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Research Article

Calculation of Binding Energy and Wave Function for Exotic Hidden-Charm Pentaquark

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In this study, pentaquark $P_c(4380)$ composed of a baryon Σ_c, and a $\bar{D}^*$ meson is considered. Pentaquark is as a bound state of two-body systems composed of a baryon and a meson. The calculated potential will be expanded and replaced in the Schrödinger equation until tenth sentences of expansion. Solving the Schrödinger equation with the expanded potential of Pentaquark leads to an analytically complete approach. As a consequence, the binding energy E_B of pentaquark P_c and wave function is obtained. The results E_B will be presented in the form of tables so that we can review the existence of pentaquark P_c. Then, the wave function will be shown on diagrams. Finally, the calculated results are compared with the other obtained results, and the mass of observing pentaquark P_c and the radius of pentaquark are estimated.

1. Introduction

In the first researches, the existence of multiquark states is illustrated in its simplest possible form, in which baryons are made of three fundamental quarks and mesons from a quark and an antiquark [1]. Indeed, from the point of view of the mathematics and physics, there was no QCD theorem opposing the existence of exotic multiquark. In gauge field theory, the QCD principle allows the existence of multiquarks and hybrids, which include quark and gluonic degrees of freedom [2].

The search for the pentaquark and its probing has a long history. About ten years ago, in the study of pentaquark, a big progress was occurred when LEPS collaborations started a search with a claim to find out strong evidence of the pentaquark with the mass of about 1.540 GeV [3]. Then, many theoretical and experimental approaches were pursued, and many ideas were proposed in this field. For example, Zou and his colleagues suggested that the components of pentaquark can be included in nucleon. Since the heavy quarks play an important role in stabilization of multiquark systems and these play exactly the same role as the hydrogen molecule in QED [4–7], there are theoretical predictions about exotic hidden-charm pentaquark. In particular, the possibility of the existence of hidden-charm molecular baryons composed of an anticharmed meson and a charmed baryon was systematically studied within the one boson exchange model [3].

In 2003, LEPS collaboration reported the evidence of pentaquark s state with content of quark $uudd\bar{s}$ having a very narrow width [8]. Unfortunately, this exotic flavor was not confirmed in subsequent experiments [9, 10]. In fact, the possible theoretical arguments are presented for the nonexistence of a stable pentaquark s in the references [11, 12]. Also, this mode has not been found with a light flavor yet. However, the baryons with light flavor may be able to have a significant pentaquark components [2].

A decade ago, numerous researches were conducted around the world to find exotic particles. The result of these efforts was the observation of the mesonic X, Y, and Z particles in Belle, BESIII, BABAR, and LHCb. Some of these were considered as candidates for exotic states because they do not fit in a regular mesonic structure [1]. The common point of the exotic states is that all of these contain the heavy quarks and antiquarks.

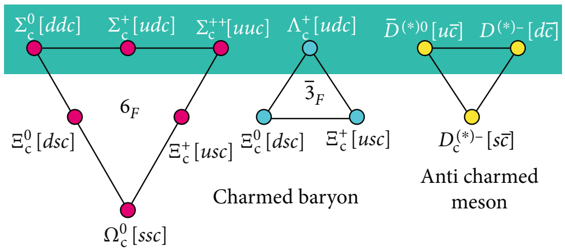

FIGURE 1: (color line) The s-wave charmed baryons with $J^P = 1/2^+$ and pseudoscalar or vector s-wave charmed mesons which form the molecular baryons with double charm [6].

Due to the heavy quarks, the exotic states can be stable, and light modes can be combined with a regular mode [13]. This estimation is consistent with the fact that all of the exotic states have a hidden c or b which this is experimentally observed. If this claim is valid that the heavy components stabilize multiquark systems by particle physics scientists, it will be a natural proposal from their experimental colleagues to search for exotic states consisting clear quarks c and b, for example, $b\bar{c}qq'$ [2].

Here, one hidden-charm molecular baryon composed of an anticharmed meson and a charmed baryon has been studied. This can have one of the two flavors, i.e., symmetric 6_F or antisymmetric flavor $\bar{3}_F$ as shown in Figure 1. So, spin parity of s-wave charmed baryon is $J^P = 1/2^+$ or $J^P = 3/2^+$ for 6_F and $J^P = 1/2^+$ for $\bar{3}_F$. The pseudoscalar or vector anticharmed meson is made of s-wave anticharmed mesons. In Figure 1, hidden-charm molecular states composed of the anticharmed mesons and the charmed baryons are placed inside the green range [6].

