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On a Class of Universal Probability Spaces

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Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

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ABSTRACT

Aims/ Objectives: The objective of this paper is to introduce a class of probability spaces that include several exceptions introduced by Dieudonné [3], Anderson and Jessen [4], and Doob and Jessen [5]. The class of alternative probability space is called the Universal Probability Space (UPS). The UPS consists of Borel sets, elements of which are tensors. It is proven that indeed such tensor sets represent a more general probability space. Given the properties of tensors, it is shown that the exceptions introduced by Dieudonné, Anderson, Jessen, and Doob are merely special events that can occur in the UPS.
Study Design: Methodological study.
Place and Duration of Study: Research Unit of Economics Traffic Clinic - ETC, Paris, France, between June 2015 and September 2015.
Methodology: Borel tensor sets were used in constructing a more general probability space.
Results: Some basic definitions and properties of Borel tensor sets in the context of the UPS are given. It is shown that the UPS has a defined metric. Some elements of the UPS are given such as conditional probability and independence property.
Conclusion: The UPS is a more generalized probability space.

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1 INTRODUCTION

Kolmogorov [1] describes a model for probability theory, which is based on a probability measure (P), and on a Borel field (**B**) of subsets of a space (Ω). This model is universally accepted and used in probability and statistic works. However few researchers since have produced examples that show that the Kolmogorov's model is too general, and there is a need for a more restricted probability space, Blackwell [2]. Such examples are given by Dieudonné [3], Anderson and Jensen [4], and Doob and Jensen, [5], [6]. Dieudonné [3] states that there exists a pair (Ω, **B**), a probability measure (P) on (**B**), and a Borel subfield (**A** $\subset$ **B**), for which there is no function Q(ω,E) defined for all ($\omega \in \Omega$) and ($E \in$ **B**) with the following properties: (Q) is for fixed (E) an (A-measurable) function of (ω) and for fixed (ω) a probability measure on (**B**), and for every (A $\in$ **A**), ($E \in$ **B**), we have

$$P(A \cap E) = \int_A Q(\omega, E) dP(\omega). \qquad (1.1)$$

To explain the Dieudonné's example, he mainly states that the probability of the occurrence of two events as is defined in (1.1) strictly depends on the existence of Borel sets, the existence of probability measure on Borel sets, and the existence of measurable (real-valued) functions on Borel sets. Borel sets must be both countable and closed, and continuous and open so that both events could occur in any two Borel sets of a Borel field (**B**) of a probability space (Ω). In general, a standard probability space is considered to be isomorphic to interval [0,1] with a Lebesgue measure for a finite or countable set of events or a combination (disjoint union) of both. In the probability space thus defined, the probability measure of occurrence of two countable sets can not be found by a continuous measure.

The only feasible space that contains both closed and open sets is the tensor space which is a mapping or transformation of any event ($\omega \subset \mathbb{R}$) to a space ($\mathbb{R}^d$), where ($d < \infty$) represents a finite dimension. This means that an event is considered to be made of several dimensions. For example any event ($\omega \subset \mathbb{R}^d$) in (i, j)th dimension is written as ($\omega^i_j = \omega_{ij} \cdot e^{ij}$) and in (k, l)th dimension as ($\omega^k_l = \omega^{kl} \cdot e_{kl}$). Thus the concept of open and closed sets is transformed into tensor sets which are both open and closed. Open tensor sets contain manifolds (M) made out of linear combination, or inner and/or outer product of tensors. An example of such a manifold would be [$\omega^{ij}_{kl} = (\omega_{ij} \otimes \omega^{kl}) \cdot (e^{ij} \otimes e_{kl})$]. Since open tensor sets form modules in tensor space, they satisfy the closure condition and therefore contain closed sets. Closed tensor sets contain tensors themselves, for example a tensor set (ω^i_j, ω^k_l) is a closed set since tensors are considered to be affine manifolds and by themselves respect the closure condition. The transformation of an event ($\omega \subset \mathbb{R}$) from ($\mathbb{R}$) to ($\mathbb{R}^d$) has repercussions on how the probability of that event is calculated and this is a significant evolution.

