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Modelling Bubble Lifetime of Thin Film Surfactants Solution on Fuel Spillage

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Authors' contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

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Abstract

Aims / Objectives: To find the lifetime of the bubble by plotting the rate of mass flow rate change against time.
Place and Duration of Study: Department of Mathematics and Applied Science, Catholic University of Eastern Africa, Nairobi, Kenya, between February 2020 and March 2021.
Methodology: The maximum lifetime of the bubble is assumed to match the time when the mass flow rate change is zero. The study also assumes the velocity of flow rate and other fluid properties at the interface of fuel-surfactant constant other than Re. Re is varied from 0.01 to 100.
Results: The graphical plots show that for $Re \ll 1$, and $Re \gg 1$, the stability depends on diffusive viscosity and linearized convection, respectively. The simulation suggested that the bubble formed at the fuel-surfactant interface may have $Re = 1$ and its lifetime is $t_b \simeq 0.28$.
Conclusion: The lifetime of surfactant depends on Re while assuming other interface properties constant.

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Recommendation: Future studies in the area need to consider the effect of variation in temperature, velocity, and Reynolds number in determining the lifetime of a bubble in the thin foam of the surfactant-fuel interface.

Keywords: Fuel-Surfactant interface; Bubble lifetime; interface thickness.

2010 Mathematics Subject Classification: 53C25; 83C05; 57N16.

1 Introduction

Surfactants help in many human activities such as coating, cosmetics, environmental protection, food, and medical industry [1]. Surfactants, for instance, on fuel, allow multi-phases and formation of colloids [2]. The thin film spread specifically on fuel has numerous advantages, such as the prevention of fuel explosion. Of importance is the determination of the lifetime of the thin film given the real-world problem of fuel spillage [3]-[20].

Surfactants thin film spread on fuel is governed by surface tension [21], which spread uniformly across the surface of the fuel spill. The surface tension induces shear stresses at the fuel-air interface [22]. The stresses distribute the surfactants evenly over the surface of the fuel from low to high areas of surface tension. The variation in surface tension due to variation in stresses results in variation in spill heights, known as Marangoni flow [23, 24, 25]. Surfactants spreads are fundamental in the attainment of this phenomenon by fuel spillage. Literature on Marangoni flow are covered in a lot of existing materials [23, 24, 25, 26, 27, 28, 29].

Marangoni stresses causes non-uniform spreading hence depletion of the thin spread [30, 31]. These stresses cause the bubbles on the thin film surfactants on fuel (see Fig. 1). The spread depletion is analogous to the life of the bubble formed on the fuel due to surfactant spread on the fuel surface during spillage [32].

Cavitation is the formation and dynamic life of bubbles in or on the surface upon destabilization due to pressure difference causing stresses [33]. Bremond *et al.* [34], Chahine [35], Golykh [36], Kiyama *et al.* [37], Kyriazis *et al.* [38], and Reynolds [39] widely studies cavitation under various configurations like near the solid walls, and gas-liquid interface, and so on. Existing studies on the dynamics of bubbles growth and collapse in thin film or liquid are presented in the following literature [39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49]. A recent study by Li *et al.* [50] studied the effect of surfactant and evaporation on the thin liquid film spreading in the presence of surface acoustic waves. Li *et al.* [50] study is based on a theoretical model of liquid film flow in the presence of surface acoustic waves (SAWs). Li *et al.* [50] assumed SAWs affect insoluble surfactant and leading to evaporation on the spreading process of the partially wetting thin liquid film. However closer review of Li *et al.* [50] study on cavitation reveal that the study it did not consider a lifetime of bubble on thin foam or surface of fuel. Nonetheless, none of the existing studies have modeled a lifetime of bubble based on fuel-surfactant thin film layer, which has prompted the proposed study [51]-[70].

The first research on cavitation by Reynolds [39] suggested that the lifetime of a bubble depends on the boundary or surface actions of fluids, which is affected by viscosity, velocity, temperature difference, and Reynolds number. The bubble rises due to buoyancy force at the free surface leading to the ejection of aerosols into the surface, making their use dynamic in many fields [71, 72]. This tendency makes bubble formation important in fuel-surfactant thining since its lifetime determines the time before the potential explosion [73]-[100]. Thus, the proposed paper takes an interest in lifetime of bubbles on a thin layer between fuel and surfactant. The paper focuses on modeling the lifetime of the bubble without considering the fuel-surfactant properties. The paper considers the variation in Reynolds number in the thin film to model the life of the bubble on the surfactant-fuel interface. Modeling the lifetime of the bubble is essential in estimating the time before fuel vapor is released into the atmosphere. Thus, the proposed research seeks to inform firefighters on the time needed to prevent explosions after surfactant spread.

