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Existence of Periodic Solution for a Non-Autonomous Stage-Structured Predator-Prey System with Impulsive Effects

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Abstract

In this paper, we studied a non-autonomous predator-prey system where the prey dispersal in a two-patch environment. With the help of a continuation theorem based on coincidence degree theory, we establish sufficient conditions for the existence of positive periodic solutions. Finally, we give numerical analysis to show the effectiveness of our theoretical results.

Keywords: Periodic Solution, Coincidence Degree Theory, Stage-Structured, Impulsive

1. Introduction

In recent years, non-autonomous predator-prey systems have been widely studied [1-6]. There has been a growing interest in the study of mathematical models of populations dispersing among patches in the nature world [3,7-9].

In the classical predator-prey models it is usually assumed that each individual predator admits the same ability to feed on prey. However, it is different for some species whose individuals have a life history that takes them through two stages, immature and mature, where immature predators are raised by their parents, so many models with time delays and stage structure for both prey and predator were investigated and rich dynamics have been observed [4,6,10-12].

In this paper, we are considered the effects of prey diffusion in two patches and maturation delay for predator on the dynamics of an impulsive predator-prey model. We discuss the differential equation: (See 1.1)

$$\begin{cases} \left.\begin{aligned} \dot{x}_1(t) &= x_1(t)\big(a_1(t) - r_1(t)x_1(t)\big) \\ &\quad + d_1(t)\big(x_2(t) - x_1(t)\big), \\ \dot{x}_2(t) &= x_2(t)\big(a_2(t) - r_2(t)x_2(t)\big) \\ &\quad - k(t)x_2(t)y_2(t) \\ &\quad + d_2(t)\big(x_1(t) - x_2(t)\big), \\ \dot{y}_1(t) &= c(t)x_2(t)y_2(t) \\ &\quad - c(t-\tau)e^{\int_{t-\tau}^{t} -r(s)ds}x_2(t-\tau)y_2(t-\tau) \\ &\quad - d(t)y_1(t) - q_1(t)y_1^2(t), \\ \dot{y}_2(t) &= c(t-\tau)e^{\int_{t-\tau}^{t} -r(s)ds}x_2(t-\tau)y_2(t-\tau) \\ &\quad - q_2(t)y_2^2(t), \end{aligned}\right\} t \neq t_k, \\ \left.\begin{aligned} x_1(t_k^+) &= (1+\theta_{1k})x_1(t_k), \\ x_2(t_k^+) &= (1+\theta_{2k})x_2(t_k), \\ y_1(t_k^+) &= (1+\varphi_k)y_1(t_k), y_2(t_k^+) = y_2(t_k), \end{aligned}\right\} t = t_k, \end{cases} \tag{1.1}$$

Where we suppose that the system is composed of two patches connected by diffusion. $x_1(t)$ and $x_2(t)$ represent the densities of prey species in patch I and II at time *t*, $y_1(t)$ and $y_2(t)$ represent the densities of the immature and mature predator at time *t* in patch II, respectively. $x_1(t)$, $x_2(t)$ can diffuse between patch I and II while the predator species is confined to patch II. τ repre- sents a constant time to maturity. $a_i(t)(i=1,2)$ is the intrinsic growth rate; $\frac{r_i(t)}{a_i(t)}(i=1,2)$ is the carrying capacity; $d_i(t)(i=1,2)$ is the dispersal rate of prey species; $k(t)$ is the capture rate of mature predator. $c(t)$ is a conversion efficiency. $d(t)$ is the death rate of the immature predator. $q_i(t)(i=1,2)$ is the rate of intra-specific competition. θ_{ik} and φ_k represent the annual birth pulse of $x_i(t), y_1(t)(i=1,2)$ at $t_k\left(k\in Z^+\right)$. We make the following assumptions for our model:

1) $a_i(t), r_i(t), d_i(t), q_i(t)(i=1,2), d(t), k(t), c(t)$ and $r(t)$ are continuous positive $\omega-$periodic functions;

2) θ_{1k}, θ_{2k} and φ_k are constants and there exists a positive integer q such that

$$\theta_{1k+q}=\theta_{1k}, \theta_{2k+q}=\theta_{2k}, \varphi_{k+q}=\varphi_k, t_{k+q}=t_k+\omega .$$

2. Preliminaries

Denote by $PC(J,R)(J\subset R)$ the set of functions $\psi: J\to R$, which are piecewise continuous in $[0,\omega]$, and have points of discontinuity $t_k\in[0,\omega]$. Let $PC^1(J,R)$ denote the set of functions ψ with derivative $\dot{\psi}(t)\in PC(J,R)$. We define the Banach space of $\omega-$periodic functions $PC_\omega=\left\{\psi\in PC([0,\omega],R)\,|\,\psi(0)=\psi(\omega)\right\}$ with $\|\psi\|_{PC}=\sup\left\{|\psi(t)|: t\in[0,\omega]\right\}$ and PC_ω^1 with $\|\psi\|_{PC_\omega^1}=\max\left\{\|\psi(t)\|_{PC_\omega}, \|\dot{\psi}\|_{PC_\omega^1}\right\}$, we will considered the $PC_\omega\times PC_\omega$ with the norm

$$\|(\psi_1,\psi_2)\|_{PC}=\|\psi_1\|_{PC}+\|\psi_2\|_{PC} .$$

We define:

$$\overline{f}=\frac{1}{\omega}\int_0^\omega f(t)dt, \quad f^L=\min_{t\in[0,\omega]} f(t), \quad f^M=\max_{t\in[0,\omega]} f(t).$$

3. Existence of Positive Periodic Solutions

In this section, we study the existence of positive periodic solutions of system (1.1).