The second example comes from Anderson and Jensen [4], who show that given a sequence of probability spaces (Ω_n) and Borel fields (**B**$_n$), (Ω_n,**B**$_n$), there exist probability measures (P_n) for all sets (**A**$_n$) where (**A**$_n$) is in the multiplicative probability space ($\prod_{i=1}^n \Omega_i$), given as (**A**$_n \subset \prod_{i=1}^n \Omega_i$), and where the Borel fields (**B**$_n$) are also in the multiplicative probability space ($\prod_{i=1}^n \Omega_i$), given as ($B_n \subset \prod_{i=1}^n \Omega_i$) such that the probability measure (P_n) is countably additive on each set (A_n) but not on the union of the sets ($\cup$**A**$_n$). The observations of Anderson and Jensen are similar to Dieudonné. They observe that even though individually probability measures may exist, they may not exist for the whole probability space (Ω) where ($\Omega = \prod_{i=1}^n \Omega_i$).

The third example is provided by Doob and Jensen [5], [6]. They state that given any pair (Ω, **B**) and a probability measure (P), on a Borel field (**B**), then for any real-valued **B**-measurable functions (f) and (g) such that ($f \subset F \subset$ **B** $\subset \Omega$)

and ($g \subset G \subset$ **B** $\subset \Omega$), where (F) and (G) are two linear Borel sets, the following exists:

$$P[\omega : f \in F, g \in G] \neq P[\omega : f \in F] \times P[\omega : g \in G]. \quad (1.2)$$

Expression (1.2) states that the probability of occurrence of an event (ω) in two continuous sets with two **B**-measurable functions $(f(\omega))$ and $(g(\omega))$, $(P(\omega) : \omega \in f(\omega) \bigcap g(\omega))$ is not equal to the probability of occurrence of the event (ω) in $(f(\omega))$ multiplied by the probability of occurrence of the event (ω) in $(g(\omega))$, $([P(\omega) : \omega \in f(\omega) \bigcap g(\omega)] \neq [P(\omega) : \omega \in f(\omega) \subset F] \times [P(\omega) : \omega \in g(\omega) \subset G])$. The existence and occurrence of each example is proved by Doob, Kolmogorov and Hartman [7] for the case when the probability space $(\Omega \subset \cup_{i=1}^{n} B_i)$ contains the union of Borel sets and the Borel field (**B** $\subset \Omega \subset \cup_{i=1}^{n} B_i$) consists of the union of Borel subsets (B_i). All (3) examples suggest that the probability space (Ω) is not well defined and more specific probability spaces should be identified.

Gnedenko and Kolmogorov [8] propose the perfect probability space $(\Omega, \mathbf{B}, P)$ with real-valued **B**-measurable function (f); where given a linear set(A) there exists a (ω) such that ($\omega : f(\omega) \in A \subset$ **B**). There exists a Borel subset $(B \subset A)$ such that $([P(\omega) : f(\omega) \in B] = [P(\omega) : f(\omega) \in A])$. The perfect probability space proposed by Gnedenko and Kolmogorov, provides a more restricted probability space that does not allow for special cases such as the (3) examples introduced. The proof of this is given in Doob. Blackwell identifies a more restricted probability space, called the Lusin space $(\Omega, \mathbf{B})$ which is more restricted than the perfect probability space of Gnedenko and Kolmogorov. The Lusin space is based on Gnedenko and Kolmogorov's perfect probability space. The particularities of Lusin space are: (1) The Borel field (**B**) is separable and contains all sub-sets (B_n), (2) the range of any real-valued **B**-measurable function (f) for which ($\omega : f(\omega) \in$ **B** $\in \Omega$) is an analytic set. An analytic set is a continuous image of the set of irrational numbers [2]. The advantage of analytic sets is that all the elements of these sets are separable.

In essence, the Lusin space restricts the Borel field (**B**) to sets of irrational numbers where the probabilities are in continuous real sets (B). Blackwell uses the properties of the analytic sets to prove the existence of the Lusin space $(\Omega, \mathbf{B})$ and the existence of a metric in this space. This metric constitutes the probability measure. Since the Lusin space contains countable and separable subsets that determine the set of Borel fields, therefore, the (3) probabilities discussed earlier can occur in this space. The Lusin space is Homomorphism to the perfect Kolmogorov probability space, thus is not an evolution of the perfect space but rather an extension. The concept of mapping introduced by Blackwell is the right approach, but the use of analytic sets to solve the (3) examples is somewhat ambiguous, since the analytic sets are naturally included in the Gnedenko and Kolmogorov's perfect probability space.