The rest of the paper is organized as follows. Section 2 presents the dynamics of the model formulated and parameter transformation. The parameters were transformed into dimensionless form and contains fluid flow governing equations and interface coupling conditions for the bubble. Section 3 presents graphical simulations and results in line with the discussion of the findings. Section 4 presents findings conclusion and possible future studies.

2 Problem Formulation Model

We consider bubbles formed on thin layer of the surfactant-fuel foam (see Fig. 1). The fluids have constant density ϱ_{b_s} and ϱ_{b_f}, velocity component u_{b_s} and u_{b_f}, viscosity υ_{b_s} and υ_{b_f}, where subscript s and f stand for surfactants and fuel respectively. We assume the two components of velocity $\mathbf{U_b}$ by u_{b_s}, u_{b_f} and $\mathbf{V_b}$ by $\upsilon_{b_s}, \upsilon_{b_f}$ so that the fluid velocity in all the coordinates are described by

$$\frac{dx}{dt} \equiv u = u_b(x, y, z, t), \tag{1}$$

then,

$$\frac{df}{dt} \equiv \mathbf{U_b} = \frac{d}{dt} f\Bigg[\ \big(u_{b_s}(x, y, t), u_{b_f}(x, y, t)\big)\ \Bigg], \tag{2}$$

where t is time. The atmospheric pressure leading to buoyancy forces are all assumed constant at g. We also assume a uniform spread of surfactant over the fuel spillage. We consider the dimensional form of incompressible Navier-Stokes equations

$$\frac{\partial \mathbf{U_b}}{\partial \mathbf{x_b}} = 0 \tag{3a}$$

$$\frac{\partial \mathbf{U_b}}{\partial t} = -\frac{\partial (u_{b_s} u_{b_f})}{\partial \mathbf{x_b}} + 2\nu \frac{\partial D_b}{\partial \mathbf{x_b}} - \frac{1}{\rho}\left(\frac{\partial p_b}{\partial \mathbf{x_b}} - s_b\right) \tag{3b}$$

$$= -\frac{\partial (u_{b_s} u_{b_f})}{\partial \mathbf{x_b}} + \nu \frac{\partial^2 u_b}{\partial \mathbf{x_b}^2} - \frac{1}{\rho}\left(\frac{\partial p_b}{\partial \mathbf{x_b}} - s_b\right), \tag{3c}$$

with the Non-dimensionalized parameters as

$$t^* = \frac{tU_b}{L} \tag{4a}$$

$$x_b^* = \frac{x_b}{L} \tag{4b}$$

$$p^* = \frac{p}{\rho \mathbf{u_b}^2} \tag{4c}$$

$$\frac{1}{Fr} = \frac{gL}{u_b^2} \tag{4d}$$

$$u_b^* = \frac{u_b}{\mathbf{u_b}} \tag{4e}$$

$$\mathbf{P_b} = p_b^* + \frac{x_2^*}{Fr^2} s = \begin{bmatrix} 0 \\ -\rho_b g \end{bmatrix} \tag{4f}$$

$$2\frac{\partial D_b}{\partial x_b} = \frac{\partial^2 u_b}{\partial x_b^2} \tag{4g}$$