Before stating our result on positive $\omega-$periodic solutions of system (1.1), we need the following lemma:

Lemma 3.1 ([13]). Let $\Omega\in X$ be an open bounded set. Let L be a Fredholm mapping of index zero and N be $L-$compact on $\overline{\Omega}$. Assume

1) for each $\lambda\in(0,1)$, x is any solution of $Lx=\lambda Nx$ such that $x\notin\partial\Omega$;

2) for each $QNx\neq 0$ for each $x\in\partial\Omega\cap KerL$;

3) $\deg\left\{JQN, \Omega\cap KerL, 0\right\}\neq 0$.

Then the equation $Lx=Nx$ has at least one solution in $\overline{\Omega}\cap DomL$.

Theorem 3.1 If the system (1.1) satisfies

(H1) $$a\omega+\ln\left[\prod_{k=1}^{q}(1+\theta_k)\right]>0,$$

$$\overline{d}\,\omega-\ln\left[\prod_{k=1}^{q}(1+\varphi_k)\right]>0,$$

(H2) $$\overline{a_1-d_1}\,\omega+\ln\left[\prod_{k=1}^{q}(1+\theta_k)\right]>0,$$

$$\overline{a_2-d_2}\,\omega-k^M e^{M_4}\omega+\ln\left[\prod_{k=1}^{q}(1+\theta_{2k})\right]>0,$$

(H3) $$c^L e^{m_2+m_4}-c^M e^{-r^L+M_2+M_4}>0,$$

then the system (1.1) has at least one $\omega-$periodic positive solution.

Proof. Let $x_1(t)=e^{u_1(t)}, x_2(t)=e^{u_2(t)}, y_1(t)=e^{u_3(t)}$, $y_2(t)=e^{u_4(t)}$, then

$$\left.\begin{cases}\dot{u}_1(t)=a_1(t)-r_1(t)e^{u_1(t)}+d_1(t)e^{u_2(t)-u_1(t)}\\ \qquad -d_1(t),\\ \dot{u}_2(t)=a_2(t)-r_2(t)e^{u_2(t)}-k(t)e^{u_4(t)}\\ \qquad +d_2(t)e^{u_1(t)-u_2(t)}-d_2(t),\\ \dot{u}_3(t)=c(t)e^{u_2(t)+u_4(t)-u_3(t)}-d(t)-q_1(t)e^{u_3(t)}\\ \qquad -c(t-\tau)e^{\int_{t-\tau}^{t}-r(s)ds}e^{u_2(t-\tau)+u_4(t-\tau)-u_3(t)},\\ \dot{u}_4(t)=c(t-\tau)e^{\int_{t-\tau}^{t}-r(s)ds}e^{u_2(t-\tau)+u_4(t-\tau)-u_4(t)}\\ \qquad -q_2(t)e^{u_4(t)},\end{cases}\right\} t\neq t_k,$$

$$\left.\begin{cases}u_1(t_k^+)=u_1(t_k)+\ln(1+\theta_{1k}),\\ u_2(t_k^+)=u_2(t_k)+\ln(1+\theta_{2k}),\\ u_3(t_k^+)=u_3(t_k)+\ln(1+\varphi_k), u_4(t_k^+)=u_4(t_k),\end{cases}\right\} t=t_k, \tag{3.1}$$

One can easily see that if system (3.1) has one $\omega-$periodic solution $\left(u_1(t),u_2(t),u_3(t),u_4(t)\right)^T$, then

$$\left(e^{u_1(t)},e^{u_2(t)},e^{u_3(t)},e^{u_4(t)}\right)^T=\left(x_1^*(t),x_2^*(t),y_1^*(t),y_2^*(t)\right)^T$$

is a positive $\omega-$periodic solution of system(1.1). Thus, in what follows our goal is to show that system (3.1) has at least one $\omega-$periodic solution.

Here, we rewrite

$$f_1(t)=\dot{u}_1(t),\ f_2(t)=\dot{u}_2(t),$$

$$f_3(t)=\dot{u}_3(t),\ f_4(t)=\dot{u}_4(t).$$

Let

$$DomL=PC_\omega^1\times PC_\omega^1\times PC_\omega^1$$

and

$$N: PC_\omega^1\times PC_\omega^1\times PC_\omega^1\to Z,$$

with

$$N\begin{pmatrix}u_1\\u_2\\u_3\\u_4\end{pmatrix}=\left(\begin{pmatrix}f_1(t)\\f_2(t)\\f_3(t)\\f_4(t)\end{pmatrix},\begin{Bmatrix}\ln(1+\theta_{1k})\\\ln(1+\theta_{2k})\\\ln(1+\varphi_k)\\0\end{Bmatrix}_{k=1}^{q}\right)$$

and

$$KerL=\left\{\begin{pmatrix}u_1\\u_2\\u_3\\u_4\end{pmatrix}:\begin{pmatrix}u_1\\u_2\\u_3\\u_4\end{pmatrix}=\begin{pmatrix}C_1\\C_2\\C_3\\C_4\end{pmatrix}\in R^4,t\in[0,\omega]\right\}.$$

Where Q is defined by

$$QZ=\left(\frac{1}{\omega}\begin{pmatrix}\int_0^\omega f(t)dt+\sum_{k=1}^{q}a_k\\\int_0^\omega g(t)dt+\sum_{k=1}^{q}b_k\\\int_0^\omega h(t)dt+\sum_{k=1}^{q}c_k\\\int_0^\omega j(t)dt+\sum_{k=1}^{q}d_k\end{pmatrix},\left\{\begin{pmatrix}0\\0\\0\\0\end{pmatrix}\right\}_{k=1}^{q}\right).$$