A new angle to pursue would be to find out whether it is possible to have a probability space where an event can occur if it is in a Borel tensor field $(\boldsymbol{B})$ that contains a group of finite Borel sets that are mappings in a tensor space (Ω), i.e. $(\boldsymbol{B} = \bigcup_{l=1}^{M} B_l^i)$ and $(\boldsymbol{B} = \prod_{l=1}^{M} B_l^i)$, the superscript (i) represents the tensor indexing of coordinates, and (l) represents the number of Borel tensor sets contained in a tensor field $(\boldsymbol{B})$. Each Borel tensor set can be identified as $(B^i \in \Omega : B^i = B_{j_s}^{i_r} \otimes e_{i_1,\ldots,i_r}^{j_1,\ldots,j_s})$, where $(e_{i_1,\ldots,i_r}^{j_1,\ldots,j_s})$ constitutes the basis in the tensor space (Ω). The advantage of each Borel set to be a mapping in a tensor space is that it acquires the following properties; (a) closure, the product of every two mappings of the Borel set (B) is a mapping of the Borel set; (b) Inverse: for every mapping of the Borel set, there is an inverse mapping that is in the Borel set. The class of such alternative probability space is called the Universal Probability Space (UPS). The UPS consists of Borel sets, elements of which are mappings or tensors. It can be argued that indeed such tensor sets represent a more general probability space. Given the properties of tensors, it is shown that the exceptions introduced by [3], [4], [5] are merely events that can occur in the UPS. Some basic definitions and properties of Borel tensor sets in the context of the UPS are given. It is shown that the UPS has a defined metric. Some elements of the UPS are given such as conditional probability, and independence property.

2 THE UNIVERSAL PROBABILITY SPACE, UPS: SOME BASIC DEFIN - ITIONS

Borel tensor sets (B^i) make up a Borel field ($\boldsymbol{B}$). It is relevant to give some definitions and properties of such sets. The advantage of a Borel set as a mapping or a tensor is that both the countable-separable sets and the continuous sets are in the same Borel field. In this section it is illustrated that any space consisting of Borel tensor fields (Ω,$\boldsymbol{B}$) is a complete space. Furthermore, it is demonstrated that the complete space of Borel fields has a metric that is equivalent to a probability measure (P). A complete space of Borel fields with probability measures is a UPS.

Theorem 2.1. *If (Ω) is a tensor space, where a class of Borel tensor fields ($\boldsymbol{B}$) can be defined, then the ensemble (Ω,$\boldsymbol{B}$) constitutes a complete space.*

Proof. It suffices to show that the class of Borel tensor field ($\boldsymbol{B}$) is convex, [9], [10], [11], [12], [13]. Convex sets are complete; therefore a class of convex Borel tensor sets constitutes a complete tensor space. Divide ($\boldsymbol{B}$) into (m) partitions. Let a partition be represented by ($K_i; i = 1, ..., m$). Let each partition contain a set of tensor points or weights ($\lambda_i \in K_i$) such that ($\lambda_i = \sum_\alpha l_\alpha \cdot e^\alpha$). ($e^\alpha$) is a basis for the Borel tensor field,(l_α) is a set of non-negative integers. Since each Borel set is the smallest set with a minimum (0), and a maximum (1), then the partition that contains (0) has a weight ($\lambda_0 < \lambda_i : i = 1, ..., m$). The partition that contains (1), has a weight ($\lambda_1 > \lambda_0$) and ($\lambda_1 > \lambda_i : i = 1, ..., m - 1$). Any partitions between the two partitions (K_0)and (K_1) must be convex partitions such that for any set of non-negative integers (n_α) not equal to (l_α), the following inequality exists: ($\sum_\alpha l_\alpha \cdot e^\alpha < \sum_\alpha n_\alpha \cdot e^\alpha$). Thus the tensor field ($\boldsymbol{B}$) is convex, and the ensemble (Ω,$\boldsymbol{B}$) constitutes a complete or universal space. □