Recently, LHCb collaboration has observed two resonance structures $P_c(4380)$ and $P_c(4450)$ with mass and width decay $M_{P_c(4380)} = 4380 \pm 8 \pm 29 MeV$, $\Gamma_{P_c(4380)} = 205 \pm 18 \pm 86$MeV and $M_{P_c(4450)} = 4449.8 \pm 1.7 \pm 2.5 MeV$, $\Gamma_{P_c(4450)} = 39 \pm 5 \pm 19$MeV in the invariant mass spectrum $J/\psi p$ from $\Lambda_b \rightarrow J/\psi pK$. According to the final state $J/\psi p$, it is concluded that two observing states P_c cannot be isosinglet, and these consist of hidden-charm quantum numbers. Also, it is suggested that structures for each of the states P_c are considered for $P_c(4380)$, $\Sigma_c(2455)\bar{D}^*$ and $P_c(4450)$, $\Sigma_c^*(2520)\bar{D}^*$ [2, 3].

Binding energy may be calculated analytically by solving Schrödinger equation with the expanded potential of Pentaquark. This approach offers advantages over numerical solution of the Schrödinger equation for pentaquark. First, all values for binding energy can be calculated, and this is done more accurately. Second wave function can be computed and presented graphically ($\phi(r)$ versus r), from which useful particle characteristics and data could be extracted.

In this work, this is described by analytical solution of pentaquark $\Sigma_c\bar{D}^*$ in four sections: where apart from the introduction in section 1, the potential of pentaquark P_c(4380) is discussed in section 2. Then, using the potential of pentaquark, the Schrödinger equation is analytically solved in section 3. Finally, important conclusions are discussed in section 4.

TABLE 1: $(E_B(\text{MeV}), M_{P_c}(\text{MeV}))$ for $P_c(4380)$ and $g_1 = 0.94$.

Λ(MeV)	$g = 0.51$	$g = 0.59$	$g = 0.67$
800	(-63, 4400.32)	(-40, 4423.32)	(-17, 4446.32)
850	(-62, 4401.32)	(-34, 4429.32)	(-6.7, 4456.62)
900	(-59, 4404.32)	(-26, 4437.32)	—
1000	(-45, 4418.32)	(-0.11, 4463.21)	—
1100	(-22, 4441.32)	—	—

TABLE 2: $(E_B(\text{MeV}), M_{P_c}(\text{MeV}))$ for $P_c(4380)$ and $g_1 = 0.75$.

Λ(MeV)	$g = 0.51$	$g = 0.59$	$g = 0.67$
800	(-90, 4373.32)	(-73, 4390.32)	(-56, 4407.32)
850	(-95, 4368.32)	(-75, 4390.32)	(-53, 4407.32)
900	(-99, 4364.32)	(-74, 4389.32)	(-49, 4414.32)
1000	(-102, 4361.32)	(-67, 4369.32)	(-32, 4431.32)
1100	(-98, 4365.32)	(-51, 4412.32)	(-3.14, 4460.18)
1200	(-86, 4377.32)	—	—

TABLE 3: $(E_B(\text{MeV}), M_{P_c}(\text{MeV}))$ for $P_c(4380)$ and $\Lambda = 800$MeV.

g_1	$g = 0.51$	$g = 0.59$	$g = 0.67$
0.75	(-15.99, 4447.33)	(-15.80, 4447.52)	(-15.62, 4447.7)
0.94	—	(-15.47, 4447.85)	(-15.27, 4448.05)
1.95	—	(-14.01, 4449.32)	—

2. The Potential of Pentaquark P_c and Its Expansion

Pentaquark is considered to be composed of one baryon and one meson. For pentaquark, with such a structure, the potential is given as follows [2]:

$$V_{\Sigma_c\bar{D}^*}(r) = \frac{1}{3}\frac{gg_1}{f_\pi}\nabla^2 Y(\Lambda, m_\pi, r)\mathcal{J}_0\mathcal{G}_0, \tag{1}$$

where the coupling constant $g = 0.59 \pm 0.07 \pm 0.01$ is extracted from the width of D^* [14, 15], $g_1 = 0.75, 0.94, 1.95$ [6, 16]. Also, it is the mass of pion $m_\pi = 135 MeV$ and pion decay constant $f_\pi = 132 MeV$ [2]. The amount of phenomenological cutoff parameter is considered $\Lambda = 0.8 GeV - 2.5 GeV$ [2, 12]. Finally, the $Y(\Lambda, m_\pi, r)$ is [2]:

$$Y(\Lambda, m_\pi, r) = \frac{1}{4\pi r}\left(e^{-m_\pi r} - e^{-\Lambda r}\right) - \frac{\Lambda^2 - m_\pi^2}{8\pi\Lambda}e^{-\Lambda r}. \tag{2}$$

