Hindawi
Advances in High Energy Physics
Volume 2021, Article ID 6628693, 13 pages
https://doi.org/10.1155/2021/6628693

Review Article

Aspects of Semiclassical Black Holes: Development and Open Problems

Alexander Y. Yosifov

Space Research and Technology Institute, Bulgarian Academy of Sciences, Akad. G. Bonchev Street, Building 1, Sofia 1113, Bulgaria

Correspondence should be addressed to Alexander Y. Yosifov; alexanderyyosifov@gmail.com

Received 20 November 2020; Accepted 11 February 2021; Published 25 February 2021

Academic Editor: Rong Gen Cai

The current work is a review, dedicated to the study of semiclassical aspects of black holes. We begin by briefly looking at the main statements of general relativity. We then consider the Schwarzschild, Kerr, and Reissner-Nordstrom black hole solutions and discuss their geometrical properties. Later, the thermodynamic nature of black holes is established. In light of this, we formulate the information loss problem and present the most promising approaches for addressing it with emphasis on introducing low-energy quantum corrections to the classical general relativity picture. Finally, in the context of multimessenger astronomy, we look at naked singularities as possible gravitational collapse endstates and their role in the unitarity of quantum mechanics and discuss their observational prospects.

1. Introduction

Black holes are the most exotic objects in the universe. Despite their apparent simplicity, motivated by the handful of parameters required to describe them, namely, mass M, angular momentum J, and charge Q, to this day, black holes continue to have a central role in theoretical physics. Our understanding of black holes has greatly improved over the last decades. They were initially viewed as entirely *black* objects out of which nothing can escape and were thus thought to have neither entropy nor temperature. Today, we know that black holes are not really black. They have entropy, and in fact, we have a strong reason to believe that they are the most entropic objects in the universe. Although the microscopic origin of the black hole microstates remains a matter of debate, Bekenstein and Hawking identified it with a geometrical quantity, the horizon area, through the Bekenstein-Hawking entropy bound. (In 2019, the Event Horizon Telescope Collaboration took the first image of a black hole, demonstrating the robustness of general relativity.)

Assigning entropy to black holes led to the important realization that they should have some nonzero temperature and hence radiate particles to asymptotic infinity. Associating temperature to black holes is what took us from the classical to the semiclassical realm. Black hole radiation emerges in the semiclassical regime, where we consider classical geometry, immersed in quantum fields. We now know that the temperature is inversely proportional to the black hole's mass. This inverse proportionality is assumed to play a major role in the late-time evolution of the system. The derivation of Hawking radiation presented a great challenge to quantum mechanics as it suggested that the evolution of quantum states is nonunitary. Lack of unitarity would mean initially that pure quantum states evolve into mixed ones, which would lead to loss of information. The question of whether quantum mechanics is unitary gave rise to the well-known information loss paradox which has been one of the main driving forces behind much of the progress in the field.

The paper is organized as follows. In Section 2, we put forward the basic concepts of general relativity and show

how they lead to the formulation of the Einstein equations. Later, in Section 3, we show the most widely used black hole solutions to the Einstein equations and discuss their basic properties. Section 4 is devoted to the transition from the classical to the semiclassical regime as we present the laws of black hole thermodynamics. Then, in Section 5, we put forward an intuitive formulation of the information loss paradox. Later, in Section 6, we review the candidate models for resolving the information paradox, where we point the main shortcomings and open questions. We then emphasize a toy model which makes use of quantum corrections to the classical general relativistic picture as a possible way out of the paradox and argue that it is the most efficient in terms of both deviations from the standard prescription and possible detection. Finally, in Section 7, we discuss naked singularities as a possible endstate of generic collapse scenarios and examine their observational distinguishability to classical black holes in light of multimessenger astronomy.

2. Basic Ideas of General Relativity

General relativity is a dynamical theory which views gravity in terms of spacetime curvature. The Einstein equations are nonlinear second-order partial differential equations [1–4]. Two essential principles follow from that, namely, the equivalence principle and general covariance.

The equivalence principle states that the motion of a test particle in a gravitational field is independent of its mass. Thus, there is equivalence between the gravitational mass and the inertial mass of the test particle. That is to say, gravity does not discriminate and treats all masses equally.

General covariance, on the other hand, says that the laws of physics should be identical to all observers, meaning the theory is coordinate invariant. The principle of general covariance suggests that the field equations are tensorial and relate the spacetime curvature to the matter content, *i.e.*, the Ricci tensor $R_{\mu\nu}$ to the stress-energy tensor $T_{\mu\nu}$.

In particular, suppose we have a (3 + 1)-dimensional Riemannian geometry with Minkowskian signature (-,+,+,+). For some general coordinates x^{μ}, where $\mu \in 0, 1, 2, 3$, the metric admits a line element

$$ds^2 = g_{\mu\nu} dx^{\mu} \otimes dx^{\nu}, \tag{1}$$

where for each spatial point we can define a symmetric metric tensor

$$g_{\mu\nu}(x) = \eta_{\alpha\beta} \frac{\partial X_x^{\alpha}(x)}{\partial x^{\mu}} \frac{\partial X_x^{\beta}(x)}{\partial x^{\nu}}, \tag{2}$$

where $g_{\mu\nu}$ allows us to determine the length between a pair of points on a manifold and $\eta_{\alpha\beta}$ denotes the Minkowski metric. The metric tensor thus brings a notion of causality. Equation (1) gives the infinitesimal distance between a pair of neighboring spatial coordinates. Note that in this case, the coefficients vary smoothly from point to point.

Here, the metric tensor, despite the smooth geometry, is nontrivially related to the spatial coordinates, and so, the spacetime, spanned by it, has a much richer structure than the Minkowski metric with its nonvarying connection terms $\eta_{\mu\nu}$. Every manifold has a unique connection which makes the covariant derivative with respect to that connection vanish:

$$\nabla_{\mu} g^{\mu\nu} = 0. \tag{3}$$

That unique connection is given in terms of the Christoffel symbols which, to first order, read

$$\Gamma^{\lambda}_{\mu\nu} \coloneqq \frac{1}{2} g^{\lambda\sigma} \left(\partial_{\mu} g_{\nu\sigma} + \partial_{\nu} g_{\mu\sigma} - \partial_{\sigma} g_{\mu\nu} \right). \tag{4}$$

Let us now define the Riemann tensor which plays an important role in general relativity:

$$R^{\rho}_{\sigma\mu\nu} = \partial_{\mu} \Gamma^{\rho}_{\nu\sigma} - \partial_{\nu} \Gamma^{\rho}_{\mu\sigma} + \Gamma^{\rho}_{\mu\lambda} \Gamma^{\lambda}_{\nu\sigma} - \Gamma^{\rho}_{\nu\sigma} \Gamma^{\lambda}_{\mu\sigma}, \tag{5}$$

where by contracting an index, we obtain the so-called Ricci tensor

$$R_{\mu\nu} = R^{\lambda}_{\mu\nu\lambda}. \tag{6}$$

The trace of (6) and coordinate changes leave invariant a quantity which is defined at every point on a manifold, called the Ricci scalar

$$R = R^{\mu}_{\mu}. \tag{7}$$

The Ricci (curvature) scalar quantifies, at each point, the metric deviation from flat spacetime.

Having defined the metric tensor for some curved spacetime region (2) and knowing the Christoffel symbols at each point (4), we can now consider the distance between a pair of points by the geodesic equation (Note that for Minkowski spacetime, the Christoffel symbols vanish, and $g_{\mu\nu=}\eta_{\mu\nu}$ which is also a valid solution.)

$$\frac{d^2 x^{\lambda}}{ds^2} + \Gamma^{\lambda}_{\mu\nu}(x) \frac{dx^{\mu}}{ds} \frac{dx^{\nu}}{ds} = 0. \tag{8}$$

Note that (8) describes the trajectory of a freely moving particle.