Furthermore, $K_P:\operatorname{Im}L\to KerP\cap DomL$ is given by

$$K_PZ=\begin{pmatrix}\int_0^\omega f(t)dt+\sum_{0<t_k<t}a_k-\frac{1}{\omega}\int_0^\omega\int_0^t f(s)dsdt-\sum_{k=1}^{q}a_k\\\int_0^\omega g(t)dt+\sum_{0<t_k<t}b_k-\frac{1}{\omega}\int_0^\omega\int_0^t g(s)dsdt-\sum_{k=1}^{q}b_k\\\int_0^\omega h(t)dt+\sum_{0<t_k<t}c_k-\frac{1}{\omega}\int_0^\omega\int_0^t h(s)dsdt-\sum_{k=1}^{q}c_k\\\int_0^\omega j(t)dt+\sum_{0<t_k<t}d_k-\frac{1}{\omega}\int_0^\omega\int_0^t j(s)dsdt-\sum_{k=1}^{q}d_k\end{pmatrix}.$$

Thus,

$$K_P(I-Q)N\begin{pmatrix}u_1\\u_2\\u_3\\u_4\end{pmatrix}=\begin{pmatrix}\int_0^\omega f_1(t)dt+\sum_{0<t_k<t}\ln(1+\theta_{1k})\\\int_0^\omega f_2(t)dt+\sum_{0<t_k<t}\ln(1+\theta_{2k})\\\int_0^\omega f_3(t)dt+\sum_{0<t_k<t}\ln(1+\varphi_{1k})\\\int_0^\omega f_4(t)dt\end{pmatrix}$$

$$-\begin{pmatrix}\frac{1}{\omega}\int_0^\omega\int_0^t f_1(s)dsdt+\sum_{k=1}^{q}\ln(1+\theta_{1k})\\\frac{1}{\omega}\int_0^\omega\int_0^t f_2(s)dsdt+\sum_{k=1}^{q}\ln(1+\theta_{2k})\\\frac{1}{\omega}\int_0^\omega\int_0^t f_3(s)dsdt+\sum_{k=1}^{q}\ln(1+\varphi_k)\\\frac{1}{\omega}\int_0^\omega\int_0^t f_4(s)dsdt\end{pmatrix}$$

$$-\begin{pmatrix}\left(\frac{1}{t}-\frac{1}{2}\right)\int_0^t f_1(s)dt+\sum_{k=1}^{q}\ln(1+\theta_{1k})\\\left(\frac{1}{t}-\frac{1}{2}\right)\int_0^t f_2(s)dt+\sum_{k=1}^{q}\ln(1+\theta_{2k})\\\left(\frac{1}{t}-\frac{1}{2}\right)\int_0^t f_3(s)dt+\sum_{k=1}^{q}\ln(1+\varphi_k)\\\left(\frac{1}{t}-\frac{1}{2}\right)\int_0^t f_4(s)dt\end{pmatrix}$$

In order to apply the Lemma 3.1, we also need to find an appropriate open and bounded subset Ω. Corrsponding to the operator equation $Lu=\lambda Nu$, here, $\lambda\in(0,1)$, $u=(u_1,u_2,u_3,u_4)^T$, we can get

$$\begin{cases}\left.\begin{array}{l}\dot u_1(t)=\lambda f_1(t),\dot u_2(t)=\lambda f_2(t),\\\dot u_3(t)=\lambda f_3(t),\dot u_4(t)=\lambda f_4(t),\end{array}\right\}t\neq t_k,\\\left.\begin{array}{l}u_1(t_k^+)=u_1(t_k)+\lambda\ln(1+\theta_{1k}),\\u_2(t_k^+)=u_2(t_k)+\lambda\ln(1+\theta_{2k}),\\u_3(t_k^+)=u_3(t_k)+\lambda\ln(1+\varphi_k),\\u_4(t_k^+)=u_4(t_k),\end{array}\right\}t=t_k,\end{cases}\tag{3.2}$$

Suppose $u=(u_1,u_2,u_3,u_4)^T$ is a $\omega-$periodic solution to (3.2). By integrating over $[0,\omega]$,

$$\begin{cases}\overline{a_1-d_1}+\frac{1}{\omega}\ln\left[\prod_{k=1}^{q}(1+\theta_{1k})\right]\\=\frac{1}{\omega}\int_0^\omega\left(r_1(t)e^{u_1(t)}-d_1(t)e^{u_2(t)-u_1(t)}\right)dt,\\\overline{a_2-d_2}+\frac{1}{\omega}\ln\left[\prod_{k=1}^{q}(1+\theta_{2k})\right]\\=\frac{1}{\omega}\int_0^\omega\left(r_2(t)e^{u_2(t)}+k(t)e^{u_4(t)}-d_2(t)e^{u_1(t)-u_2(t)}\right)dt,\\\overline{d}-\frac{1}{\omega}\ln\left[\prod_{k=1}^{q}(1+\varphi_k)\right]\\=\frac{1}{\omega}\int_0^\omega\left(c(t)e^{u_2(t)+u_4(t)-u_3(t)}-q_1(t)e^{u_3(t)}\right)dt\\\quad-\frac{1}{\omega}\int_0^\omega c(t-\tau)e^{\int_{t-\tau}^{t}-r(s)ds}e^{u_2(t-\tau)+u_4(t-\tau)-u_3(t)}dt,\\\frac{1}{\omega}\int_0^\omega q_2(t)e^{u_4(t)}dt\\=\frac{1}{\omega}\int_0^\omega c(t-\tau)e^{\int_{t-\tau}^{t}-r(s)ds}e^{u_2(t-\tau)+u_4(t-\tau)-u_4(t)}dt,\end{cases}\tag{3.3}$$