Theorem 2.2. *Given that (Ω,$\boldsymbol{B}$) is a complete space, then there is a metric on the tensor field ($\boldsymbol{B}$), [14], [15], [16], [17]. This metric is the probability measure (P). The complete space (Ω,$\boldsymbol{B}$) with the probability measure (P) is a UPS.*

Proof. Let ($c_\alpha = \sum_\alpha l_\alpha \cdot e^\alpha$) with be a tensor event point on a Borel tensor set ($B^i \subset \boldsymbol{B}$). An event point is an event in tensor space. (e^α) is a standard basis for tensors in (B^i). A metric on (B^i) is defined as the distance between the tensor point (0), and (c_α), (P($c_\alpha \geq 0$)), such that it is the greatest lower bound of the length of all pairs of points (d((c_α, μ_α)) belonging to the Borel tensor set (B^i). The metric (P(c_α)) is equal to (P(c_α) = $\mid c_\alpha \mid$) satisfies the standard axioms of probability. 1) The metric is between (0) and (1), ($0 \leq P(c_\alpha) \leq 1$). 2) To Each set ($B^i \subset \boldsymbol{B}$) is assigned a non-negative metric P(c_α) which is called the probability of a tensor event point (c_α). 3) P($\bigcup B^i$) is equal to 1. 4) P (c_α+μ_α)= P(c_α) + P(μ_α), if (c_α) and (μ_α), the two tensor event points are not on a same linear transformation. If the tensor set (B^i) contains a manifold, then the probability P(c_α, μ_α) must be positive ($P(c_\alpha, \mu_\alpha) \geq 0$), symmetric ($P(c_\alpha, \mu_\alpha) = P(\mu_\alpha, c_\alpha)$), and ($P(c_\alpha, \mu_\alpha) \leq P(c_\alpha) + P(\mu_\alpha)$), and is non degenerate, if ($P(c_\alpha, \mu_\alpha) = 0$), then ($c_\alpha = \mu_\alpha$) due to the continuity of the Borel tensor set(B^i). □

Theorem 2.3. *Given that (Ω,$\boldsymbol{B}$) is a UPS, and let (B^i) and (B^j) for ($i \neq j$) be two Borel tensor subsets such that ($B^i \bigcap B^j = 0$), the two tensor subsets are separable, then there exists a sub Borel tensor field ($\boldsymbol{C} \subset \boldsymbol{B}$) such that (($B^i \cup B^j$) $\in$ $\boldsymbol{C}$) the union of the two Borel tensor subsets is in the sub Borel tensor field ($\boldsymbol{C}$). Let ($c_\alpha = \sum_\alpha l_\alpha \cdot e^\alpha$) be any tensor event point in either one of the Borel tensor sets (B^i) or (B^j), such that ($c_\alpha \in \boldsymbol{B}$), then ($c_\alpha$) is also in the sub Borel tensor field ($\boldsymbol{C}$), i.e. ($c_\alpha \in \boldsymbol{C}$).*

Proof. Let ($c_\alpha = \sum_\alpha l_\alpha \cdot e^\alpha$) be a tensor event point on a Borel tensor set (B^i). (e^α) is a standard basis for tensors in (B^i). Now let ($\nu^\alpha_{\bar{\alpha}} = \sum_{(\alpha,\bar{\alpha})} n_{(\alpha,\bar{\alpha})} \cdot (e^\alpha, e^{\bar{\alpha}})$), be a tensor event point constructed from the basis of (B^i) and (B^j).($e^{\bar{\alpha}}$) is a standard basis for the Borel tensor set (B^j). The tensor event point (c_α)can be modified by ($\nu^\alpha_{\bar{\alpha}}$) in the following way: ($c_\alpha = det(\nu^\alpha_{\bar{\alpha}}) \cdot \sum_\alpha l_\alpha \cdot e^\alpha$), where the determinant of ($\nu^\alpha_{\bar{\alpha}}$) is positive ($det(\nu^\alpha_{\bar{\alpha}}) \geq 0$). The modified ($c_\alpha$) is in the union of the two Borel tensor sets (B^i) and (B^j), ($c_\alpha \in (B^i \bigcup B^j)$), and since (($B^i \bigcup B^j$) $\in$ $\boldsymbol{C}$), then ($c_\alpha \in \boldsymbol{C}$). □