Let us now turn our attention to the Einstein equations. As is well known, general relativity relates geometrically the spacetime curvature to the matter (energy and momentum) content, given by the stress tensor $T_{\mu\nu}$ which is conserved

$$\nabla^{\mu} T_{\mu\nu} = 0. \tag{9}$$

The stress tensor $T_{\mu\nu}$ obeys the following energy conditions which are expected to hold for generic matter fields (We should note that within quantum field theory, both conditions can be violated.):

(i) Null energy condition: $T_{\mu\nu}k^{\mu}k^{\nu} \geq 0$ for k^i null vectors

(ii) Dominant energy condition: $T_{\mu\nu}\tilde{k}^{\mu}\tilde{k}^{\nu} \geq 0$ for $\tilde{k}^i$ time-like vectors

The so-called Einstein tensor $G_{\mu\nu}$, expressed in terms of the Ricci tensor (6), scalar (7), and metric tensor (2), satisfies (9) and reads

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}. \tag{10}$$

Having defined $G_{\mu\nu}$, we can now write the Einstein equations:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi GT_{\mu\nu}, \tag{11}$$

where $R_{\mu\nu}$ is calculated from the connection coefficients. In case $R_{\mu\nu} = 0$, the Weyl curvature remains nonzero. The l.h.s. of (11) gives the metric curvature, more intuitively, the deviation from Minkowski geometry, while the r.h.s. is the matter content. Not surprisingly, (11), in the weak-field limit, reduces to the Newton equation:

$$\nabla^2\Phi = 4\pi G\rho. \tag{12}$$

We can now summarize the basic principles underlying general relativity. Namely,

(i) the laws of physics are coordinate invariant, which is to say, there is no preferred coordinate system

(ii) $T_{\mu\nu}$ is the curvature source. That is due to the relation between inertial mass and energy, on the one hand, and the equivalence between inertial and gravitational mass, on the other

(iii) in a vacuum, where the Christoffel symbols vanish, there should also be a well-defined solution to the field equations

One of the great predictions of general relativity is the formation of black holes, and causal boundaries, *i.e.*, horizons. This will motivate the use of semiclassical physics (classical gravity coupled to quantum field theory) later in Section 4.

3. Black Hole Solutions

In this section, we outline the most widely used black hole solutions and discuss their basic geometric properties [5–7].

3.1. Schwarzschild Black Hole. The Schwarzschild solution to the Einstein equations (11) describes a static spherically symmetric black hole, which is fully characterized by just one parameter, its mass M., Figure 1. The metric reads

$$\begin{aligned} ds^2 = g_{\mu\nu}dx^{\mu} \otimes dx^{\nu} = -\left(1 - \frac{2M}{r}\right)dt^2 + \left(\frac{dr^2}{1 - 2M/r}\right) \\ + r^2\left(d\theta^2 \sin^2\theta d\varphi^2\right), \end{aligned} \tag{13}$$

where $r = 0$ denotes the singularity.

By examining the metric, we see that it admits two pathological regions, where only one can be cured by suitable coordinate change. In particular, notice that the $(1 - 2M/r)$ term vanishes when $r = 2M$. This implies that the horizon is singular. However, this has been shown to be only a coordinate singularity which is easily treatable. The second singular region is at $r = 0$, and the singularity there is spacelike (*i.e.*, physical), meaning the Kretschmann, $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} = 48G^2M^2/r^6$, diverges. Unlike the coordinate one, dealing with this curvature singularity requires physics which still remains beyond our reach.

The Schwarzschild coordinates are not "penetrating" coordinates and are thus only meaningful in the region $r \in (2M,\infty)$. Note that within the black hole interior, in the region $r \in (0, 2M)$, all future-directed timelike geodesics will eventually reach $r = 0$, while all the outgoing trajectories for $r > 2M$ will be moving away from $r = 2M$ as t increases.

3.2. Kerr Black Hole. The Kerr solution describes a stationary axisymmetric black hole, where (i) the Killing horizon and the event horizon need not coincide and (ii) is characterized, in addition to its mass M, by angular momentum J. The Kerr metric, in Boyer-Lindquist coordinates, is the following:

$$ds^2 = \left(1 - \frac{2Mr}{\Sigma}\right)dt^2 - \frac{4Mra\ \sin^2\theta}{\Sigma}dtd\phi - \frac{\Sigma}{\Delta}dr^2 - \Sigma d\theta^2, \tag{14}$$

where

$$\begin{aligned} a &= \frac{J}{M}, \\ \Sigma &= r^2 + a^2\cos^2\theta, \\ \Delta &= r^2 - 2Mr + a^2, \end{aligned} \tag{15}$$

where a is the rotation parameter which gives the spin-to-mass ratio.

The metric admits two horizons, respectively, at $r_{\pm} = M \pm \sqrt{M^2 - a^2}$, where $r{:=}\{r_- = \mathscr{CH}^+\}$ is the inner (Cauchy) horizon and $r{:=}\{r_+ = \mathscr{H}^+\}$ is the outer (event) horizon, which is generated by a null hypersurface. (The Cauchy horizon plays an important role in terms of extendability and hyperbolicity properties of the geometry of rotating solutions to the Einstein equations (11), and we will return to it in Section 7.)

The Boyer-Lindquist coordinates are a generalization of the Schwarzschild and Kerr systems of coordinates. They are very useful because (i) they reduce the number of off-diagonal terms to just a single one, (ii) as $a \longrightarrow 0$, they reduce

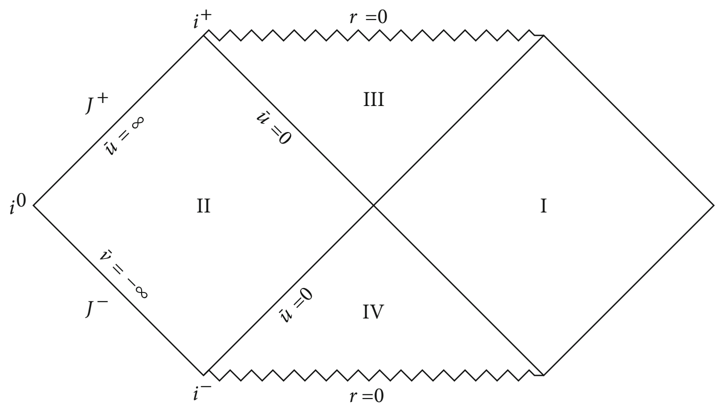

Figure 1

to the usual Schwarzschild coordinates, and (iii) for $M \longrightarrow 0$, the Boyer-Lindquist coordinates describe flat Minkowski space. Therefore, one can intuitively think of the Kerr metric as a perturbed (distorted) Minkowski space. If we are at the equatorial plane, $\Theta = \pi/2$, the metric (14) is ill-defined for $\Sigma = 0$ as we encounter a ring singularity.

3.3. Reissner-Nordstrom Black Hole. The Reissner-Nordstrom solution describes a charged nonrotating black hole. Its line element is given as

$$ds^2 = -\left(1 - \frac{2M}{r} + \frac{r_Q^2}{r^2}\right)dt^2 + \left(\frac{dr^2}{1 - 2M/r + r_Q^2/r^2}\right) + r^2\left(d\theta^2 \sin^2\theta d\varphi^2\right), \tag{16}$$

where $r_Q^2 = Q^2/4\pi\varepsilon$ denotes the characteristic length scale and $(1/4)\pi\varepsilon$ is the Coulomb force.