According to (3.2) and (3.3), we have

$$\begin{aligned}\int_0^{\omega}\left|\dot u_1(t)\right|dt &< \int_0^{\omega}\left|a_1(t)-d_1(t)\right|dt \\ &+\int_0^{\omega}\left|r_1(t)e^{u_1(t)}-d_1(t)e^{u_2(t)-u_1(t)}\right|dt \\ &< 2\overline{a_1-d_1}\omega+\ln\left[\prod_{k=1}^{q}(1+\theta_{1k})\right]\end{aligned} \tag{3.4}$$

$$\int_0^{\omega}\left|\dot u_2(t)\right|dt < 2\overline{a_2-d_2}\omega+\ln\left[\prod_{k=1}^{q}(1+\theta_{2k})\right] \tag{3.5}$$

$$\int_0^{\omega}\left|\dot u_3(t)\right|dt < 2\bar d\omega+\ln\left[\prod_{k=1}^{q}(1+\varphi_k)\right] \tag{3.6}$$

$$\int_0^{\omega}\left|\dot u_4(t)\right|dt < 2\int_0^{\omega}q_2(t)e^{u_4(t)}dt \tag{3.7}$$

Scince $u_i(t)\in PC_\omega$, $\exists\xi_i,\eta_i\in[0,\omega]\,(i=1,2,3,4)$, such that

$$u_i(\xi_i)=\min_{t\in[0,\omega]}u_i(t),\quad u_i(\eta_i)=\max_{t\in[0,\omega]}u_i(t).$$

Let $v(t)=\max\{u_1(t),u_2(t)\}$, then $v(t)\in PC_\omega$

1) if $u_1(t)>u_2(t)$ or $u_1(t)=u_2(t)$, but $\dot u_1(t)\ge\dot u_2(t)$, then $v(t)=u_1(t)$ and

$$\dot u_1(t)\le\lambda\left(a_1(t)-r_1(t)e^{u_1(t)}\right)\le\lambda\left(a_1^M-r_1^Le^{u_1(t)}\right);$$

2) if $u_2(t)>u_1(t)$ or $u_1(t)=u_2(t)$, but $\dot u_2(t)\ge\dot u_1(t)$, then $v(t)=u_2(t)$ and

$$\dot u_2(t)\le\lambda\left(a_2(t)-r_2(t)e^{u_2(t)}\right)\le\lambda\left(a_2^M-r_2^Le^{u_2(t)}\right).$$

Dnote

$a=\max\{a_1^M,a_2^M\}$, $p=\min\{r_1^L,r_2^L\}$, $\theta_k=\max\{\theta_{1k},\theta_{2k}\}$, then

$$\begin{cases}D^+v(t)\le\lambda\left(a-pe^{v(t)}\right), & t\ne t_k,\\ \Delta v(t_k)\le\lambda\ln(1+\theta_k), & t=t_k,\end{cases} \tag{3.8}$$

Integrating (3.8) over $[0,\omega]$, we get

$$-\ln\left[\prod_{k=1}^{q}(1+\theta_k)\right]\le a\omega-p\int_0^{\omega}e^{v(t)}dt.$$

Therefore,

$$\int_0^{\omega}e^{u_i(\xi_i)}dt\le\int_0^{\omega}e^{u_i(t)}dt\le\frac{a\omega+\ln\left[\prod_{k=1}^{q}(1+\theta_k)\right]}{p} \tag{3.9}$$

$$u_i(\xi_i)\le\ln\frac{a\omega+\ln\left[\prod_{k=1}^{q}(1+\theta_k)\right]}{p\omega}\quad(i=1,2),$$

$$\begin{aligned}u_i(t)&\le u_i(\xi_i)+\int_0^{\omega}\left|\dot u_i(t)\right|dt+\left|\ln\left[\prod_{k=1}^{q}(1+\theta_{ik})\right]\right| \\ &\le\ln\frac{a\omega+\ln\left[\prod_{k=1}^{q}(1+\theta_k)\right]}{p\omega} \\ &+2\left(\overline{a_i-d_i}\omega+\left|\ln\left[\prod_{k=1}^{q}(1+\theta_{ik})\right]\right|\right)\underline{\underline{\Delta}}M_i\ (i=1,2),\end{aligned} \tag{3.10}$$

According to the fourth equation of (3.3), we have

$$\int_0^{\omega}q_2(t)e^{2u_4(t)}dt=\int_0^{\omega}c(t-\tau)e^{\int_{t-\tau}^{t}-r(s)ds}e^{u_2(t-\tau)+u_4(t-\tau)}dt, \tag{3.11}$$

$$\begin{aligned}q_2^L\int_0^{\omega}e^{2u_4(t)}dt&\le c^M\int_0^{\omega}e^{-r^L\tau+M_2+u_4(t-\tau)}dt \\ &=c^Me^{-r^L\tau+M_2}\int_0^{\omega}e^{u_4(t)}dt,\end{aligned} \tag{3.12}$$

Due to

$$\left(\int_0^{\omega}e^{u_4(t)}dt\right)^2\le\omega\int_0^{\omega}e^{2u_4(t)}dt \tag{3.13}$$

From (3.11) and (3.12), we have

$$\int_0^{\omega}e^{u_4(t)}dt\le\frac{\omega}{q_2^L}c^Me^{-r^L\tau+M_2} \tag{3.14}$$

$$u_4(\xi_4)\le\ln\frac{c^Me^{-r^L\tau+M_2}}{q_2^L}$$

According to (3.7) and (3.14), we get

$$\begin{aligned}\int_0^{\omega}\left|\dot u_4(t)\right|dt&<2\int_0^{\omega}q_2(t)e^{u_4(t)}dt \\ &<2q_2^M\int_0^{\omega}e^{u_4(t)}dt<\frac{2q_2^M\omega c^Me^{-r^L\tau+M_2}}{q_2^L},\end{aligned}$$

$$\begin{aligned}u_4(t)&\le u_4(\xi_4)+\int_0^{\omega}\left|\dot u_4(t)\right|dt \\ &\le\ln\frac{c^Me^{-r^L\tau+M_2}}{q_2^L}+\frac{2q_2^M\omega c^Me^{-r^L\tau+M_2}}{q_2^L}\underline{\underline{\Delta}}M_4,\end{aligned} \tag{3.15}$$