Now, we are calculating and expanding $\nabla^2 Y$:

$$\begin{aligned}\nabla^2 Y(\Lambda, m_\pi, r) &= \frac{m_\pi^2}{4\pi r}\left(e^{-m_\pi r} - e^{-\Lambda r}\right) - \frac{\Lambda^3 - m_\pi^2\Lambda}{8\pi}e^{-\Lambda r} \\ &= \frac{1}{4\pi}\left(b_0 + b_1 r + b_2 r^2 + b_3 r^3 + b_4 r^4 + b_5 r^5 + b_6 r^6 + b_7 r^7 \right. \\ &\quad \left. + b_8 r^8 + b_9 r^9 + \cdots\right),\end{aligned} \tag{3}$$

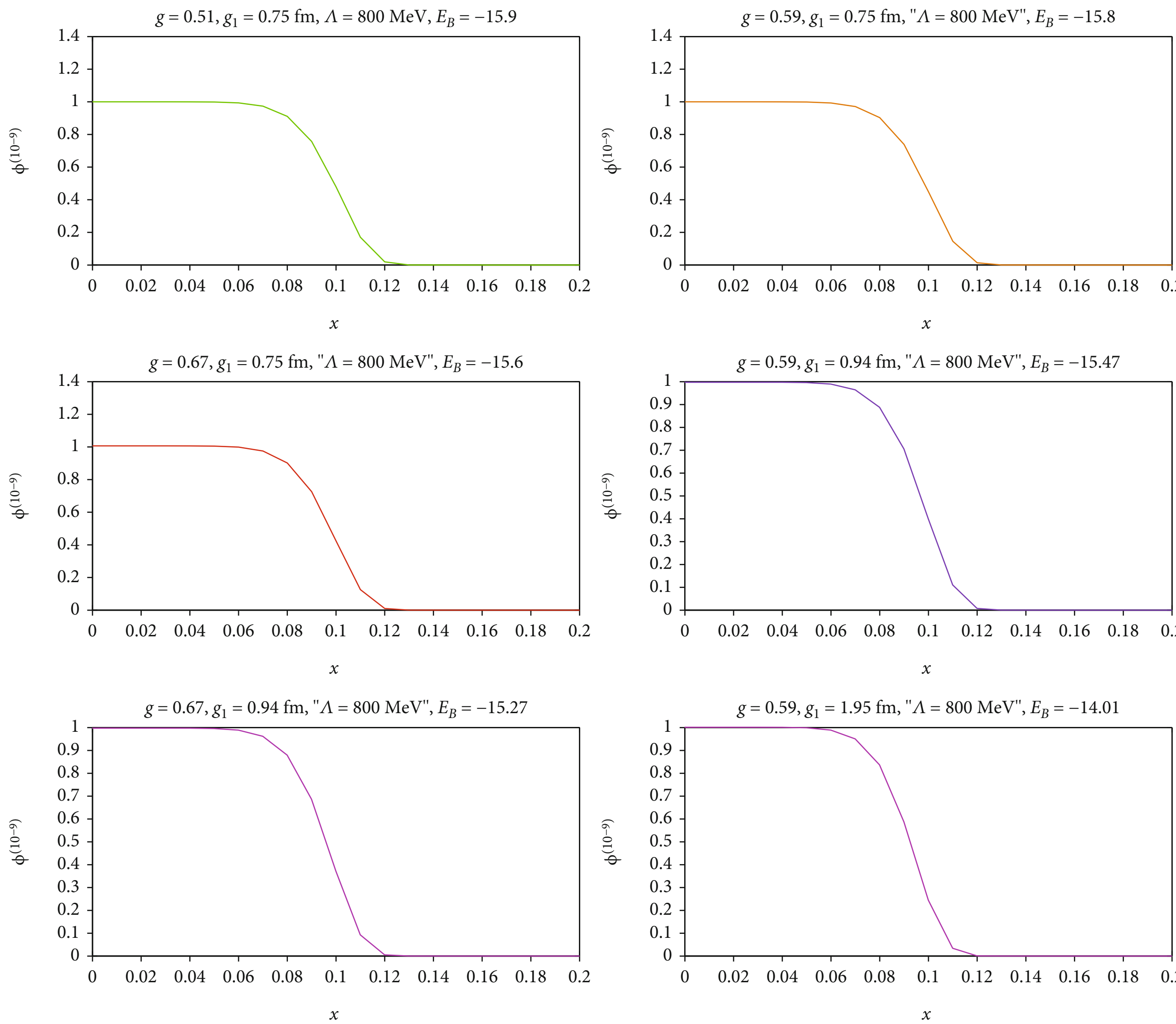

FIGURE 2: $P_c(4380)$ at r in $\Lambda = 800\text{MeV}$ for different values of g and g_1.