Evidently, this is a charged, $Q > 0$, Schwarzschild black hole in asymptotically flat spacetime. The metric (16) differs, however, from the noncharged $Q = 0$ solution in one important respect. Namely, unlike Schwarzschild (13) and similar to Kerr (14), it admits two horizons, inner r_- and outer r_+, denoted by

$$r_\pm = M \pm \sqrt{M^2 - r_Q^2}. \tag{17}$$

Having once crossed the outer horizon, ingoing rays become timelike and can no longer escape to asymptotic infinity. Depending on the ratio between M and r_Q^2, we can face one of the three scenarios. That is, for $M > 2r_Q^2$, the metric has two horizons which need not coincide. In this case, as far as an asymptotic observer is concerned, infalling geodesics crossing r_+ will be infinitely redshifted. For $M < 2r_Q^2$, a naked singularity forms. Namely, the ring-like singularity is no longer hidden behind the outer horizon and can be probed by asymptotic observers. (The recent advancements in multimessenger astronomy coming from the Event Horizon Telescope (EHT) and LIGO/Virgo have made it possible to probe the region close to the would-be horizon and distinguish the type of compact object. So far, the measurements we have done match the general relativity predictions. We look at naked singularities in more detail in Section 7). When $M = 2r_Q^2$, we have an extremal Reissner-Nordstrom black hole. In this case, $r_- = r_+ = M$, and so, the black hole has only one horizon. In this scenario, an observer who has crossed the horizon does not need to approach the singularity at $r = 0$. This solution, however, is highly unstable since even the slightest perturbation will break the equality and will take us to one of the two other possible ratios.

4. Semiclassical Black Holes

The classical laws of black hole mechanics were long appreciated for being reminiscent of the laws of thermodynamics, and one was thus tempted to make a parallel between them. A step in this direction was motivated by the apparent ease of violating the second law of thermodynamics which led Bekenstein to formulate the generalized second law and hence assign entropy to black holes, which was assumed to be proportional to the horizon area. Later, a big step towards facilitating the thermodynamic properties of black holes was made by Hawking when he considered the semiclassical case of a black hole surrounded by quantum fields. After his derivation of the thermal flux, which was found to be related to the surface gravity, the interpretation of black holes as thermodynamical systems was established.

4.1. Generalized Second Law. In the early 1970s, the robustness of the second law of thermodynamics was not yet realized. It seemed trivial to be violated in any spacetime region which included a black hole. In particular, an outside observer could simply throw matter into the black hole, and from her perspective, the entropy of the outside region (*i.e.*, the rest of the universe) would decrease. This motivated Bekenstein to assign nonzero entropy to black holes, proportional to their horizon area [8–10]:

$$S_{BH} = \frac{A}{4G\hbar}. \tag{18}$$

Bekenstein then formulated the generalized second law of thermodynamics which states that the total entropy, black hole entropy plus the entropy of the matter field outside the black hole, never decreases:

$$\Delta(S_{\mathrm{BH}} + S_{\mathrm{out}}) \geq 0. \quad (19)$$

This way, the entropy of matter thrown into a black hole is no longer unaccounted for. Although simple at first sight, this entropy relation is very fundamental and has deep roots in both gravity and quantum field theory.

4.2. Hawking Radiation. Assuming the validity of the generalized second law, black holes must have some nonzero temperature, and thus, they must radiate particles to asymptotic infinity. Following those nonclassical assumptions, in [11], Hawking used a quantum field theory in a curved spacetime framework and considered a static black hole with a scalar field in the background. The scalar field, considered to be in a vacuum state at early times, was decomposed into positive and negative frequency modes which he then traced to the asymptotic past. As a result, he discovered that, due to the strong gravitational field in the black hole background, particle pairs were created. (We are not going to repeat the derivation of Hawking radiation as it is outside the scope of this paper. For a detailed discussion, see [12–14].) Imagine an observer, carrying a measuring apparatus, approaches a black hole from asymptotic infinity. Far away from the black hole, the apparatus will not measure any particles. Suppose now that the observer passes closely by the black hole and accelerates to infinity again. If she now checks her measuring apparatus, it will have recorded some particles, created in the vicinity of the black hole, which are emitted to asymptotic infinity. Hence, the black hole emits blackbody radiation with a number operator expectation value:

$$\langle N \rangle = \frac{1}{\left(e^{8\pi M\omega} + 1\right)}, \quad (20)$$

where ω is the mode frequency at temperature

$$T = \frac{\kappa}{2\pi} \quad (21)$$

and κ is the surface gravity. In the Schwarzschild metric, $\kappa = 1/4M$.

4.3. Black Hole Thermodynamics. Hints about the thermodynamic properties of black holes came prior to the discovery of Hawking radiation with the formulation of the area theorem. The area theorem says that the horizon area should increase in any general physical situation. Of course, it was developed entirely on a classical gravity basis.

4.3.1. The Zeroth Law. The zeroth law of black hole thermodynamics states that the surface gravity κ of a stationary solution is uniform throughout the horizon. The statement can be proven without using the field equations. Consider the Killing field which can be interpreted as a generator of the horizon [10, 15]

$$\chi_a = \xi_t + \Omega_H \xi_\phi, \quad (22)$$

where Ω_H is the angular velocity of the horizon, which vanishes for static solutions.

In Schwarzschild coordinates, the Killing field χ_a is timelike for $r > 2M$, null for $r = 2M$, and spacelike for $r < 2M$. Since the Killing field on the horizon is null

$$\chi_a \chi^a = 0, \quad (23)$$

we can write

$$\chi^b \chi_{a;b} = \kappa \chi_a, \quad (24)$$

where κ is the surface gravity. Taking the Lie derivative of (24) yields

$$\kappa, a\xi^a = 0. \quad (25)$$

Evidently, κ is constant along the generated null Killing horizon. It is straightforward to see the similarity with the zeroth law of thermodynamics which states that the temperature of a system in thermal equilibrium is constant.

4.3.2. The First Law. The first law of black hole thermodynamics makes a statement about the constancy of the stress tensor, where for the case of a rotating charged black hole it reads

$$dM = \kappa \frac{dA}{8\pi G} + \Omega dJ + \Phi dQ, \quad (26)$$

where all the terms are defined locally on the horizon and κ plays the role of temperature.

This is a dynamical equation which shows how the macroscopic black hole parameters react to perturbations. In its simplest form, where $\Phi = J = 0$, (26) can be written as

$$\Delta M = \int T_{\mu\nu} \xi^\mu k^\nu d\lambda dA, \quad (27)$$

where we can expand (27) as

$$\begin{aligned} \Delta M &= \left(\frac{\kappa}{8\pi G}\right) \int R_{\mu\nu} k^\mu k^\nu \lambda d\lambda dA = \left(\frac{\kappa}{8\pi G}\right) \int \frac{d\rho}{d\lambda} \lambda d\lambda dA \\ &= \left(\frac{\kappa}{8\pi G}\right) \int (-\rho) d\lambda dA = \left(\frac{\kappa}{8\pi G}\right) \Delta A. \end{aligned} \quad (28)$$

Here, A is the horizon area, λ is the affine connection term, and k^ν is a tangent vector to the horizon generators. Moreover, the stress tensor is given in terms of the Riemann tensor and the tangent and Killing vectors.