According to the third equation of (3.3), we have

$$\int_0^{\omega}c(t)e^{u_2(t)+u_4(t)-u_3(t)}dt\ge\bar d\omega-\ln\left[\prod_{k=1}^{q}(1+\varphi_k)\right],$$

Duo to $u_2(t)<M_2, u_4(t)<M_4$, we have

$$c^Me^{M_2+M_4}\int_0^{\omega}e^{-u_3(t)}dt\ge\bar d\omega-\ln\left[\prod_{k=1}^{q}(1+\varphi_k)\right],$$

$$u_3(\xi_3)\le\ln\frac{c^Me^{M_2+M_4}\omega}{\bar d\omega-\ln\left[\prod_{k=1}^{q}(1+\varphi_k)\right]},$$

and

$$u_3(t) \le u_3(\xi_3) + \int_0^\omega |\dot{u}_3(t)|dt + \left|\ln\left[\prod_{k=1}^q (1+\varphi_k)\right]\right|$$
$$\le \ln \frac{c^M e^{M_2+M_4}\omega}{\bar{d}\omega - \ln\left[\prod_{k=1}^q (1+\varphi_k)\right]} + 2\left(\bar{d}\omega + \left|\ln\left[\prod_{k=1}^q (1+\varphi_k)\right]\right|\right) \triangleq M_3, \quad (3.16)$$

From the first equation of (3.3), we have

$$\int_0^\omega r_1(t)e^{u_1(\eta_1)}dt \ge \int_0^\omega r_1(t)e^{u_1(t)}dt \ge \overline{a_1 - d_1}\omega + \ln\left[\prod_{k=1}^q (1+\theta_{1k})\right],$$

So,

$$u_1(\eta_1) \ge \ln \frac{\overline{a_1 - d_1}\omega + \ln\left[\prod_{k=1}^q (1+\theta_{1k})\right]}{\bar{r}_1\omega},$$

$$u_1(t) \ge u_1(\eta_1) - \int_0^\omega |\dot{u}_1(t)|dt - \left|\ln\left[\prod_{k=1}^q (1+\theta_{1k})\right]\right|$$
$$\ge \ln \frac{\overline{a_1 - d_1}\omega + \ln\left[\prod_{k=1}^q (1+\theta_{1k})\right]}{\bar{r}_1\omega} - 2\left(\overline{a_1 - d_1}\omega + \left|\ln\left[\prod_{k=1}^q (1+\theta_{1k})\right]\right|\right) \triangleq m_1, \quad (3.17)$$

From the second equation of (3.3), we have

$$\int_0^\omega r_2(t)e^{u_2(t)}dt \ge \overline{a_2 - d_2}\omega + \ln\left[\prod_{k=1}^q (1+\theta_{2k})\right] - k^M e^{M_4}\omega,$$

$$u_2(\eta_2) \ge \ln \frac{\overline{a_2 - d_2}\omega + \ln\left[\prod_{k=1}^q (1+\theta_{2k})\right] - k^M e^{M_4}\omega}{\bar{r}_2\omega},$$

$$u_2(t) \ge u_2(\eta_2) - \int_0^\omega |\dot{u}_2(t)|dt - \left|\ln[\prod_{k=1}^q (1+\theta_{2k})]\right|$$
$$\ge \ln \frac{\overline{a_2 - d_2}\omega + \ln[\prod_{k=1}^q (1+\theta_{2k})] - k^M e^{M_4}\omega}{\bar{r}_2\omega} - 2\left(\overline{a_2 - d_2}\omega + \left|\ln[\prod_{k=1}^q (1+\theta_{2k})]\right|\right) \triangleq m_2, \quad (3.18)$$

From (3.11) we have

$$q_2^M e^{u_4(\eta_4)} \int_0^\omega e^{u_4(t)}dt \ge \int_0^\omega q_2(t)e^{2u_4(t)}dt \ge c^L e^{-r^M\tau + m_2} \int_0^\omega e^{u_4(t)}dt,$$

$$u_4(\eta_4) \ge \ln \frac{c^L e^{-r^M\tau+m_2}}{q_2^M},$$

$$u_4(t) \ge u_4(\eta_4) - 2q_2^M \int_0^\omega e^{u_4(t)}dt \ge \ln \frac{c^L e^{-r^M\tau+m_2}}{q_2^M} - \frac{2\omega q_2^M c^M e^{-r^L\tau+M_2}}{q_2^L} \triangleq m_4, \quad (3.19)$$

According to the third equation of (3.3), we have

$$\left(c^L e^{m_2+m_4} - c^M e^{-r^L+M_2+M_4}\right)\int_0^\omega e^{-u_3(t)}dt \le \bar{d}\omega + q_1^M e^{M_3}\omega + \ln\left[\prod_{k=1}^q (1+\varphi_k)\right],$$

Similarly, we have

$$u_3(\eta_3) \ge \ln \frac{c^L e^{m_2+m_4} - c^M e^{-r^L+M_2+M_4}}{\bar{d}\omega + q_1^M e^{M_3}\omega + \ln\left[\prod_{k=1}^q (1+\varphi_k)\right]},$$

$$u_3(t) \ge u_3(\eta_3) - \int_0^\omega |\dot{u}_3(t)|dt - \left|\ln\left[\prod_{k=1}^q (1+\varphi_k)\right]\right|$$
$$\ge \ln \frac{c^L e^{m_2+m_4} - c^M e^{-r^L+M_2+M_4}}{\bar{d}\omega + q_1^M e^{M_3}\omega + \ln\left[\prod_{k=1}^q (1+\varphi_k)\right]} - 2\left(\bar{d}\omega + \left|\ln\left[\prod_{k=1}^q (1+\varphi_k)\right]\right|\right) \triangleq m_3, \quad (3.20)$$