Corollary 2.4. *Given that (Ω,$\boldsymbol{B}$) is a UPS, and if* *two Borel tensor sets (B^i) or (B^j), contain the same tensor event points, then the two tensor sets (B^i) and (B^j), are identical.*

Corollary 2.5. *Given that (Ω,$\boldsymbol{B}$) is a UPS, then any Borel tensor set (B^i) that consists of all event points that are affine transformations is in the Borel tensor field ($\boldsymbol{B}$).*

Corollary 2.6. *Given that (Ω,$\boldsymbol{B}$) is a UPS, and let the Borel tensor field ($\boldsymbol{D}$) be a group set of all tensor event points ($c^t_\alpha \in \boldsymbol{D}$), such that ($c^t_\alpha = \sum_\alpha f_\alpha(t) \cdot e^\alpha$), where ($f_\alpha(t) \in R^d; d < \infty$), is a manifold, and (t) is a real-valued parameter in ($t \in [0, 1]$). The Borel tensor field ($\boldsymbol{D}$) is a subset of ($\boldsymbol{B}$); ($\boldsymbol{D} \subset \boldsymbol{B}$), and ($\Omega$,$\boldsymbol{D}$) is a UPS.*

Proof. Let (Z) Be a Borel tensor set in the Borel tensor field ($\boldsymbol{D}$), ($Z \subset \boldsymbol{D}$). Let the tensor event point ($c^t_\alpha = \sum_\alpha f_\alpha(t) \cdot e^\alpha$) be in the Borel tensor set (Z), ($c^t_\alpha \in Z$). If the manifold ($f_\alpha(t)$) is affine, then it is possible to find (t=t*) such that ($Z \bigcap B^i) = (c_\alpha, c^{t*}_\alpha$)) for some ($B^i \in \boldsymbol{B}$), and ($c^{t^*}_\alpha = \sum_\alpha f_\alpha(t^*) \cdot e^\alpha$) . By Corollary 2.5, the tensor event points, (c_α), and ($c^{t^*}_\alpha$) are identical, and thus the two Borel tensor sets (Z) and (B^i) are identical. Since ($Z \subset \boldsymbol{D}$), the couple (Ω,$\boldsymbol{D}$)is a UPS. If ($f_\alpha(t)$) is a non-linear manifold, then let ($f_\alpha(t)$) be a tensor multiplicity product ($f_\alpha(t) \in (B^i \otimes B^j), i \neq j$). ($c^t_\alpha$) is in the intersection of the two Borel tensor sets with (Z), [$c^t_\alpha \in (Z \bigcap (B^i \otimes B^j))$]. Since (($B^i \otimes B^j) \subset \boldsymbol{B}$), then ($Z \in \boldsymbol{B}$). Since ($Z \in \boldsymbol{D}$), then ($\boldsymbol{D} \subset \boldsymbol{B}$), and ($\Omega$,$\boldsymbol{D}$) is a UPS. ☐

3 CONDITIONAL PROBABILITY DISTRIBUTION UNDER THE UNIVERSAL PROBABILITY SPACE

In this section the example introduced by Dieudonné is represented as a theorem in the UPS.

Theorem 3.1. *Given that (Ω,$\boldsymbol{B}$) is a UPS. Let (P) be a probability measure on the Borel tensor field ($\boldsymbol{B}$). Let ($\boldsymbol{A}$) be a Borel tensor subfield ($\boldsymbol{A} \subset \boldsymbol{B}$). Let ($\boldsymbol{M}$) be a manifold on ($\boldsymbol{B}$) such that ($\boldsymbol{A} \subset \boldsymbol{M}$), then there exists a function ($\gamma(\omega, \boldsymbol{E}) \in \boldsymbol{A}$) defined for all ($\omega \in \Omega$) and any other Borel tensor subfield ($\boldsymbol{E} \subset \boldsymbol{B}$). We have*

$$P(\boldsymbol{A} \cap \boldsymbol{E}) = \int_{\omega \in \boldsymbol{A}} \gamma(\omega, \boldsymbol{E}) \cdot T_\alpha(\gamma\omega), \qquad (3.1)$$

where ($T_\alpha(\gamma\omega)$) is a tangent tensor in the Borel tensor field ($\boldsymbol{B}$) for a tensor type (α).