4.3.3. The Second Law. The second law of black hole thermodynamics is related to the well-known Hawking area theorem. It states that, assuming the validity of the cosmic censorship conjecture and the positive energy condition, the horizon area of a black hole can never decrease *classically*. It is generally stated as

$$dA \geq 0. \tag{29}$$

Quantum mechanically, of course, black holes do decay via Hawking radiation. As it was shown in the generalized second law, this thermal spectrum radiation has some nonzero entropy; thus, the total entropy never decreases. Apparently, this is reminiscent of the second law of thermodynamics.

4.3.4. The Third Law. The third law of black hole thermodynamics says one cannot reduce the horizon surface gravity κ to zero in a fine number of steps. At first sight, one is tempted to conclude that given $\kappa = 1/4M$, in order for κ to vanish, infinite mass has to be added to the black hole. Or said otherwise, the surface gravity can only be reduced to zero in an infinite number of steps. Let us look at the case of a Reissner-Nordstrom metric. Its surface gravity is defined as

$$\kappa = \frac{4\pi\mu}{A}, \tag{30}$$

where μ is the mass-to-charge ratio, where for an extremal black hole, $\mu = 0$.

When $\mu = 0$, the horizon area is given as

$$A = 4\pi\left(2M^2 - Q^2\right). \tag{31}$$

Clearly, the above equation is ill-defined since it implies vanishing temperature but a nonzero area. So if $M^2 < Q^2$, this would mean that there is a naked singularity which violates the censorship conjecture.

5. The Information Paradox

With the derivation of Hawking radiation, it was clear that there was a clash of principles. On the one hand, traditional general relativistic description suggests that when a star collapses to a black hole, a singularity forms which, by the censorship conjecture, is hidden from the rest of the universe by an event horizon. The event horizon is a causal boundary which prevents outside observers from probing the black hole interior as well as particles from the inside to escape. Restricting ourselves to the general relativistic picture, matter falls inside a black hole and reaches the singularity in some finite proper time, and thus, the information about its quantum state is lost. Quantum mechanically, however, the evolution of quantum states is unitary. Hence, once matter enters the black hole, and later the system evaporates, radiating quanta to asymptotic infinity, the information about the quantum state of the perturbed matter should be preserved. An eventual loss of information would imply, given the emitted Hawking quanta are in a maximally mixed state, that a black hole turns a pure state into a density matrix (36). Following quantum mechanics, the combined state of the black hole and the emitted Hawking radiation is given by the tensor product of their Hilbert spaces

$$\mathscr{H} = \mathscr{H}_{\mathrm{BH}} \otimes \mathscr{H}_{\mathrm{rad}}, \tag{32}$$

where

$$\begin{aligned} \Psi_A &\in \mathscr{H}_{\mathrm{BH}}, \\ \Psi_B &\in \mathscr{H}_{\mathrm{rad}}. \end{aligned} \tag{33}$$

Hence, the general pure state Ψ takes the following form

$$|\Psi\rangle = |\Psi\rangle_A \otimes |\Psi\rangle_B. \tag{34}$$

Alice, an outside observer, only having access to the emitted Hawking radiation, sees it as being in a thermal state, described by a reduced density matrix

$$\rho_B = \mathrm{Tr}_A \rho_{AB}. \tag{35}$$

That is, as far as she is concerned, a pure state has evolved into a density matrix

$$|\Psi\rangle \longrightarrow \rho. \tag{36}$$

The basic framework suggests that pairs of entangled particles are created in the near-horizon region. The positive energy one is radiated to asymptotic infinity I^+, while the negative energy partner falls inside, causing the black hole to lose mass. This process continues until the black hole radiates away all of its mass. Alice, collecting the emitted Hawking particles, sees them in a thermal state (35). That is because as far as she is concerned, the part of the density matrix, describing the remaining black hole degrees of freedom, is traced out (hidden behind the horizon), and so, the emitted quanta are described by a reduced density matrix. By the no-hair conjecture, nothing about the black hole, apart from M, J, and Q, can be known by an outside observer. Therefore, information is effectively lost.

6. Approaches to the Paradox

Through the years, there have been many different proposals for addressing the information loss problem [16, 17]. Some models advocate for more radical modifications to the standard black hole description, while others suggest that small corrections to the Hawking emission may be enough to safe unitarity of quantum theory. In the current section, we briefly review some of the more prominent proposals.

6.1. Fuzzballs. The fuzzball proposal [18–21] motivated by superstring theory models suggests that black holes are in fact giant fuzzball objects. The general framework proposes that there is a membrane (fuzzball surface) just outside the would-be horizon

$$r_f = 2M + \delta, \tag{37}$$

where δ is a small positive constant.

Since this surface is physical, and cannot be omitted by suitable coordinate change, the fuzzball has no interior region. Effectively, the whole black hole is replaced by a horizon-area-sized fuzzball via extremely unlikely quantum tunneling effects which in this case are associated with gross violations of energy/momentum conservation. So, the infalling matter would hit the fuzzball surface and be absorbed. Even though nothing prevents infalling particles from being reflected back to asymptotic infinity, the large microscopic entropy of the system makes reflection nontrivial, meaning not every degree of freedom is reflected in a mirror-like fashion.

Unlike classical black holes, here, the microstate of the system is not given by the generic semiclassical state, up to a constant, but rather by the fuzzballs forming the object. Similar to any hot object, the black-hole-sized fuzzball radiates. There is an important difference to regular black holes, however. In this case, the emitted radiation does not come from particle creation in the vicinity of the fuzzball. But rather, since there is no causal boundary, the emitted particles are reflected off the fuzzball surface. Because the region just outside the fuzzball is not in a vacuum state but has a physical surface, the region close to the would-be horizon can be probed by outside observers. Similar to generic collapsing models within classical general relativity, the outside region of neutrally charged fuzzball is given by the Schwarzschild metric. Thus, if one probes the near-fuzzball region using massless particles, the dynamics should be well defined.

The conjectured fuzzball model may seem appealing because of the way it deals with singularities. Simply put, since there is no interior spacetime, no singularity forms.

6.2. Black Hole Remnants. One way of addressing the information paradox is redefining the late-time Hawking evaporation process [22, 23]. In the classical picture, a massive star collapses to form a black hole, which then evaporates monotonically through emitting Hawking particles to asymptotic infinity until it has radiated all of its initial entropy. The black hole remnant scenarios suggest that black hole evaporation comes to a halt when either $M_{\mathrm{BH}} \longrightarrow m_P$ or $M_{\mathrm{BH}} \longrightarrow M_r$, where $M_{\mathrm{BH}} \gg M_r \gg m_P$.

We may generally define a remnant as a late-time black hole stage, localized with respect to outside observers, which could be either eternal or long-lived (*i.e.*, metastable). Note that the long-lived remnants have a lifespan which is very long even compared to the age of the universe. For instance, a nonrotating uncharged black hole of mass M has a lifetime $\mathcal{O}(M^3)$, while a remnant of the same mass has a lifetime, exponential in n for $n \geq 4$. So, even though the metastable remnants are, strictly speaking, not eternal, they could be easily taken as such for all practical purposes.

The metastable remnants emit radiation through a strictly non-Hawking-like mechanism, where the emission is associated with their nonzero temperature. The remnant is assumed to remain maximally entangled with its environment, resulting in a pure state. Similar to black holes, when they radiate away their initial entropy, they disappear. Because remnants were developed to save unitarity of quantum mechanics, they have to be able to contain all the information of the black hole. Emitting that information, however, requires energy, and remnants are considered to be low-energy objects. Releasing all of the initial black hole information would take an exponentially long time, hence motivating their long life spans. The eternal remnants, however, may still radiate some particles, but they do so extremely slowly. Generally, the eternal remnants are assumed to store the initial black hole information forever.