where

$$b_0 = \left(-m^3 + \frac{\Lambda^3}{2} + \frac{m^2\Lambda}{2}\right); b_1 = \left(\frac{m^4}{2} - \frac{\Lambda^4}{2}\right),$$
$$b_2 = \left(-\frac{m^5}{3!} + \frac{\Lambda^5}{2\times 2!} - \frac{m^2\Lambda^3}{6\times 2!}\right); b_3 = \left(\frac{m^6}{4!} - \frac{\Lambda^6}{2\times 3!} + \frac{m^2\Lambda^4}{4\times 3!}\right),$$
$$b_4 = \left(-\frac{m^7}{5!} + \frac{\Lambda^7}{2\times 4!} - \frac{3m^2\Lambda^5}{6\times 4!}\right); b_5 = \left(\frac{m^8}{6!} - \frac{\Lambda^8}{2\times 5!} + \frac{m^2\Lambda^6}{3\times 5!}\right),$$
$$b_6 = \left(-\frac{m^9}{7!} + \frac{\Lambda^9}{2\times 6!} - \frac{5m^2\Lambda^7}{14\times 6!}\right); b_7 = \left(\frac{m^{10}}{8!} - \frac{\Lambda^{10}}{2\times 7!} + \frac{3m^2\Lambda^8}{8\times 7!}\right),$$
$$b_8 = \left(-\frac{m^{11}}{9!} + \frac{\Lambda^{11}}{2\times 8!} - \frac{7m^2\Lambda^9}{18\times 8!}\right); b_9 = \left(\frac{m^{12}}{10!} - \frac{\Lambda^{12}}{2\times 9!} + \frac{2m^2\Lambda^{10}}{5\times 9!}\right). \tag{4}$$

3. Solving the Schrödinger Equation for Pentaquark P_c

To investigate the existence of one bound state of pentaquark, we solve the Schrödinger equation with calculated potential in previous section for pentaquark P_c.

The radial Schrödinger equation for two-body systems is

$$\left(\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} - \frac{l(l+1)}{r^2}\right)R_{n,l}(r) + \frac{2\mu}{\hbar}(E - V(r))R_{n,l}(r) = 0. \tag{5}$$

Taking $\hbar = 1$ and changing the variable $\phi(r) = rR_{n,l}(r)$, Eq. (5) becomes

$$\frac{d^2}{dr^2}\phi(r) + 2\mu\left(E - V(r) - \frac{l(l+1)}{2\mu r^2}\right)\phi(r) = 0. \quad (6)$$

By placing the potential $V(r) = V_{\Sigma_c \bar{D}^*}(r)$ and the expanded shape of $\nabla^2 Y$ in Eq. (6), an equation is expressed as follows:

$$\frac{d^2}{dr^2}\phi(r) + 2\mu\Big(E - C_0 - C_1 r - C_2 r^2 - C_3 r^3 - C_4 r^4 - C_5 r^5 - C_6 r^6 - C_7 r^7 - C_8 r^8 - C_9 r^9 + \cdots - \frac{l(l+1)}{2\mu r^2}\Big)\phi(r) = 0, \quad (7)$$

where

$$C_n = B_1 b_n\,;\, n = 0, 1, \cdots, 9\,;\, B_1 = \frac{1}{12\pi}\frac{g g_1}{f_\pi^2}\mathcal{J}_0\mathcal{G}_0. \quad (8)$$

In Eq. (8), for $\Sigma_c \bar{D}^*$ with $I = 1/2$, $J = 3/2$, it is placing the numerical value of $\mathcal{J}_0\mathcal{G}_0 = 1$ product from literature [2].