Thus, we have

$$\sup_{t\in(0,\omega)} |u_i(t)| \le \max\{|M_1|,|M_2|,|M_3|,|M_4|,|m_1|,|m_2|,|m_3|,|m_4|\} \triangleq D_i\ (i = 1,2,3,4),$$

Denote $M = \max\{D_1, D_2, D_3, D_4\} + D_0$,where D_0 may be taken sufficiently large such that each solution to Equations (3.21)

$$\begin{cases} \overline{a_1 - d_1} - \bar{r}_1 e^{u_1} + \bar{d}_1 e^{u_2 - u_1} = \frac{1}{\omega}\ln\left[\prod_{k=1}^q (1+\theta_{1k})\right], \\ \overline{a_2 - d_2} - \bar{r}_2 e^{u_2} - \bar{k}e^{u_4} + \bar{d}_2 e^{u_1 - u_2} = \frac{1}{\omega}\ln\left[\prod_{k=1}^q (1+\theta_{2k})\right], \\ \bar{c}e^{u_2+u_4-u_3} - c(t-\tau)e^{\int_{t-\tau}^t -r(s)ds} e^{u_2(t-\tau)+u_4(t-\tau)-u_3(t)} \\ -\bar{d} - \bar{q}_1 e^{u_3} = \frac{1}{\omega}\ln\left[\prod_{k=1}^q (1+\varphi_k)\right], \\ c(t-\tau)e^{\int_{t-\tau}^t -r(s)ds} e^{u_2(t-\tau)+u_4(t-\tau)-u_3(t)} = \bar{q}_2 e^{u_4}, \end{cases} \quad (3.21)$$

satisfies $\left\|\left(u_1^*,u_2^*,u_3^*,u_4^*\right)^T\right\| < D_0$, then $\|u\| < M$.

Denote $\phi : DomL \times [0,1] \to X$ as the form

$$\phi(u_1,u_2,u_3,u_4,\mu)=$$
$$\begin{pmatrix} \overline{a_1-d_1}-\overline{r_1}e^{u_1}+\frac{1}{\omega}\ln\left[\prod_{k=1}^{q}(1+\theta_{1k})\right] \\ \overline{a_2-d_2}-\overline{r_2}e^{u_2}+\frac{1}{\omega}\ln\left[\prod_{k=1}^{q}(1+\theta_{2k})\right] \\ -\overline{d}-\overline{q_1}e^{u_3}+\frac{1}{\omega}\ln\left[\prod_{k=1}^{q}(1+\varphi_k)\right] \\ c(t-\tau)e^{\int_{t-\tau}^{t}-r(s)ds}e^{u_2(t-\tau)+u_4(t-\tau)-u_3(t)}-\overline{q_2}e^{u_4} \end{pmatrix}$$
$$+\mu\begin{pmatrix} \overline{d_1}e^{u_2-u_1} \\ -\overline{k}e^{u_4}+\overline{d_2}e^{u_1-u_2} \\ \overline{c}e^{u_2+u_4-u_3}-c(t-\tau)e^{\int_{t-\tau}^{t}-r(s)ds}e^{u_2(t-\tau)+u_4(t-\tau)-u_3(t)} \\ 0 \end{pmatrix},$$

Where $\mu \in [0,1]$ is a parameter. With the mapping ϕ, we have $\phi(u_1,u_2,u_3,u_4,\mu) \neq 0$ for $(u_1,u_2,u_3,u_4)^T \in \partial\Omega \cap KerL$. So we know that $\|u\| < M$.

Obviously, the algebraic Equation (3.22) has a unique solution $\left(u_1^*,u_2^*,u_3^*,u_4^*\right)$.

$$\begin{cases} \overline{a_1-d_1}-\overline{r_1}e^{u_1}+\frac{1}{\omega}\ln\left[\prod_{k=1}^{q}(1+\theta_{1k})\right]=0, \\ \overline{a_2-d_2}-\overline{r_2}e^{u_2}+\frac{1}{\omega}\ln\left[\prod_{k=1}^{q}(1+\theta_{2k})\right]=0, \\ -\overline{d}-\overline{q_1}e^{u_3}+\frac{1}{\omega}\ln\left[\prod_{k=1}^{q}(1+\varphi_k)\right]=0, \\ c(t-\tau)e^{\int_{t-\tau}^{t}-r(s)ds}e^{u_2(t-\tau)+u_4(t-\tau)-u_3(t)}-\overline{q_2}e^{u_4}=0, \end{cases} \quad (3.22)$$

From the coincidence degree theory, we can obtain

$$\deg\left(JQNu,\Omega\cap KerL,0\right) = \deg\left(\phi(u_1,u_2,u_3,u_4,\mu),\Omega\cap KerL,0\right)=1.$$

4. Numerical Analysis

In this paper, we have focused on the dynamics complexity of a stage-structured system with diffusion and impulsive effects. By using the method of coincidence degree, we obtain the sufficient condition for the existence of at least one positive $\omega-$periodic solution. In this section, we give the numerical results.