Non-dimensionalized mass conservation equations

$$\frac{U_b}{L_b}\frac{\partial u_b^*}{\partial x_b^*} \tag{4h}$$

$$\implies \frac{\partial u_b^*}{\partial x_b^*} \tag{4i}$$

$$\begin{aligned}
\tfrac{U^2}{L_b}\tfrac{\partial u_i^*}{\partial t^*} &= -\tfrac{U_b^2}{L_b}\tfrac{\partial u_{b_s}^* u_{b_f}^*}{\partial x_b} + \nu\tfrac{U_b}{L^2}\tfrac{\partial^2 u_{b_s}^*}{\partial^2 x_{b_f}^{2*}} - \tfrac{1}{\rho}\left(\tfrac{\rho U_b^2}{L_b}\tfrac{\partial p_b^*}{\partial x_{b_s^*}} - \begin{bmatrix} 0 \\ -\rho g \end{bmatrix}\right) \\
\longrightarrow \tfrac{\partial u_{b_s}^*}{\partial t^*} &= -\tfrac{\partial (u_{b_s^*} u_{b_f^*})}{\partial x_{b_f}^*} + \tfrac{1}{Re}\tfrac{\partial^2 u_{b_s}^*}{\partial x_{b_f}^{2*}} - \tfrac{\partial p^*}{\partial x_{b_s}^*} + \tfrac{L_b}{\rho U_b^2}\begin{bmatrix} 0 \\ -\rho g \end{bmatrix} \\
\longrightarrow \tfrac{\partial u_{b_s}^*}{\partial t^*} &= -\tfrac{\partial (u_{b_s^*} u_{b_f^*})}{\partial x_{b_f}^*} + \tfrac{1}{Re}\tfrac{\partial^2 u_{b_s}^*}{\partial x_{b_f}^{2*}} - \tfrac{\partial p^*}{\partial x_{b_s}^*} - \begin{bmatrix} 0 \\ \frac{gL_b}{u_b^2} \end{bmatrix} \\
\longrightarrow \tfrac{\partial u_{b_s}^*}{\partial t^*} &= -\tfrac{\partial (u_{b_s^*} u_{b_f^*})}{\partial x_{b_f}^*} + \tfrac{1}{Re}\tfrac{\partial^2 u_{b_s}^*}{\partial x_{b_f}^{2*}} - \tfrac{\partial p^*}{\partial x_{b_s}^*} \\
\longrightarrow \tfrac{\partial u_{b_s}^*}{\partial t^*} &= -\tfrac{\partial (u_{b_s^*} u_{b_f^*})}{\partial x_{b_f}^*} + \tfrac{1}{Re}\tfrac{\partial^2 u_{b_s}^*}{\partial x_{b_f}^{2*}} - \tfrac{\partial P_b}{\partial x_{b_s}^*} \\
\longrightarrow \tfrac{\partial u_{b_s}^*}{\partial t^*} &= H_b - \tfrac{\partial P_b}{\partial x_{b_s}^*}
\end{aligned} \tag{5}$$

Atasi et al.[101] established that the largest single drainage bubble has a thickness of h_b of the thin film. The bubble decays at an exponential rate due to competition between stresses arising from drainage forces.

$$\frac{h_b}{h_{b_0}} = e^{\frac{-a_b t_b}{\tau_b}}, \tag{6}$$

where h_{b_0} is the initial film thickness, a_b is thinning rate, t_b bubble lifetime and $\tau_b = \frac{\mu_b}{\rho g R_b}$ is the extensional flow based on viscous-gravity forces. ρ is the density at the interface, g is gravitational acceleration, and R_b is the bubble assumed radius also defined as $R_b = \left(\frac{3}{4}\pi_b V_b\right)^{\frac{1}{3}}$ with V being the bubble volume [102, 101]. We combine (5) with (6) to obtain bubble non-dimensionalized mass conservation equation as

$$\frac{\partial u_{b_s}^*}{\partial t^*} = e^{\frac{-a_b t_b}{\tau_b}}\left(H_b - \frac{\partial P_b}{\partial x_{b_s}^*}\right). \tag{7}$$

The pressure at the interface leading to bubble failure at time t is derived by assuming divergence of the non-dimensionalized momentum equation and invoking continuity equation, that is, $\frac{\partial u_b^*}{\partial t} = 0$ such that,

$$\nabla.\left(\frac{\partial u_b^*}{\partial t} - H_b + \frac{\partial P_b}{\partial x_{b_s}^*}\right) = 0|_{\left(H_b = -\frac{-\partial (u_{b_s} u_{b_f})}{\partial x_b} + \frac{1}{Re}\frac{\partial^2 u_b}{\partial x_b^2}\right)} \tag{8}$$

$$\longrightarrow \frac{\partial}{\partial t}\left(\frac{\partial u_b}{\partial t} - H_{1_b} + \frac{\partial P_b}{\partial x_b}\right) + \frac{\partial}{\partial y_b}\left(\frac{\partial v_b}{\partial t} - H_{2_b} + \frac{\partial P_b}{\partial y_b}\right) = 0 \tag{9}$$

$$\longrightarrow \frac{\partial^2 P_b}{\partial y_b^2} + \frac{\partial^2 P_b}{\partial y_b^2} = \frac{\partial H_{1_b}}{\partial x_b} + \frac{\partial H_{2_b}}{\partial y_b} \tag{10}$$