By considering the following, the proposed reply (cf. [17, 18]) for the differential Eq. (7) yields

$$\phi(r) = N(r)e^{M(r)} = r^n e^{M(r)}. \quad (9)$$

Differentiating second degree of ϕ in Eq.(9) gives

$$\phi''(r) = \left(n(n-1)r^{-2} + 2nM' r^{-1} + M'' + M'^2\right) r^n e^M. \quad (10)$$

Here, we are solving differential equation (7) by considering expansion $V(r)$ up to the 10^{th} order for calculating binding energy E_B of pentaquark. Hence, an approximation was attempted up to the 10^{th} order, which not only expanded the potential behavior up to the 10^{th} order similar to the potential behavior in equation (1) but also the resulting of binding energy that has sufficient precision compared to numerical literature [2, 19]. This is indicating the adequacy of approximation.

Now, we can considered two position for $\phi(r)$. The first $\phi(0) = 0$ is for $n = 1$, and the second $\phi(0) = cte$ is considered for $n = 0$ that *cte* means a constant. Thus, we study them in separate subsections.

3.1. The Position $\phi(0) = 0$. To considered $\phi(0) = 0$ then $M(r)$ yields [19]:

$$M(r) = a_1 r^2 + a_2 r^3 + a_3 r^4 + a_4 r^5 + a_5 r^6 + a_6 r^7 + a_7 r^8 + a_8 r^9 + a_9 r^{10}. \quad (11)$$

Replacing Eq. (11) into Eq. (10) and comparing two Eqs. of (7) and (10), the following expression (Eq.(12)) for r^{-2} is obtained in terms of $l(l+1)$ in Eq.(8), as well as a system of 11 nonlinear equations expressed later:

$$n(n-1) = l(l+1). \quad (12)$$

In the base state, two values for n, namely, $n = 0$ and $n = 1$ are obtained that according to Eq. (9) and the condition $\phi(0) = 0$, the value $n = 0$ could be unacceptable, and $N(r) = r$ will be. Therefore, $\phi(r)$ and $\phi''(r)$ are obtained as follows:

$$\begin{aligned} \phi(r) &= r e^{M(r)}, \\ \phi''(r) &= \left(2M' r^{-1} + M'' + M'^2\right) r e^M. \end{aligned} \quad (13)$$

After replacing Eq. (11) in Eq. (13) and separately equal to the different powers of r, the following nonlinear equations are obtained:

$$\begin{aligned}
6a_1 &= -2\mu(E - C_0),\\
12a_2 &= 2\mu C_1,\\
20a_3 + 4a_1^2 &= 2\mu C_2,\\
30a_4 + 12a_1 a_2 &= 2\mu C_3,\\
42a_5 + 9a_2^2 + 16a_1 a_3 &= 2\mu C_4,\\
56a_6 + 20a_1 a_4 + 24a_2 a_3 &= 2\mu C_5,\\
72a_7 + 16a_3^2 + 24a_1 a_5 + 30a_2 a_4 &= 2\mu C_6,\\
90a_8 + 28a_1 a_6 + 36a_2 a_5 + 40a_3 a_4 &= 2\mu C_7,\\
110a_9 + 25a_4^2 + 32a_1 a_7 + 42a_2 a_6 + 48a_3 a_5 &= 2\mu C_8,\\
132a_{10} + 36a_1 a_8 + 48a_2 a_7 + 56a_3 a_6 + 60a_4 a_5 &= 2\mu C_9.
\end{aligned} \quad (14)$$

Here, by replacing the numerical values of constants for pentaquark $P_c(4380)$, $\Sigma_c = 2455MeV$, and $\bar{D}^* = 2008.32MeV$, we obtained the binding energy of pentaquark [2]. In Tables 1 and 2, different values of binding energy from Eq. (14) for $P_c(4380)$ have been shown.

In the tables above M_{P_c}, is calculated as follows:

$$M_{P_c} = M_{\Sigma_c} + M_{\bar{D}^*} + E_B. \quad (15)$$

According to the obtained values, it is observed that in $g_1 = 0.75$, the results for pentaquark mass are much closer to the $M_{P_c(4380)}$. Also, to check the results, one of the main differences between this paper and the other references [2, 20] is that the acceptable results are obtained for the $M_{P_c(4380)}$ only in the $800\text{MeV} \le \Lambda \le 1200\text{MeV}$, and these cannot be found in $\Lambda \ge 1200\text{MeV}$.