The general argument in favor of remnants suggests that if a black hole evaporates completely, a large number of its microstates would be lost. So, an outside observer, collecting the Hawking particles, will not be able to reconstruct the complete wave function

$$\left|\Psi_{\text{whole}}\right\rangle = \sum_i \alpha_i^* \left|\psi_i^{\text{rad}}\right\rangle \otimes \left|\varphi_i^{\text{interior}}\right\rangle, \tag{38}$$

where α^* is a complex amplitude.

Although remnants may seem appealing as an elegant and somewhat less exotic solution to the information problem, as of now, there are obvious shortcomings with that proposal that need to be addressed. For instance, the model suggests that it shields the singularity from outside observers, but it is not immediately clear how this would work out for Planck mass remnants. Another obvious setback comes from AdS/CFT. Namely, how can a Planck mass remnant store all of the initial black hole entropy without violating the Bekenstein entropy bound? These are some of the open questions which we expect will be addressed in the future.

6.3. Corrections to Classicality

6.3.1. Why We Need Them. Following the equivalence principle of general relativity, an infalling observer crossing the horizon of a black hole with mass $M_{\mathrm{BH}} \gg m_P$ should not feel anything out of the ordinary (the no-drama principle). In fact, the low-energy semiclassical black hole picture is based on the following statements:

(i) Quantum mechanics is unitary

(ii) There is no drama at the horizon for an infalling observer (the equivalence principle holds)

(iii) Local quantum field theory is valid in the near-horizon region

The validity of that semiclassical description is expected to hold through most of a black hole's lifetime. That is, the low-energy picture should be accurate for as long as perturbation theory is correct.

As it is well understood, static spherically symmetric black holes in asymptotically flat spacetimes exhibit inverse proportionality between their mass and their temperature, as $M_{\mathrm{BH}} \longrightarrow 0$, $T \longrightarrow \infty$ (21). In this case, divergences occur.

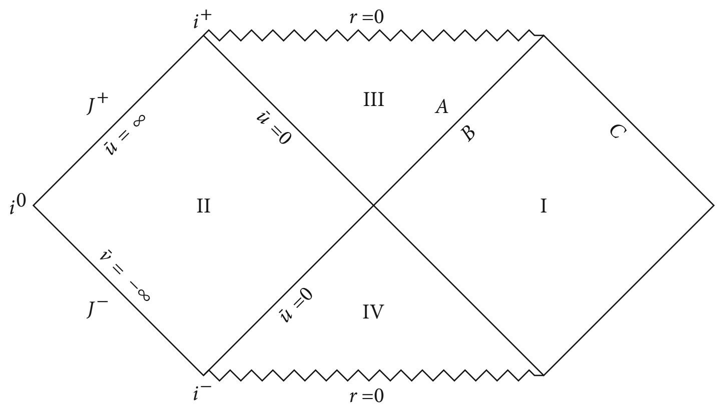

FIGURE 2

Hence, as the black hole adiabatically loses mass, the perturbative analysis breaks down at some energy scale. This line of reasoning partially motivated the recently described remnant scenarios and also the possibility of late-time black hole explosions [24].

Usually, modifications to semiclassical physics are not associated with changes in macroscopic dynamics at low temperatures. Deviations from the classical general relativity regime are expected to manifest when the mass of a black hole reaches the Planck scale.

Recently, however, AMPS [25] suggested that the equivalence principle may be violated even when the black hole is massive. They conjectured that violations to the traditional general relativistic picture do not depend on the mass of the black hole but rather on cross-horizon entanglement. The AMPS argument, in a nutshell, goes as follows. Consider a classical collapse of a pure state, and suppose the black hole is let to freely evaporate to asymptotic infinity. As it evaporates, the entanglement entropy between the black hole and the emitted Hawking cloud increases. That is, the entanglement entropy is initially low and monotonically increases until it reaches its maximum value at Page time

$$t_{\text{Page}} \sim \mathcal{O}\left(M^2\right). \tag{39}$$

At Page time, the black hole has radiated away half of its initial entropy, so it can still be fairly massive. After t_{Page}, $\dim\left(\mathcal{H}_{\text{radiation}}\right) > \dim\left(\mathcal{H}_{\text{BH}}\right)$.

Imagine now an infalling observer after Page time, Figure 2. Following the basic principles of semiclassical black holes above, an infalling observer should measure that the early Hawking radiation C is purified by the late-time radiation B, as it is demanded by unitarity of quantum theory. By the monogamy of entanglement, however, if B and C are maximally entangled, then there cannot be entanglement between B and the interior degrees of freedom A. The lack of entanglement between A and B yields a highly nontrivial state at the horizon, as far as an infalling observer is concerned, i.e., *firewall*. To protect the equivalence principle, we need to have maximum cross-horizon entanglement. And this cannot be the case if unitarity is to be preserved. Suppose we have a Schwarzschild black hole (13) whose metric is factorized as in ((32)–34). Here, strong subadditivity reads

$$S_{BC} + S_{AB} \geq S_{ABC} + S_B, \tag{40}$$

where C and B are early and late Hawking radiation, respectively, and A denotes the remaining modes inside the black hole. The no-drama statement above implies that $S_{AB} = 0$, which would make $S_{ABC} = S_C$. Then, (40) would become

$$S_C \geq S_C + S_B, \tag{41}$$

which is evidently false.

We can furthermore suppose that A, B, and C are individual qubits and state the monogamy of entanglement as

$$E(C \mid AB) \geq E(B \mid C) + E(C \mid A), \tag{42}$$

where E is bipartite entanglement. As we commented earlier, assuming AB are maximally entangled, as it is required by the equivalence principle, then no entanglement can exist between BC as is required by unitarity of quantum mechanics.

As we can see, according to AMPS, deviations from traditional general relativity can, and even should, manifest even for massive black holes. The main takeaway from the firewall paradox is that the stated above 3 principles *cannot* simultaneously be true. So, we have to give up on either unitarity, as predicted by quantum theory, or the equivalence principle of general relativity. This apparent contradiction is, namely, where the fundamental importance of the firewall paradox stems from.

6.3.2. Low-Curvature General Relativity Corrections. The firewall paradox demonstrates that corrections to general relativity not only are justified but also should manifest in low-curvature regimes when the black hole is still massive compared to the Planck scale. Studying quantum field theory in curved spacetimes is a more conservative way for addressing the connection between gravity and quantum theory.

Consider the following toy model [26–28]. Suppose we have a pure state collapse $|\Psi$ in the metric (13). Following the Hilbert space decomposition of the system, its complete pure state is given as the tensor product of its corresponding subsystems, where each subsystem is given by a reduced density matrix.

In this setting, one can consider the Minkowski vacuum in the near-horizon region. The vacuum state, up to a constant, is given in terms of the eigenstates associated with the corresponding wedges

$$|\text{vacuum}\rangle_M = \sum_i |E_i^A\rangle \otimes |E_i^B\rangle, \tag{43}$$

where the entanglement of the closest wedge modes normalizes the stress tensor at the horizon. However, the stress tensor still fluctuates. We are interested in the metric backreaction to the stress tensor fluctuations. One may assume that those fluctuations can alter the emitted Hawking spectrum or modify the near-horizon metric. We will now briefly look at the backreaction of the background metric (in the horizon vicinity) to linear perturbations of the gravity field, *i.e.*, graviton $h_{\mu\nu}$. We believe that this is the most fruitful approach since it does not introduce gross violations of general relativity but rather tightly confined ones which can potentially be probed by the EHT and LIGO.