Proof. Let (Ω,$\boldsymbol{B}$) be a UPS. Let (P) be a probability measure on the Borel tensor field ($\boldsymbol{B}$). ($\boldsymbol{B}$) contains the union and the intersection of non-empty Borel tensor sets ($B^i \in \boldsymbol{B}$). It is given that the Borel tensor subfield ($\boldsymbol{A} \subset \boldsymbol{B}$) is a subset of ($\boldsymbol{B}$), and ($\boldsymbol{M} \in \Re^d$) is a manifold in ($\boldsymbol{B}$), ($\boldsymbol{M} \subset \boldsymbol{B}$). Since ($\boldsymbol{A} \subset \boldsymbol{B}$), then ($\boldsymbol{A} \subset \boldsymbol{M}$). By Corollary 2.6, there exists a function ($\gamma(\omega, \boldsymbol{E}) \in \boldsymbol{M}$), where ($\boldsymbol{E} \in \boldsymbol{B}$) is any other Borel tensor field. By Theorem 2.3, ($\boldsymbol{E} \subset \boldsymbol{B}$) and ($\boldsymbol{A} \subset \boldsymbol{B} \subset \boldsymbol{M}$), then ($\boldsymbol{E} \subset \boldsymbol{M}$) and (($\boldsymbol{A} \cap \boldsymbol{E}) \subset \boldsymbol{M}$). The change in the probability measure (dP) for (ω) is a tangent tensor ($T_\alpha(\gamma\omega)$) for a tensor type (α) in the Borel tensor field ($\boldsymbol{B}$). The probability of the intersection of the two Borel tensor fields ($\boldsymbol{A}$) and ($\boldsymbol{E}$) can be formulated by (3.1). ☐

4 INDEPENDENCE UNDER THE UNIVERSAL PROBABILITY SPACE

Theorem 4.1. *Let (Ω,$\boldsymbol{B}$) be a UPS. Let (Ω_n,$\boldsymbol{B}_n$) be a sequence of UPSs. Let ($\boldsymbol{A}_n \subset (B^1 \otimes B^2 ..... \otimes B^n)$). The probability measure (P) on ($\boldsymbol{B}$) is countably additive for ($\cup \boldsymbol{A}_n$), i.e., P($\cup \boldsymbol{A}_n$) = P($\boldsymbol{A}_1$)+ P($\boldsymbol{A}_2$)+......+P($\boldsymbol{A}_n$) where ($\boldsymbol{A}_1 = \boldsymbol{B}_1$),($\boldsymbol{A}_2 = (\boldsymbol{B}_1 \otimes \boldsymbol{B}_2$), and ($\boldsymbol{A}_n = (\boldsymbol{B}_1 \otimes \boldsymbol{B}_2 ...... \otimes \boldsymbol{B}_n)$).*

Proof. ($\cup \boldsymbol{A}_n = \boldsymbol{A}_1 + \boldsymbol{A}_2 + ... + \boldsymbol{A}_n = \boldsymbol{B}_1 + (\boldsymbol{B}_1 \otimes \boldsymbol{B}_2) + ... + (\boldsymbol{B}_1 \otimes \boldsymbol{B}_2 ...... \otimes \boldsymbol{B}_n)$). By Theorem 2.2, there exists a metric equivalent to a probability measure for the UPS (Ω,$\boldsymbol{B}$), therefore there exists a probability measure for each sequence of UPSs (Ω_n,$\boldsymbol{B}_n$). Let (ω^i) be a sequence of event points, such that ($\omega^i \subset \boldsymbol{A}_i$). Let ($P_n$) be a sequence of probability measures for the sequence of Borel tensor subfields ($\boldsymbol{A}_n$). By Corollary 2.6, if all sequences of event points (ω^i) are identical, then all the Borel tensor subsets ($\boldsymbol{A}_n$) are identical. By Corollary 2.5, ($P_n(\cup \boldsymbol{A}_n) =$

