenstein bound.

2.11 The Black Holes Entropic Information Formula and Gravitational Fine-grained Entropy

"However, as the black hole evaporates more and more, its area will shrink, and we run into trouble when the entropy of radiation is bigger than the thermodynamic entropy of the black hole. The reason is that now it is not possible for the entropy of radiation to be entangled with the quantum system describing the black hole because the number of degrees of freedom of the black hole is given by its thermodynamic entropy, the area of the horizon. [17]

This assumption is wrong because it neglects the fact that black holes entropic information formula is able to express the entropy of the entangled radiation based on the degrees of freedom of the black hole given by this formula: $k \frac{c^3 \ln(2) t_{evap}}{16\pi^2 GM}$ where $t_{evap} = \frac{64\pi^3 M^3 G^2}{\hbar \ln(2) c^4}$ independently of the area law, which is based on the area of the horizon.

Diving into Casini's work with the black holes entropic information formula, we obtain new enlightening about the gravitational fine grained black hole entropy. The ingenious proposal of casini [31] is to replace 2 π R E, by:

$$K_V = \text{tr} (K \rho_V) - \text{tr} (K \rho_V^0).$$

Indeed, in [22-30], Casini's work, on the right-hand side of the Bekenstein bound, a difficult point is to give a rigorous interpretation of the quantity 2 π R E, where R is a characteristic length scale of the system and E is a characteristic energy. This product has the same units as the generator of a Lorentz boost, and the natural analog of a boost in this situation is the modular Hamiltonian of the vacuum state K = − log (ρ_V^0).

With these definitions, the bound reads

$$S_V \leq K_V,$$

The version of the Bekenstein bound is $S_V \leq K_V$, namely

$$S(\rho_V) - S(\rho_V^0) \leq \text{Tr}(K \rho_V) - \text{Tr}(K \rho_V^0)$$

is equivalent to

$$S_V \equiv S(\rho_V | \rho_V^0) \equiv \text{Tr} (\rho_V (\log \rho_V - \log \rho_V^0)) \geq 0$$

Where the black holes entropic information formula is equal to Sv where S (ρ_V) is the Von Neumann entropy of the reduced density matrix ρ_V associated with V, V in the excited state ρ, and S (ρ_V^0) is the corresponding Von Neumann entropy for the vacuum state ρ^0.

$$S_V \equiv S (\rho_V | \rho_V^0) \equiv S(\rho_V) - S(\rho_V^0) \equiv \text{Tr} (\rho_V (\log \rho_V - \log \rho_V^0)) \equiv k \frac{c^3 \ln(2) t_{evap}}{16\pi^2 GM} \geq 0$$

As black holes entropic information formula, is equal to Bekenstein universal bound

$$\frac{kc^3 \ln(2) t_{evap}}{16\pi^2 GM} = \frac{2\pi k R E}{\hbar c}$$

As the difference between the expectation value of the modular Hamiltonian in the excited state and the vacuum state

$$K_V = \text{tr} (K \rho_V) - \text{tr} (K \rho_V^0).$$

is equal to Bekenstein universal bound.

Finally, we obtain :

$$K_V = \text{tr} (K \rho_V) - \text{tr} (K \rho_V^0) = \frac{kc^3 \ln(2) t_{evap}}{16\pi^2 GM} = Sv = S (\rho_V | \rho_V^0) = S(\rho_V) - S(\rho_V^0) = \text{Tr} (\rho_V (\log \rho_V - \log \rho_V^0)) = \frac{2\pi k R E}{\hbar c}$$

Naive definitions of entropy and energy density in Quantum Field Theory suffer from ultraviolet divergences. In the case of the Bekenstein bound, ultraviolet divergences can be avoided by taking differences between quantities computed in an excited state and the same quantities computed in the vacuum state [32].

We must take note that the first version of the fine-grained entropy formula discovered by Ryu and Takayanagi is a general formula for the fine-

grained entropy of quantum systems coupled to gravity [17].

The black holes entropic Information formula can calculate the entangled Hawking radiation down to the quantum system, describing black hole independently of the area law of the entropy of Bekenstein-Hawking. The black holes entropic information formula is equal to the universal bound originally found by Jacob Bekenstein [20,21]. Which is equal by Casini's work [18] to the difference between the expectation value of the modular Hamiltonian in the excited state and the vacuum state, itself equal to the Von Neumann entropy. The ultraviolet divergences can be avoided by taking differences between quantities computed in an excited state and the same quantities computed in the vacuum state; this must be put in relation to Ryu and Takayanagi conjecture, a general formula for the fine-grained entropy of quantum systems coupled to gravity [17].