Suppose the matter fields in the near-horizon region coupled universally to $h_{\mu\nu}$. Those nonvanishing interactions will contribute to the stress tensor which fluctuates and hence perturbs the metric. Of course, away from the black hole $r \gg 1$, the overall effect of those metric fluctuations is weak. Keep in mind, however, that still $h_{\mu\nu}^2 \neq 0$. As the graviton fluctuates, it perturbs the neighboring metric

$$ds^2 = \left(g_{\mu\nu} + h_{\mu\nu}\right) dx^\mu \otimes dx^\nu. \tag{44}$$

If we concentrate on a small part of the black hole horizon, the backreaction will be negligible since $h_{\mu\nu}$ is sourced by the local energy density. When evaluated on $\mathcal{O}(r_S)$, however, the effects of the metric backreaction accumulate and become nontrivial. As the 2-point function, the strength of $h_{\mu\nu}$ can be expressed as [29, 30]

$$\left\langle h_{\mu\nu}^2 \right\rangle = \frac{1}{2} h_{\mu\nu} \int dt dt' \frac{dx^\mu}{dt} \frac{dx^\nu}{dt} \frac{dx'^\rho}{dt'} \frac{dx'^\sigma}{dt'} \left\langle h_{\mu\nu}^2(x) h_{\rho\sigma}\left(x'\right) \right\rangle. \tag{45}$$

As a result, the radial distance between a pair of close nearby points x and x' varies stochastically, sourced by the fluctuating graviton

$$\left\langle h_{\mu\nu}(x) h^{\mu\nu}\left(x'\right) \right\rangle = 2\varphi(x)\varphi\left(x'\right), \tag{46}$$

where $\varphi(x^i)$ denotes the fluctuating at a given point graviton.

Considering the accumulated effect may result in $\mathcal{O}(1)$ deviations from general relativity which, importantly, could have important implications for the information loss problem.

In addition, any such metric fluctuations should affect test particles by producing nontrivial deviations from the classical geodesic equation. As a result, the geodesic equation (8) receives a linear metric correction $\gamma^\mu_{\alpha\beta}$ and becomes the Langevin equation

$$\frac{dx^\mu}{d\tau} = -\Gamma^\mu_{\alpha\beta} x^\alpha x^\beta - \gamma^\mu_{\alpha\beta} x^\alpha x^\beta, \tag{47}$$

where x^i is a 4-vector.

Using Langevin, we can, in principle, calculate some deviations from the classical trajectories that test particles follow. That quantum correction accumulates over the black hole's lifetime and eventually becomes $\mathcal{O}(1)$. For instance, one can assume that the fluctuations produce nonlocal effects which carry the information out of the black hole. (It should be noted, however, that the question of the validity of the proposed "soft" quantum corrections to the black hole geometry is very much open as some have suggested [31, 32] that the proposed corrections are not strong enough to restore unitarity to black hole evaporation. Moreover, some have criticized the model due to the apparent lack of testable predictions in the context of AdS/CFT (see [33] and the references therein).)

The conjectured fluctuations of the stress tensor in the vicinity of the horizon may be considered to lead to a form of stimulated Hawking emission. In some models [34], restoring unitary evolution necessitates the emission of extra particles beyond the traditional Hawking spectrum. An important condition that has to be met in this scenario is that the additional Hawking particles need to respect the equivalence principle. Hence, the quanta should be low temperature with wavelength of order the Schwarzschild radius and should only appear as a small correction to the overall spectrum. For this criteria to be met, one can assume that the extra radiation begins early, *i.e.*, much before Page time.

7. Naked Singularities

The discussion in Section 6 was focused on different approaches to the information loss problem, assuming, of course, that a black hole is the endstate of a gravitational collapse. In parallel, however, there has been an interesting line of research exploring different possible scenarios for gravitational collapse endstates, the most promising of which are the so-called naked singularities, namely, singularities which are not cloaked behind a horizon.

The Penrose-Hawking singularity theorems [35] dictate that, following the Einstein equations (11), at the end of a gravitational collapse of a physically reasonable matter, a spacelike singularity forms (For comprehensive reviews, see [36–38].). Although we may not be completely sure about the nature of singularities, it is well established that they lead to breakdown of classical physics, and any eventual treatment of the pathologies should be within the realm of quantum gravity. The *weak* and *strong* cosmic censorship conjectures are very useful tools for dealing with singularities. Let us take a brief look at them before returning to the main part of the section.

The weak cosmic censorship [39, 40] is a statement about naked singularities and suggests that a spacelike singularity, formed in gravitational collapse, is hidden behind a horizon. Simply put, given generic conditions (the details of which are beyond the scope of the current paper) applying to both, the matter fields, and the asymptotic flatness of spacetime, a faraway observer cannot probe a singularity. Let us be more precise. Imagine $(\mathscr{M}, g_{\mu\nu})$ is an asymptotically flat spacetime, and $\mathscr{B}$ is a black hole region within that spacetime. More formally,

$$\mathscr{B} \coloneqq \mathscr{M} - I^{-}(\mathscr{I}^{+}), \quad (48)$$

where $I^{-}(\mathscr{I}^{+})$ denotes the region in the past of future null infinity, and the black hole horizon is the boundary of $\mathscr{B}$:

$$\mathscr{H} = \partial\mathscr{B}. \quad (49)$$

Given this spacetime region, the weak cosmic censorship can be defined as follows.

7.1. Weak Cosmic Censorship. Suppose $(h_{\mu\nu}, K_{\mu\nu}, \psi)$ (Note that the stress tensor of ψ should satisfy generic energy conditions.) is nonsingular asymptotically flat initial data defined on a hypersurface Σ. Then, the maximal Cauchy development of this initial data is complete and thus yields an asymptotically flat spacetime.

To put it vaguely, an asymptotic observer at future null infinity will never encounter a singularity.

The strong cosmic censorship [41–43] is roughly the statement that infalling observers in spacetimes which admit Cauchy horizons (*e.g.*, Kerr (14) or Reissner-Nordstrom (16)) *cannot* probe the timelike singularity. That is, infalling observers cannot cross the Cauchy horizon. The conjecture generally suggests that.

7.2. Strong Cosmic Censorship. Given smooth asymptotically flat vacuum initial data (*i.e.*, globally hyperbolic solutions to (11)), defined on a codimension-1 submanifold Σ within the black hole interior, the maximal Cauchy evolution is *inextendable* beyond the Cauchy horizon. (The precise formulation is very subtle and is outside the objective of the paper. For a detailed take on the matter, the reader is referred to [41–44] and the references therein.)

That is, the spacetime is globally hyperbolic, meaning there is intrinsic predictability, given some initial data. Interestingly, the validity of the strong cosmic censorship was recently put into question. It was demonstrated by Dafermos and Luk [44] for the case of a subextremal Kerr black hole that when perturbing the initial data, the metric is *continuously extendable* across the Cauchy horizon. Without going into much details, in [44], Dafermos and Luk proved the following theorem.

Theorem 1 (see [44]) (Moreover, their results have also been considered for Einstein-Maxwell-real-scalar-field [45, 46] and Einstein-Maxwell-Klein-Gordon systems [47].). *Consider general vacuum initial data corresponding to the expected induced geometry of a dynamical black hole settling down to Kerr (with parameters $0 < |a| < M$) on a suitable spacelike hypersurface Σ within the black hole. Then, the maximal evolution of the spacetime $(\mathscr{M}, g_{\mu\nu})$, corresponding to Σ, is globally covered by a double null foliation and has a nontrivial Cauchy horizon across which the metric is continuously extendable.*

We should note that the two conjectures are not directly related and need not hold simultaneously. Below, we focus on the weak censorship and, in particular, on the observational distinguishability between naked singularities and black holes.