The discretized wall functions $H_b \in H_{b_s}, H_{b_f}$ are Navier-Stokes equation in $2D$. The vector product of the normal vector to the wall and momentum equations helps us establish the boundary conditions for the pressure. Assuming the bubble is formed due to the change in momentum equation in the fuel f and surfactant s.

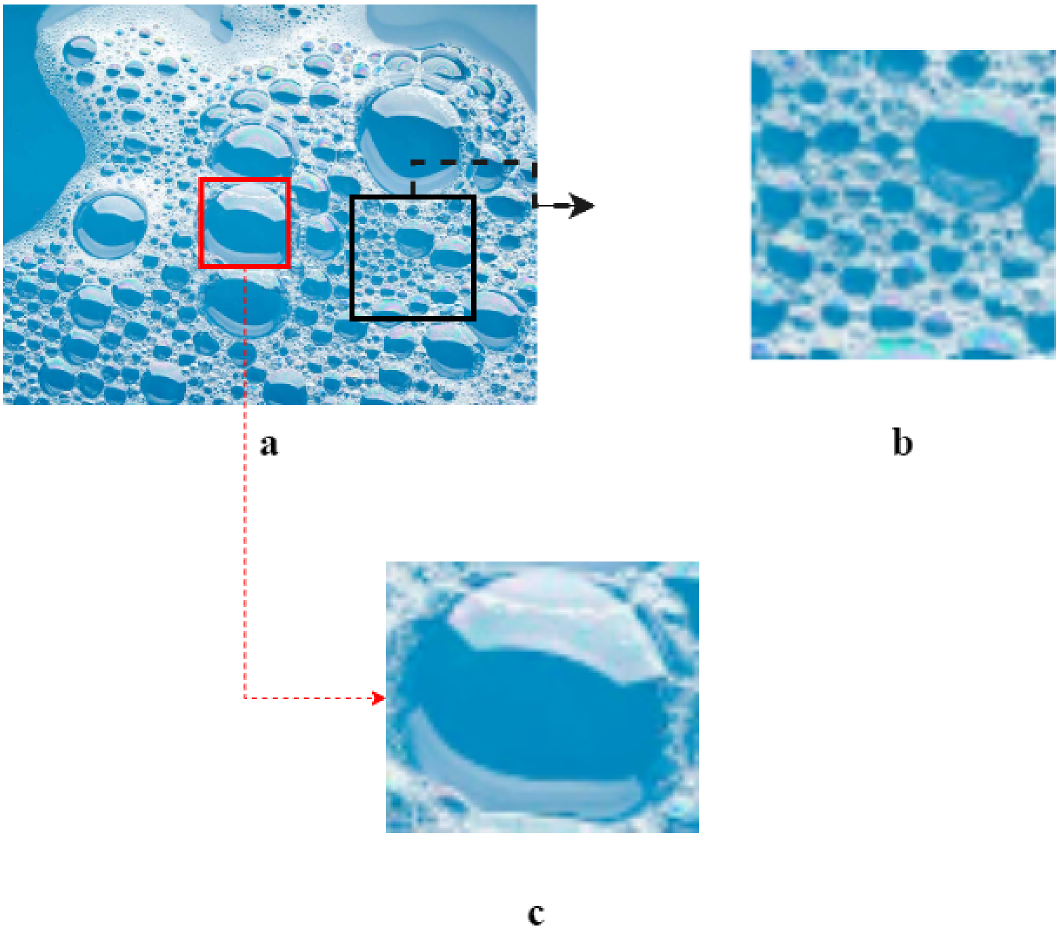

Fig. 1. (a) Thin film foam on fuel. (b) Microscopic view of thin film bubble of (a). (c) extract of the largest bubble formed on the surfactant-fuel thin film with thickness h_b and exponential decay postulated in (6)