2.12 Ryu and Takayanagi Conjecture

The Ryu–Takayanagi conjecture is a conjecture within holography that posits a quantitative relationship between the entanglement entropy of a conformal field theory and the geometry of an associated anti-de Sitter spacetime [33,34]. The formula characterizes "holographic screens" in the bulk; that is, it specifies which regions of the bulk geometry are "responsible to particular information in the dual CFT" [35].

The Ryu–Takayanagi formula calculates the entropy of quantum entanglement in conformal field theories on Bekenstein-Hawking entropy of black holes in the context of Juan Martín Maldacena's holographic principle, in which conformal field theories on a surface form a gravitational theory in a closed volume [36].

"The Bekenstein–Hawking area law, while claiming that the area of the black hole horizon is proportional to the black hole's entropy, fails to provide a sufficient microscopic description of how this entropy arises. The holographic principle provides such a description by relating the black hole system to a quantum system which does admit such a microscopic description. In this case, the CFT has discrete eigenstates, and the thermal state is the canonical ensemble of these states [19]. The entropy of this ensemble can be calculated through normal means, and yields the same result as predicted by the area law. This turns out

to be a special case of the Ryu–Takayanagi conjecture" [16].

"The first version of the fine-grained entropy formula was discovered by Ryu and Takayanagi [37]. It was subsequently refined and generalized by several authors [38-45]. Originally, the Ryu-Takayanagi formula was proposed to calculate holographic entanglement entropy in anti-de Sitter spacetime, but the present understanding of the formula is much more general. It requires neither holography, nor entanglement, nor anti-de Sitter spacetime. Rather it is a general formula for the fine-grained entropy of quantum systems coupled to gravity" [17].

The black-hole entropy is proportional to the area of its event horizon A. The black hole entropy horizon law which turns out to be a special case of the Ryu–Takayanagi conjecture. The black-hole entropy area relationship was generalized to arbitrary regions via the Ryu–Takayanagi formula, which relates the entanglement entropy of a boundary conformal field theory to a specific surface in its dual gravitational theory [19] but the current understanding of the formula is much more general. As being a general formula for the fine-grained entropy of quantum systems coupled to gravity [17].

3. RESULTS, DISCUSSION, GENERALIZATION

During the exploration of the black holes scale, following our results, we have found that the black holes entropic information formula for a mass of bit of information at the Hawking temperature is equal to the Bekenstein-Hawking entropy of the black hole, result materialized by the rapport of both formulas. We have taken extremal data from the black holes scale analysis with the two most extreme value (GW170817, TON618) giving the value of the rapport of both formulae matching perfectly, indeed the result is one, the perfect egality. For the two other data used (Cygnus X1, Messsier87) we obtain a maximum difference of $4 * 10^{-16}$ in regard to the rapport of both formulas. We have use two different relations for the black holes entropic information formula. One is to put in relation to the Planck area relation formula showed at Fig. 6 and the other is to put in relation to the informational expression seen in Fig. 7. We have new formulation of the Bekenstein-Hawking entropy additionally including the information notion and the time of evaporation of the