Broadly speaking, naked singularities can be either *locally* or *globally* naked. A singularity is locally naked if there are null or timelike geodesics which can reach an outside observer. For example, an observer who has crossed the horizon of a massive Reisner-Nordstrom black hole. As far as such observer is concerned, she is timelike separated from the singularity. At the same time, no rays from the singularity can reach spacelike separated (*i.e.*, outside) observers. Globally naked singularities are those which could be probed by observers at future null infinity. Here, no horizons are present, and null or light rays can reach asymptotic observers. Since the globally naked singularities can, in principle, be probed by asymptotic observers, one naturally asks whether those singularities are point-like or are extended. Of course, that cannot be accurately addressed and depends on the matter fields in the given collapse model and on the equation of state. Basically, we assume that if a timelike singularity forms, it will typically be extended, and thus, light rays will be emitted. However, it has been argued [48–52] that it is possible even for point-like singularities to have outgoing geodesics at some time slice.

In addition, an observer may be interested in measuring the mass of a naked singularity. Although one may be tempted to consider an exotic case where the mass is negative, usually, certain generality conditions apply which put tight constraints on such scenarios. Those constraints, like positive mass/energy density, and nonsingular initial conditions, for instance, render negative-mass cases physically unreasonable (unless one is willing to employ very complicated and fine-tuned scenarios which rely on ad hoc considerations).

Another interesting possibility to consider is the relation between naked singularities and causality. Generally, causality violations in general relativity are associated with the presence of closed timelike curves. In fact, the presence of a singularity necessitates closed curves in the near future geometry. The Penrose-Hawking singularity theorems, however, dictate that if there is a singularity, then causality should be safely clocked behind a horizon, given the semiclassical metric. Usually, establishing any relation between naked singularities and causality violation is highly nontrivial. This is partly because of the different naked singularity configurations one may consider.

In light of the recent advancements in multimessenger astronomy, we naturally ask questions such as the following: what are the observational signatures of such exotic objects, will the neighboring geometry be affected, and can we distinguish astrophysical black holes from naked singularities? We have so far seen that naked singularities come in different forms, according to classical general relativity. Hence, their observational signatures, if present at all, should also differ. Intuitively, we assume that either the stress tensor blows up at the naked singularity or some nontrivial quantum gravity effects become relevant in the immediate vicinity of the naked singularity. However, this line of reasoning does not exclude the possibility of observing traditional general relativity in the neighboring geometry. For instance, suppose we have a collapse in its endstate. In such scenarios, one might expect energetic photons to escape the highly dense region at small scales (where quantum gravity effects may be present) and hence produce observable effects which would differ from the standard black hole collapse models. On the other hand, we can employ ray tracing and look for deviations from the classical geodesic equation in the vicinity of the object or for certain particle collision patterns yielded from the singularity which deviate from the general relativity prescriptions.

One possible place to look for observational discrepancies is the accretion disk. Usually, accretion disks are studied in terms of their luminosity and energy flux. For astrophysical black holes, streams of quanta are emitted from the polar regions due to the strong electromagnetic field. As it has been argued in [53, 54], for naked singularities, there should be particular observational differences. Moreover, the lack of horizon leads to repulsive forces which yield high energy quanta emission not only from the poles but also from the equatorial plane $\Theta = \pi/2$. Such characteristic emission should have distinguishable spectrum differences to the usual black hole one.

Another possibility for finding observational differences comes from studying the photon sphere. Due to the conjectured irregular geometry in the vicinity of the naked singularity, the photon ring should have different bending angles for the closest outgoing trajectories [55]. And finally, another possible source of discrepancy is again related to the irregular horizonless of the naked singularity metric. As a result, families of infalling particles may be ejected to asymptotic infinity or outgoing ones may be deviated towards the singularity. That mixing of trajectories will, inevitably, lead to particle collisions which should not be observed in classical black hole solutions. Of course, a similar argument could be made for the waveform of the emitted gravitational radiation, too. Namely, the difference in the near-singularity geometry should, in principle, lead to deviations from the general relativistic predictions, especially for the higher frequency early-emitted gravitational waves.

8. Conclusions

We examined the main black hole solutions and outlined their geometrical properties. We then showed that the derivation of the thermal Hawking emission spectrum and the relation of the black hole entropy to the area of its horizon solidified the thermodynamic nature of the system. Later, we demonstrated how the interplay between gravity and quantum field theory in a black hole background has lead to a profound puzzle. The fundamental relation between geometry and quantum information, on the one hand, and the robustness of quantum theory, on the other, have demonstrated the deep roots of the information loss paradox. We commented on the more popular proposals for addressing it in Section 6. Interestingly, what they show is that the paradox cannot be trivially addressed, and any potential attempt at resolving it must rely on radically new physical principles. Moreover, the proposals over the years have unambiguously shown that we cannot continue relying on our good old cherished principles, namely, effective field theory, unitarity, and the equivalence principle, and that at least one of them has to be modified. In the paper, we focused our attention on a toy model, which introduces quantum corrections to the horizon geometry. It was argued the conjectured quantum corrections, although *soft*, are sufficiently strong to accumulate over the lifetime of the black hole and ultimately lead to $\mathcal{O}(1)$ effects. Finally, we touched upon an alternative collapse endstate which has recently gained popularity with the advances in multimessenger astronomy, namely, naked singularities. We first discussed the properties of the different kinds of naked singularities and then the observational prospects, where we concentrated on the possible observational discrepancies between naked singularities and black holes.

Conflicts of Interest

The author declares that they have no conflicts of interest.

Acknowledgments

The work was done during the author's affiliation with the Department of Physics and Astronomy, Shumen University, and was partially supported under project RD-08-112/2018.

References

[1] C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation*, W. H Freeman, San Francisco, 1973.

[2] R. M. Wald, *General Relativity*, The University of Chicago Press, Chicago, 1984.

[3] R. Adler, M. Bazin, and M. Schiffer, *Introduction to General Relativity*, McGraw-Hill, New York, 2nd edition, 1975.

[4] T. Damour, "General relativity today," https://arxiv.org/abs/0704.0754.

[5] C. Heinicke and F. W. Hehl, "Schwarzschild and Kerr solutions of Einstein's field equation: an introduction," *International Journal of Modern Physics D*, vol. 24, article 1530006, 2015.

[6] P. T. Chruściel, J. L. Lopes Costa, and M. Heusler, "Stationary black holes: uniqueness and beyond," *Living Reviews in Relativity*, vol. 15, no. 7, p. 169, 2012.

[7] M. T. Anderson, "On stationary vacuum solutions to the Einstein equations," *Annales Henri Poincaré*, vol. 1, no. 5, pp. 977–994, 2000.

[8] J. D. Bekenstein, "Black holes and the second law," *Lettere Al Nuovo Cimento Series 2*, vol. 4, no. 15, pp. 737–740, 1972.

[9] J. D. Bekenstein, "Black holes and entropy," *Physics Review D*, vol. 7, no. 8, pp. 2333–2346, 1973.

[10] J. D. Bekenstein, "Generalized second law of thermodynamics in black hole physics," *Physics Review D*, vol. 9, p. 3292, 1974.

[11] S. W. Hawking, "Particle creation by black holes," *Communications in Mathematical Physics*, vol. 43, no. 3, pp. 199–220, 1975.

[12] T. Jacobson, "Introduction to quantum fields in curved spacetime and the Hawking effect," in *Lectures on Quantum Gravity*Springer, Boston, MA.