$$\begin{cases} \left.\begin{aligned} &\dot{x}_1(t)=x_1(t)[3-1.6\cos(\omega t)-1.5x_1(t)] \\ &\quad +(2-\cos(\omega t)[x_2(t)-x_1(t)], \\ &\dot{x}_2(t)=x_2(t)[5.2-3.2\sin(\omega t)-2.4x_2(t)] \\ &\quad -(3-2.5\sin(\omega t))x_2(t)y_2(t) \\ &\quad +(2-1.2\sin(\omega t))[x_1(t)-x_2(t)], \\ &\dot{y}_1(t)=(1.2-\sin(\omega t))x_2(t)y_2(t)- \\ &\quad (1.2-\sin(\omega(t-\tau))e^{-0.8}x_2(t-\tau)y_2(t-\tau) \\ &\quad -0.2y_1(t)-(1-0.5\cos(\omega t))y_1^2(t), \\ &\dot{y}_2(t)=-(1-0.75\cos(\omega t))y_2^2(t)+ \\ &\quad (1.2-\sin(\omega(t-\tau))e^{-0.8}x_2(t-\tau)y_2(t-\tau), \end{aligned}\right\} t\neq t_k, \\ \left.\begin{aligned} &x_1(t_k^+)=(1+\theta_1)x_1(t_k), \\ &x_2(t_k^+)=(1+\theta_2)x_2(t_k), \\ &y_1(t_k^+)=(1+\varphi)y_1(t_k), y_2(t_k^+)=y_2(t_k), \end{aligned}\right\} t=t_k, \end{cases} \quad (4.1)$$

Numerical analysis indicates that the complex dynamic behavior of system (1.1) depends on the values of impulsive perturbations φ_k, θ_{ik} $(i=1,2)$ in model (1.1). Our theoretical results are confirmed by numerical simulations. we can see that the dynamic behavior of the system (4.1) has obviously varied as the impulse value changing. Let $\theta_1=0.001$, $\theta_2=0.002$, $\varphi=0.003$, it is easily proved that the system (4.1) satisfies all the conditions of Theorem 3.1, that mean the system (4.1) has at least one positive periodic solution (**Figure 1**). As impulses increase, the periodic oscillation of system (4.1) will be destroyed (**Figure 2**).

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6. Conclusions

There is much previous work reported on non-autonomous stage-structured system or diffusive system. This motivates us to study a non-autonomous stage-structured predator-prey system with impulsive effects. As pointed out in Section 1, we built system (1.1). In Section 2, we give some preliminaries. In Section 3, by using the method of coincidence degree, we obtain the sufficient condition for the existence of at least one positive periodic solution. In Section 4, we give the numerical simulations on the dynamic behaviors of the system through two examples. But we did not discuss the global stability of the periodic solutions periodic solution of system (1.1). We

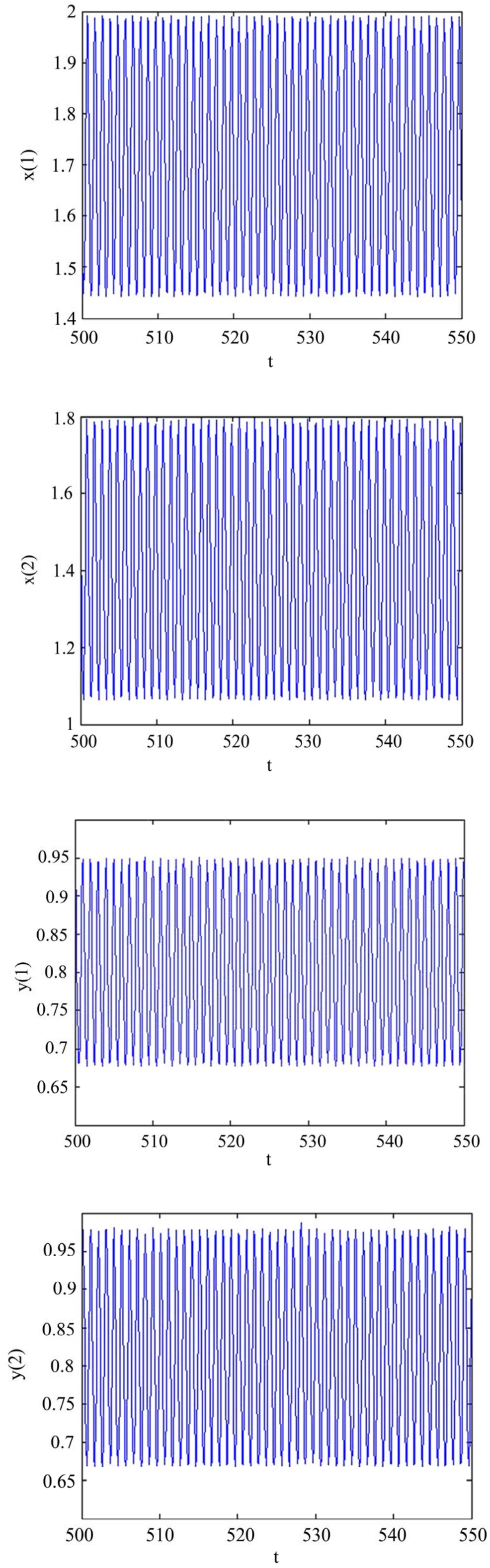

Figure 1. Dynamic behavior of the system (4.1) with initial values $(1.2, 1.2, 0.8, 0.6)$, $\tau = 0.1$ and impulsive perturbations $\theta_1 = 0.001$, $\theta_2 = 0.002$, $\varphi = 0.003$.