considered black hole. Moreover, as showed in [3], that in the framework of black hole, the black holes entropic Information formula for a mass of bit of information at the Hawking temperature is equal to the universal bound originally found by Jacob Bekenstein: $k\frac{c^3 \ln(2) t_{evap}}{16\pi^2 GM} = \frac{2\pi kRE}{\hbar c}$ because black hole saturates exactly the bound. First the work of Casini [22-30] between entropy of Von Neumann and of Bekenstein and the proof of the validity of this one in the quantum field theory framework. Second, with the equivalence between the black hole entropic information formula and the Von Neumann entropy, made possible by an ingenious proposal of Casini to equal the difference between the expectation value of the modular Hamiltonian in the excited state and the vacuum state, K_V = tr (K ρ_V) − tr (K ρ_V^0) to 2 π R E [22]. As black holes entropic information formula is equal to the universal bound, this leads to new formulation of gravitational fine-grained entropy regarding entropic information theory. The black holes entropic information formula can calculate the entropy of the black hole regardless of the area of the horizon and avoiding the ultraviolet divergences. The ultraviolet divergences can be avoided by taking differences between quantities computed in an excited state and the same quantities computed in the vacuum state. The black holes entropic information formula expresses the gravitational fine grained black hole entropy by a semiclassical gravity approach. Indeed, in a semiclassical gravity view, matter is represented by quantum matter fields that propagate according to QFTCS. We have calculated the Von Neumann entropy, the gravitational fine-grained entropy of the black hole independently of the area of the horizon law. The black holes entropic information formula expresses the black hole gravitational fine-grained entropy down to the quantum level independently of the area horizon's law permitting to the entropy of Hawking radiation to be entangled with the initial considered black hole seen as a whole quantum system. The black holes entropic information formula must be put in relation to Ryu–Takayanagi formula being a general formula for the fine-grained entropy of quantum systems coupled to gravity [17].

4. CONCLUSIONS

The black holes entropic information formula solves the problem of the black hole information paradox by explaining the information processes involved in entropy for black holes down to the quantum level independently of the area horizon's law by providing a sufficient microscopic description of how this entropy arises. The black holes entropic information formula shows that the process of black holes evaporation is consistent with the unitarity principle. Indeed, the black holes entropic information formula calculate the Hawking radiation encoding the entangled quantum information of the initial considered black hole. We have showed in [3]; that in the framework of black hole, the entropic Information formula taking in account the mass of bit of information at the Hawking temperature is equal to the universal bound originally found by Jacob Bekenstein: $k\frac{c^3 \ln(2) t_{evap}}{16\pi^2 GM} = \frac{2\pi kRE}{\hbar c}$ because black hole saturates exactly the bound. The fact is that the black-hole entropy is also the maximal entropy that can be obtained by the Bekenstein bound, wherein, the Bekenstein bound becomes an equality. Moreover, the equality between black holes entropic information formula and the Von Neumann entropy, made possible by an ingenious proposal of Casini [22-30] to equal the difference between the expectation value of the modular Hamiltonian in the excited state and the vacuum state, K_V = tr (K ρ_V) − tr (K ρ_V^0) to 2 π R E [22], as the black holes entropic information formula is also equal to the universal bound. This leads to formulation of gravitational fine-grained entropy in regard to the entropic information theory. Until now, it was not possible to express the gravitational fine-grain entropy of a black hole using the rules of gravity [46]. However, the black holes entropic information formula can calculate the entropy of the black hole independently of the area horizon's law and avoiding the ultraviolet divergences. Indeed, the ultraviolet divergences can be avoided by taking differences between quantities computed in an excited state and the same quantities computed in the vacuum state. The black holes entropic information formula expresses the gravitational fine grained black hole entropy by a semiclassical gravity approach. The black holes entropic information formula as new formulation of Bekenstein–Hawking entropy additionally including the information notion and the time of evaporation of the considered black hole permits to express the black hole entropy down to the quantum level independently of the area horizon's law permitting to the entropy of Hawking radiation to be entangled with the initial considered black hole seen as a whole quantum system. The black holes entropy which turns out to be a special case of the Ryu–Takayanagi conjecture" [16]. Ryu–Takayanagi formula being

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a general formula for the fine-grained entropy of quantum systems coupled to gravity [17]. That put the accent on the emergence quantum gravity process through the fundamentality of the entangled quantum information. All those informational processes lead to news perspectives in the approach of theoretical physics where we can see entangled quantum information as the building block of the universe, responsible of the emergent gravity, and being a keystone in the quantum entanglement processes, where entanglement doesn't create entropy, but it is entropy, in the sense of information. Moreover, we can see the implication of the concept of information in the measurement problem indeed the collapse of the wave function is due to the fact that when a measure is taken, this is taking information out…. measurer absorbs information about system so modify the system, the wave function is collapsing because information is absorbed. We can envisage information as existing by itself as information is code, and code is what is create the process but is itself the process. We can even envisage some more deep implications as: "No bits, no structure, no existence". "If consciousness exists before information, consciousness has nothing to know, indeed no consciousness without information". "You must know that universe doesn't need you to exist".

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COMPETING INTERESTS

Author has declared that no competing interests exist.

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