If we assume a constant velocity, the momentum equations describe the homogeneous Neumann boundary conditions on the interface of the surfactant and fuel. Thus, H_{b_s} describes the momentum equation on the surfactant at the interface while H_{b_f} is for the fuel. The H_{b_s} and H_{b_f} can be described by derivatives of velocity fields as

$$H_{b_s} = \frac{\partial}{\partial x_{b_s}}\left(\frac{1}{Re}\frac{\partial u_{b_s}}{\partial x_{b_s}} - u_{b_s}^2\right) + \frac{\partial}{\partial y_{b_s}}\left(\frac{1}{Re}\frac{\partial u_{b_s}}{\partial y_{b_s}} - u_{b_s}u_{b_f}\right) \tag{11a}$$

$$H_{b_f} = \frac{\partial}{\partial x_{b_f}}\left(\frac{1}{Re}\frac{\partial u_{b_f}}{\partial x_{b_f}} - u_{b_f}^2\right) + \frac{\partial}{\partial y_{b_f}}\left(\frac{1}{Re}\frac{\partial u_{b_f}}{\partial y_{b_f}} - u_{b_f}u_{b_s}\right), \tag{11b}$$

where

$$u_{b_s} = \sin\left(\alpha_1 x_{b_s}\right)\sin\left(\alpha_2 y_{b_s}\right) \tag{11c}$$

$$u_{b_f} = \sin\left(\alpha_3 x_{b_f}\right)\sin\left(\alpha_4 y_{b_f}\right). \tag{11d}$$

The problem presented in (1)-(11b) present diffusion Neumann boundary conditions within the fuel-surfactant interface. Assuming that the matrix equation of continuity of mass conservation is represented by

$$A_b p_b = \Xi_b, \tag{12a}$$

where

$$\begin{aligned} \Xi_b &= \tfrac{\partial P_b}{\partial x_b}\bigg|_{\varepsilon_b}^{\lambda_b} \nabla y_b + \tfrac{\partial P_b}{\partial x_b}\bigg|_{\eta_b}^{\psi_b} \nabla x_b, \\ &= H_{b_s}\bigg|_{\varepsilon_b}^{\lambda_b} \nabla y_b + H_{b_f}\bigg|_{\eta_b}^{\psi_b} \nabla x_b. \end{aligned} \tag{12b}$$

There is no Dirichlet boundary condition; thus, the rank of A_b is enforced to $n^2 - 1$ showing that (12b) is singular and an infinite number of solutions exists. In order to avoid fluctuations in bubble pressure during flow, the pressure value is assumed to be zero at the center of the bubble and maximum at the highest thickness, h_b. This translates to

$$\begin{aligned} H_{b_s} &= -\alpha \sin(\alpha x_{b_s}) \cos(\alpha y_{b_s}) \\ H_{b_f} &= -\alpha \cos(\alpha x_{b_f}) \sin(\alpha y_{b_f}) \\ P_{accurate} &= \cos(\alpha x_{(b_s,b_f)}) \cos(\alpha y_{(b_f,b_s)}) \end{aligned} \tag{13}$$

In order to solve (12b) taking note of (13), we employ the explicit Euler time-series method to estimate the velocity of the bubble on the thin film assuming $t_b = (m_b + 1)$

$$\begin{aligned} u_{(b_f,b_s)_{m_b+1}} &= u_{(b_f,b_s)_{m_b}} + \nabla t_b \left(H_{b_s} - \frac{\partial P}{\partial x_{(b_s,b_f)}} \right)_{m_b} \\ v_{(b_f,b_s)_{m_b+1}} &= v_{(b_f,b_s)_{m_b}} + \nabla t_b \left(H_{b_f} - \frac{\partial P}{\partial y_{(b_s,b_f)}} \right)_{m_b}. \end{aligned} \tag{14}$$

The stability condition of the system depends on the time step in the Euler series. Conrad, and Molnar [103], Molnar *et al.* [104] and Smith [105] established that stability for $Re \ll 1$ depends on diffusive viscosity, while $Re \gg 1$ stability depends on linearized convection. Therefore, the stability criterion for bubbles depends on small change in time, ∇_t, small change in horizontal distance ∇_x.