[13] N. D. Birrell and P. C. W. Davies, *Quantum Fields in Curved Space*, Cambridge University Press, Cambridge, 1982.

[14] W. G. Unruh, "Notes on black hole evaporation," *Physics Review D*, vol. 14, p. 870, 1976.

[15] R. M. Wald, "The thermodynamics of black holes," *Living Reviews in Relativity*, vol. 4, p. 6, 2001.

[16] J. Polchinski, "The black hole information problem," in *New Frontiers in Fields and Strings*, pp. 353–397, Shumen University, 2017.

[17] O. C. Stoica, "Revisiting the black hole entropy and the information paradox," *Advances in High Energy Physics*, vol. 2018, Article ID 4130417, 16 pages, 2018.

[18] S. D. Mathur, "What does the information paradox say about the universe?," https://arxiv.org/abs/1812.11641.

[19] S. D. Mathur, "A model with no firewall," https://arxiv.org/abs/1506.04342.

[20] S. D. Mathur, "What prevents gravitational collapse in string theory?," *International Journal of Modern Physics D*, vol. 25, article 1644018, 2016.

[21] S. D. Mathur, "Resolving the black hole causality paradox," *General Relativity and Gravitation*, vol. 51, no. 2, 2019.

[22] T. Banks, M. O'Loughlin, and A. Strominger, "Black hole remnants and the information puzzle," *Physical Review D*, vol. 47, no. 10, pp. 4476–4482, 1993.

[23] P. Chen, Y. C. Ong, and D. Yeom, "Black hole remnants and the information loss paradox," *Physics Reports*, vol. 603, pp. 1–45, 2015.

[24] S. W. Hawking, "Black hole explosions?," *Nature*, vol. 248, no. 5443, pp. 30-31, 1974.

[25] A. Almheiri, D. Marolf, J. Polchinski, and J. Sully, "Black holes: complementarity or firewalls?," *Journal of High Energy Physics*, vol. 2013, 2013.

[26] A. Yosifov, *Talk Given at the VI National Conference*, From the Atom to the Cosmos, From the Atom to the Cosmos, 2018.

[27] A. Yosifov and L. Filipov, "Nonlocal black hole evaporation and quantum metric fluctuations via inhomogeneous vacuum density," *Advances in High Energy Physics*, vol. 2018, Article ID 3131728, 9 pages, 2018.

[28] A. Yosifov and L. Filipov, "Entropic entanglement: information prison break," *Advances in High Energy Physics*, vol. 2017, Article ID 8621513, 7 pages, 2017.

[29] L. H. Ford and N. F. Svaiter, "Gravitons and light cone fluctuations. II. Correlation functions," *Physical Review D*, vol. 54, no. 4, pp. 2640–2646, 1996.

[30] L. H. Ford and N. F. Svaiter, "Cosmological and black hole horizon fluctuations," *Physical Review D*, vol. 56, no. 4, pp. 2226–2235, 1997.

[31] C. Callan, S. Giddings, J. Harvey, and A. Strominger, "Evanescent black holes," *Physics Review D*, vol. 45, article R1005, 1992.

[32] W. Donnelly and S. Giddings, "Observables, gravitational dressing, and obstructions to locality and subsystems," *Physical Review D*, vol. 94, article 104038, 2016.

[33] D. Marolf, "The black hole information problem: past, present, and future," *Reports on Progress in Physics*, vol. 80, no. 9, article 092001, 2017.

[34] T. Jacobson, "Black holes and Hawking radiation in spacetime and its analogues," in *Analogue Gravity Phenomenology*, vol. 870 of Lecture Notes in Physics, pp. 1–29, Springer, Cham.

[35] R. Penrose, "Gravitational collapse and space-time singularities," *Physical Review Letters*, vol. 14, no. 3, pp. 57–59, 1965.

[36] J. M. M. Senovilla, "Singularity theorems and their consequences," *General Relativity and Gravitation*, vol. 30, no. 5, 1998.

[37] R. Penrose, "Singularity theorems," *Topology and Physics*, pp. 135–171, 2019.

[38] S. Hawking and R. Penrose, "The singularities of gravitational collapse and cosmology," *Proceedings of the Royal Society of London. A. Mathematical and Physical Sciences*, vol. 314, no. 1519, pp. 529–548, 1970.

[39] R. Wald, "Gravitational collapse and cosmic censorship," in *Black Holes, Gravitational Radiation and the Universe. Fundamental Theories of Physics (An International Book Series on The Fundamental Theories of Physics: Their Clarification, Development and Application)*, vol. 100, pp. 69–86, Springer, Dordrecht.

[40] R. Penrose, "Gravitational collapse: the role of general relativity," *Nuovo Cimento Rivista Serie*, vol. 1, 252 pages, 1969.

[41] R. Penrose, "Singularities of spacetime," in *Theoretical Principles in Astrophysics and Relativity*, W. R. N. R. Liebowitz and P. O. Vandervoort, Eds., pp. 217–243, Chicago University Press, 1978.

[42] J. Isenberg, "On strong cosmic censorship," https://arxiv.org/abs/1505.06390.

[43] O. J. C. Dias, H. S. Reall, and J. E. Santos, "Strong cosmic censorship: taking the rough with the smooth," *Journal of High Energy Physics*, vol. 2018, article 1, 2018.

[44] M. Dafermos and J. Luk, "The interior of dynamical vacuum black holes I: the C^0-stability of the Kerr Cauchy horizon," https://arxiv.org/abs/1710.01722.

[45] M. Dafermos, "Stability and instability of the Cauchy horizon for the spherically symmetric Einstein-Maxwell-scalar field equations," *Annals of Mathematics*, vol. 158, no. 3, pp. 875–928, 2003.

[46] S. Aretakis, "Stability and instability of extreme Reissner-Nordström black hole spacetimes for linear scalar perturbations I," *Communications in Mathematical Physics*, vol. 307, no. 1, pp. 17–63, 2011.

[47] M. Van de Moortel, "Stability and instability of the subextremal Reissner–Nordström black hole interior for the Einstein-Maxwell-Klein-Gordon equations in spherical symmetry," *Communications in Mathematical Physics*, vol. 360, no. 1, pp. 103–168, 2018.

[48] P. Joshi and I. Dwivedi, "On the nature of naked singularities in Vaidya spacetimes," *Classical and Quantum Gravity*, vol. 6, p. 1599, 1989.

[49] P. S. Joshi and I. H. Dwivedi, "On the nature of naked singularities in Vaidya spacetimes: II," *Classical and Quantum Gravity*, vol. 8, p. 1339, 1991.

[50] S. Deshingkar and P. S. Joshi, "Structure of nonspacelike geodesics in dust collapse," *Physics Review D*, vol. 63, article 024007, 2001.

[51] F. C. Mena and B. Nolan, "Non-radial null geodesics in spherical dust collapse," *Classical and Quantum Gravity*, vol. 18, no. 21, pp. 4531–4548, 2001.

[52] B. C. Nolan and F. C. Mena, "Geometry and topology of singularities in spherical dust collapse," *Classical and Quantum Gravity*, vol. 19, no. 10, pp. 2587–2605, 2002.

[53] Z. Kovacs and T. Harko, "Can accretion disk properties observationally distinguish black holes from naked singularities?," *Physics Review D*, vol. 82, article 124047, 2010.

[54] D. Pugliese, H. Quevedo, and R. Ruffini, "Equatorial circular motion in Kerr spacetime," *Physics Review D*, vol. 84, article 044030, 2011.

[55] G. N. Gyulchev and S. S. Yazadjiev, "Gravitational lensing by rotating naked singularities," *Physics Review D*, vol. 78, article 083004, 2008.