$\sum_{i=1}^{n} P_i(\omega^i, \omega^j) = n.c : i \neq j$) for ($P_i(\omega^i, \omega^j) = c$), where (c) is a real-valued constant between [0,1]. Since (P($\boldsymbol{A}_1$)+ P($\boldsymbol{A}_2$)+......+P($\boldsymbol{A}_n$))=n.c, then the probability measure (P) is countably additive. If all sequences of event points (ω^i) are not identical, then $P_n(\cup \boldsymbol{A}_n) = (max[\mid \omega^i - \omega^j \mid] : i \neq j)$. $(P_1(\boldsymbol{A}_1) + P_2(\boldsymbol{A}_2) + ... + P_n(\boldsymbol{A}_n) = P_1(\boldsymbol{B}_1) + P_2(\boldsymbol{B}_1 \otimes \boldsymbol{B}_2) + ... + P_n(\boldsymbol{B}_1 \otimes \boldsymbol{B}_2 ...... \otimes \boldsymbol{B}_n) = 0 + max[\mid \omega^1 - \omega^2 \mid] + ... + max[\mid \omega^i - \omega^j \mid])$. The probability measure (P) is countably additive in the case where the event points (ω^i) are not identical. □

Theorem 4.2. *Let (Ω,$\boldsymbol{B}$) be a UPS, and (P) a probability measure on ($\boldsymbol{B}$). Let (f) be an affine transformation of a tensor event point (ω) in a set of all affine transformations Borel tensor set ($F \subset \boldsymbol{B}$), and let (g) be another affine transformation of the same event point (ω) in a set of all affine transformations Borel tensor set ($G \subset \boldsymbol{B}$). Let ($\omega : F \otimes G$), then the following exists and the probability measure ($P(\omega)$) is:*

$$P[\omega : F \otimes G] = P[\omega : f \in F] \times P[\omega : g \in G]. \tag{4.1}$$

.

Proof. From Corollaries 2.5, and 2.6, and Theorem 2.3, the two affine transformations (f), and (g) are duel basis. Given the duality of basis, let $((\omega = \sum_i l_i \cdot e^i) \in f : e^i = (e^1, ..., e^I))$ and $((\omega = \sum_j n_j \cdot e^j) \in g : e^j = (e^1, ..., e^J))$. The two basis ($e^i$), and ($e^j$) are the dual basis, ($\langle e^i, e^j \rangle = \delta_i^j$), where ($\delta_i^j$) is the Kronecker delta. The probability measure ($P[\omega : f \in F]$) is then equal to ($P[\omega : f \in F] =\mid \sum_i l_i \cdot e^i \mid$), and the probability measure ($P[\omega : g \in G]$) is then equal to ($P[\omega : g \in G] =\mid \sum_j n_j \cdot e^j \mid$). The product of the two probability measures is equal to ($P[\omega : f \in F] \times P[\omega : g \in G] =\mid \sum_i l_i \cdot e^i \mid \cdot \mid \sum_j n_j \cdot e^j \mid : i \neq j$). The left hand side of (4.1), by definition is equal to ($P[\omega : F \otimes G] = (\mid \sum_i \nu_i \cdot e^i) \cdot e^j \mid : i \neq j$). Therefore (4.1) exists. □

5 CONCLUSIONS

In both the perfect space of Gnedenko and Kolmogorov, and the Lusin space discussed in the introduction an event (ω) is considered to be ($\omega \in \Re$) in the space of real numbers. This implies that each event is an abstract, and the occurrence of the event is always in an abstract environment, and this is why it is possible to find exceptions to the rule. Now consider an event not as an abstract entity, but in all its dimensions which are in fact the reasons for the existence of that event. In this context, an event point is a mapping or transformation in a tensor space consisting of Borel tensor fields. The probability measure is the metric in a tensor space. The advantage is that this modification allows us to calculate a more accurate probability of an occurrence of an event. The objective in this paper is to suggest that by considering an event to be a tensor, this makes it possible to have a more in-depth view of the event, and thus be able to calculate a more realistic probability measure for that event. Where the probability of an event in a regular probability space may not exist, in a tensor space, due to mapping into a finite multidimensional space and the properties of tensor sets, the probability measure would exist. This opens a whole new set of insights into the occurrences of probabilities of events that are more realistic.

COMPETING INTERESTS

Author has declared that no competing interests exist.

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