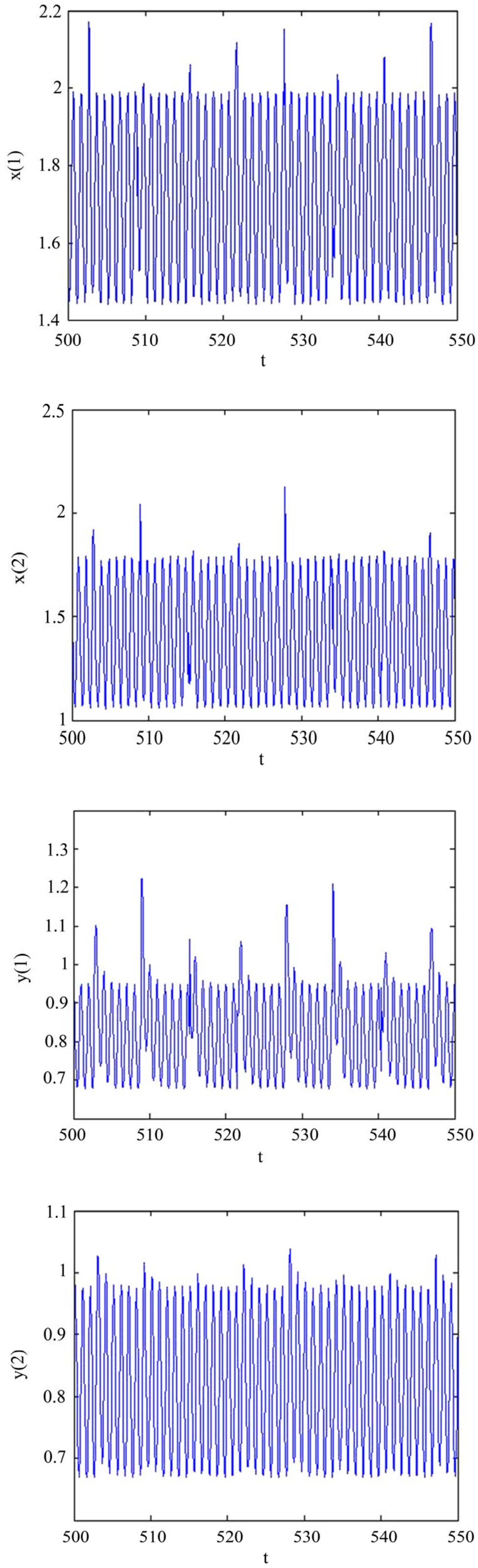

Figure 2. Dynamic behavior of the system (4.1) with initial values $(1.2, 1.2, 0.8, 0.6)$, $\tau = 0.1$ and impulsive perturbations $\theta_1 = 0.1$, $\theta_2 = 0.2$, $\varphi = 0.3$.

leave these aspects for future research.

7. References

[1] Y. Nakata, Y. Muroya, "Permanence for Nonautonomous Lotka-Volterra Cooperative Systems with Delays," *Nonlinear Anal*, Vol. 11, No. 1, 2010, pp. 528-534. doi:10.1016/j.nonrwa.2009.01.002

[2] T. V. Ton, "Survival of Three Species in a Nonautonomous Lotka-Volterra System," *Journal of Mathematical Analysis and Applications*, Vol. 362, No. 2, February 2010, pp. 427-437. doi:10.1016/j.jmaa.2009.07.053

[3] S. H. Chen, J. H. Zhang and T. Young, "Existence of Positive Periodic Solution for Nonautonomous Predator-Prey System with Diffusion and Time Delay," *Journal of Computational and Applied Mathematics*, Vol. 159, No. 2, October 2003, pp. 375-386. doi:10.1016/S0377-0427(03)00540-5

[4] Z. H. Lu, X. B. Chi and L. S. Chen, "Global Attractivity of Nonautonomous Stage-Structured Population Models with Dispersal and Harvest," *Journal of Computational and Applied Mathematics*, Vol. 166, No. 2, April 2004, pp. 411-425. doi:10.1016/j.cam.2003.08.040

[5] Z. D. Teng and L. S. Chen, "Uniform Persistence and Existence of Strictly Positive Solutions in Nonautonomous Lotka-Volterra Competitive Systems with Delays," *Computers & Mathematics with Applications*, Vol. 37, No. 7, April 1999, pp. 61-71. doi:10.1016/S0898-1221(99)00087-5

[6] J. Y. Wang, Q. S. Lu and Z. S. Feng, "A Nonautonomous Predator-Prey System with Stage Structure and Double Time Delays," *Journal of Computational and Applied Mathematics*, Vol. 230, No. 1, August 2009, pp. 283-299. doi:10.1016/j.cam.2008.11.014

[7] X. X. Liu and L. H. Huang, "Permanence and Periodic Solutions for a Diffusive Ratio-Dependent Predator-Prey System," *Applied Mathematical Modelling*, Vol. 33, February 2009, pp. 683-691. doi:10.1016/j.apm.2007.12.002

[8] Z. X. Hu, G. K. Gao and W. B. Ma, "Dynamics of a Three-Species Ratio-Dependent Diffusive Model," *Nonlinear Anal*, Vol. 217, November 2010, pp. 1825-1830.

[9] C. J. Xu, X. H. Tang and M. X. Liao, "Stability and Bbifurcation Analysis of a Delayed Predator-Prey Model of Prey Dispersal in Two-Patch Environments," *Applied Mathematics and Computation*, Vol. 216, July 2010, pp. 2920-2936. doi:10.1016/j.amc.2010.04.004

[10] S. Q. Liu, L. S. Chen and Z. J. Liu, "Extinction and Permanence in Nonautonomous Competitive System with Stage Structure," *Journal of Mathematical Analysis and Applications*, Vol. 274, No. 2, October 2002, pp. 667-684. doi:10.1016/S0022-247X(02)00329-3

[11] Z. Li and F. D. Chen, "Extinction in Periodic Competitive Stage-Structured Lotka-Volterra Model with the Effects of Toxic Substances," *Journal of Computational and Applied Mathematics*, Vol. 231, No. 1, September 2009, pp. 143-153. doi:10.1016/j.cam.2009.02.004

[12] X. W. Jiang, Q. Song and M. Y. Hao, "Dynamics Behaviors of a Delayed Stage-Structured Predator-Prey Model with Impulsive Effect," *Applied Mathematics and Computation*, Vol. 215, No. 12, February 2010, pp. 4221-4229. doi:10.1016/j.amc.2009.12.044

[13] R. E. Gaines and J. L. Mawhin, "Coincidence Degree and Nonlinear Differential Equations," Springer, Berlin, 1997.