The bubble is assumed to cease when the continuity equation is a change in flow rate, $\nabla M_b = \frac{\partial \rho}{\partial t} + \nabla(\rho u_{(b_s,b_f)}) \simeq 0$. This indicates that the Poisson equation for the velocity field is assumed to diverge freely as

$$\frac{\partial}{\partial t_b}(\nabla . u_{(b_s,b_f)}) = \nabla . H_{(b_s,b_f)} - \nabla^2 P_{(b_s,b_f)}. \tag{15}$$

Equation 15 suggests a luck of local mass conservation arises due to linear growth of local continuity errors. At the time of bubble cease (bust), that is, at h_b, the local conservation continuity is small by continue to grow. This means that simulations for the possible plot of local continuity against time can take several days before conclusion is drawn. In order to avoid this, the pressure must be suited with time-series step to match the growth in errors and stop when h_b is attained so that

$$\nabla^2 P_{(b_s,b_f)_{m_b+1}} = \nabla H_{(b_s,b_f)_{m_b}} + \frac{1}{\nabla t}(\nabla u_{(b_s,b_f)_{m_b}}). \tag{16}$$

3 Results

3.1 Numerical simulation and discussion

The paper focuses on modeling the lifetime of the bubble without considering the fuel-surfactant properties. The paper considers the velocity and Reynolds number of the thin film to model the life of the bubble on the surfactant-fuel interface. The paper focuses on the effect of varying Re numbers in a bubble due to surfactant and fuel thin film interface. The simulations are important in helping to estimate the time before bubble bust, which analogously fuel vapor purportedly released into the atmosphere. We assume the velocity of the surfactant-fuel thin layer is constant and has A plot of change in ∇M_b given (16), (14), and (15) considering varying $Re = [0.01, 0.1, 1, 10, 100, 1000]$ and $a_b = 0.2$ yield the graphical representations in Fig. 2a-2e.

Fig. 2a indicate rate of change of mass flow rate of the bubble as thickness of the bubble approaches h_b when $Re = 0.01$. The graph indicate the rate of change of mass flow rate reduces uniformly from $t_b = 0$. The change began steeply then decelerates.