3.2. The Position $\phi(0) = cte$. Now, we considered $\phi(0) = cte$; thus, $M(r)$ will be as follows [19]:

$$M(r) = a_1 r + a_2 r^2 + a_3 r^3 + a_4 r^4 + a_5 r^5 + a_6 r^6 + a_7 r^7 + a_8 r^8 + a_9 r^9. \quad (16)$$

Here, replacing Eq. (15) into Eq. (10) and comparing it to Eq. (7), a similar equation is obtained by Eq. (12). In this position, i.e., $\phi(0) = cte$, the value $n = 0$ could be acceptable, and $N(r) = 1$ will be. Therefore, $\phi(r)$ and $\phi''(r)$ are obtained as follows:

$$\begin{aligned} \phi(r) &= e^{M(r)}, \\ \phi''(r) &= \left(M'' + M'^2\right)e^M. \end{aligned} \tag{17}$$

The same as before, by replacing Eq.(15) in Eq.(16), 10 nonlinear equations are obtained, and we have

$$\begin{gathered} 2a_2 + a_1^2 = 2\mu(C_0 - E), \\ 6a_3 + 4a_1a_2 = 2\mu C_1, \\ 12a_4 + 4a_2^2 + 6a_1a_3 = 2\mu C_2, \\ 20a_5 + 8a_1a_4 + 12a_2a_3 = 2\mu C_3, \\ 30a_6 + 9a_3^2 + 10a_1a_5 + 16a_2a_4 = 2\mu C_4, \\ 42a_7 + 12a_1a_6 + 20a_2a_5 + 24a_3a_4 = 2\mu C_5, \\ 56a_8 + 16a_4^2 + 14a_1a_7 + 24a_2a_6 + 30a_3a_5 = 2\mu C_{6,} \\ 72a_9 + 16a_1a_8 + 28a_2a_7 + 36a_3a_6 + 40a_4a_5 = 2\mu C_{7,} \\ 25a_5^2 + 18a_1a_9 + 32a_2a_8 + 42a_3a_7 + 48a_4a_6 = 2\mu C_8, \\ 36a_2a_9 + 48a_3a_8 + 56a_4a_7 + 60a_5a_6 = 2\mu C_9. \end{gathered} \tag{18}$$

Also, in this position for $P_c(4380)$, we obtained the binding energy of pentaquark and the numerical coefficients of wave function. Table 3 shows the amount of binding energy from Eq. (17) and $\Lambda = 800$MeV for $P_c(4380)$.

To confirm the existence of pentaquark states, binding energy must be negative, i.e., $E_B < 0$. Also, the total mass of particles contributing of pentaquark (i.e., the sum of baryon Σ_c and meson $\bar{D}^*$ masses) in addition to binding energy is closer to the mass of pentaquark $P_c(4380)$. Here, the obtained results indicate that the binding energy ranging $-102 \le E_B \le -0.11$ for pentaquark $P_c(4380)$ is negative. Also, it conforms to calculated results in the literatures [2, 20–23]. As mentioned above, obtained results are acceptable to a great extent, and they could be considered a clear evidence for the existence of a bound of five-quark state.

Figure 2 demonstrates the wave function's diagrams for pentaquark P_c in $\Lambda = 800$MeV, at different values of g and g_1. These charts tend to zero at the given value. As shown in the graphs, wave functions become zero in $x \sim 0.12 - 0.13$, indicating that the maximum pentaquark radius ranges from 23.67 to 25.64 fm.

4. Conclusions

In this article, the pentaquark $P_c(4380)$ system consisting of baryon Σ_c and $\bar{D}^*$ meson has been considered. The obtained potential for pentaquark in reference [2] was presented and expanded. Then, expanded potential was replaced in the Schrödinger equation, and that was solved as a bound state of two-body systems. By solving this to analytically approach and according to the values of constants and cutoff, 10 nonlinear differential equations and binding energy E_B of pentaquark P_c and wave function coefficients were obtained. The resulting E_B and wave function were presented using tables and diagrams in the previous section, which could confirm the existence of a bound state of pentaquark $P_c(4380)$. Then, it is specified that the wave function plots tend to be zero at a given value. Therefore, the maximum radius of pentaquark P_c was found out which ranged from $x = 23.67$fm to $x = 25.64$fm. We observed that the calculated values matches with the findings of others regarding the mass of $P_c(4380)$. Also, the advantage of this paper lies in its method, which other references are numerically calculated, and this paper delivers the obtained results analytically. Hence, the results are more comprehensive and complete compared to them.

Data Availability

The authors confirm that the data supporting the findings of this study are available within the article.

Disclosure

This research was not receive specific funding; thus, funding agencies have no role in the design of this study, in analysis, or interpretation of the data in writing the manuscript, or in the decision to publish the results.

Conflicts of Interest

The authors also declare that there is no conflict of interests regarding the publication of this paper.

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