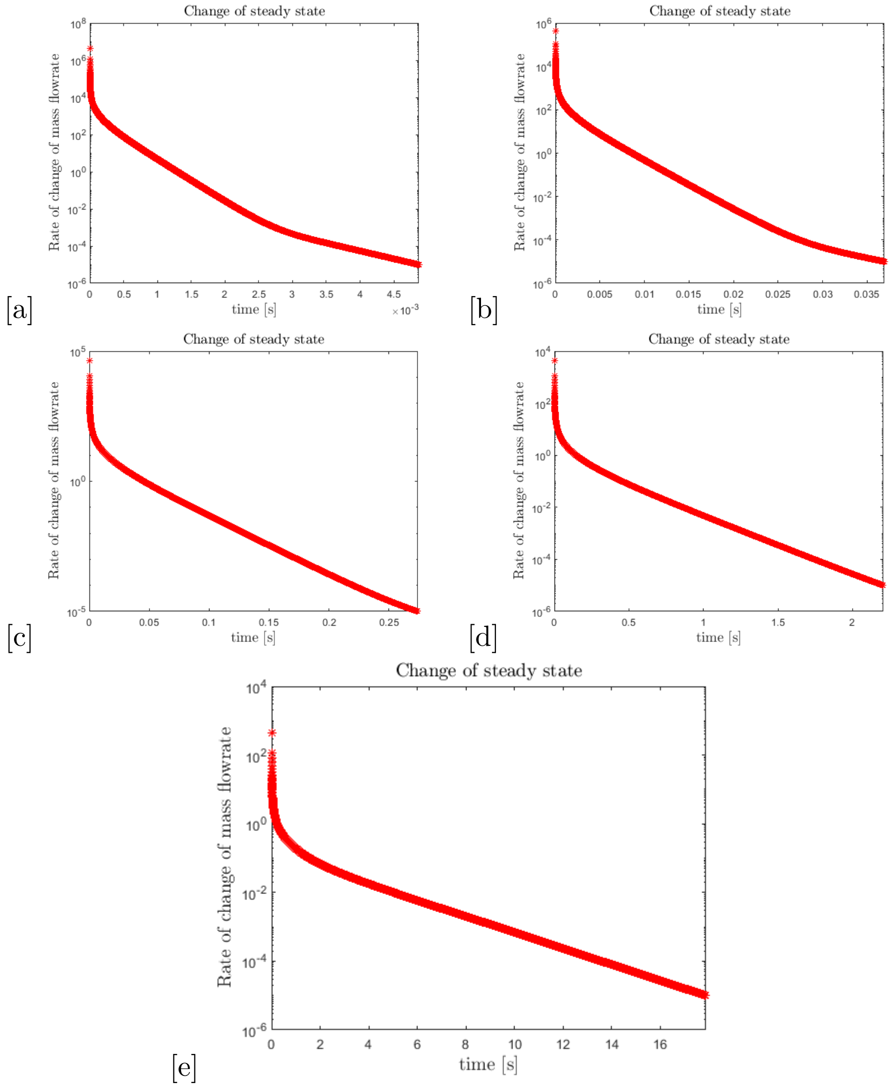

Fig. 2. Rate of change of mass flow rate at: (a) $Re = 0.01$; **(b)** $Re = 0.1$; **(c)** $Re = 1$; **(d)** $Re = 10$;**and (e)** $Re = 100$

Fig. 2b indicate rate of change of mass flow rate of the bubble as thickness of the bubble approaches h_b when $Re = 0.1$. The graph indicate the rate of change of mass flow rate posses an almost similar trend as that of Fig. 2a.

Fig. 2c indicate rate of change of mass flow rate of the bubble as thickness of the bubble approaches h_b when $Re = 1$. The graph indicate the rate of change of mass flow rate reduces uniformly from $t_b = 0$. Unlike Fig. 2a and 2b, here, the graph indicate the h_b will be reached when $t_b = 0.28s$. The observation proved the notion by [103], Molnar *et al.* [104] and Smith [105] that for $Re \ll 1$ the stability depends on diffusive viscosity, hence plot may take longer to obtain when thickness value becomes h_b.

Fig. 2d indicate rate of change of mass flow rate of the bubble as thickness of the bubble approaches h_b when $Re = 10$. The graph indicate the rate of change of mass flow rate reduces uniformly from $t_b = 0$. Similarly as in Fig. 2a and 2b and unlike in Fig. 2c the graph indicate it will take longer time for thickness to reach h_b. The observation proved the notion by [103], Molnar *et al.* [104] and Smith [105] that for $Re \gg 1$ the stability depends on linearized convection, hence plot may take longer to obtain when thickness value becomes h_b.

Fig. 2e indicate rate of change of mass flow rate of the bubble as thickness of the bubble approaches h_b when $Re = 10$. The graph indicate the rate of change of mass flow rate reduces uniformly from $t_b = 0$. Similarly as in Fig. 2a and 2b and unlike in Fig. 2c the graph indicate it will take longer time for thickness to reach h_b. The observation proved the notion by [103], Molnar *et al.* [104] and Smith [105] that for $Re \gg 1$ the stability depends on linearized convection, hence plot may take longer to obtain when thickness value becomes h_b.

4 Conclusion

Surfactants are important in many activities, such as the prevention of fuel explosion. The determination of the lifetime bubble, which depends on its thickness, is a real-world problem of fuel spillage. The proposed study aimed to establish the lifetime of the bubble. The study assumes that the rate of change of mass flow rate is proportional to the thinning rate. The study, thus, aims to find the lifetime of the bubble by plotting the rate of mass flow rate change against time. The maximum lifetime of the bubble is assumed to match the time when the mass flow rate change is zero. The study also assumes the velocity of flow rate and other fluid properties at the interface of fuel-surfactant constant other than Re. Re is varied from 0.01 to 100.

The graphical plots suggest that for $Re \ll 1$, and $Re \gg 1$, the stability depends on diffusive viscosity and linearized convection, respectively. These findings are similar to those of [103], Molnar *et al.* [104] and Smith [105]. The simulation suggested that the fuel-surfactant interface may have $Re = 1$. However, since no study has modeled a lifetime of bubble based on fuel-surfactant thin film layer, the proposed study lacks no comparison of results with existing studies. Therefore, we will assume the bubble formed at the interface of fuel-surfactant thin foam has $Re = 1$ and its lifetime is $t_b \simeq 0.28$. Future studies in the area need to consider the effect of variation in temperature, velocity, and Reynolds number in determining the lifetime of a bubble in the thin foam of the surfactant-fuel interface.

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Competing Interests

Authors have declared that no competing interests exist.

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