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On Topological Structure of the First Non-abelian Cohomology of Topological Groups

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Abstract

Let G, R, and A be topological groups. Suppose that G and R act continuously on A, and G acts continuously on R. In this paper, we define a partially crossed topological $G-R$-bimodule (A, μ), where $\mu : A \to R$ is a continuous homomorphism. Let $Der_c(G,(A, \mu))$ be the set of all (α, r) such that $\alpha : G \to A$ is a continuous crossed homomorphism and $\mu\alpha(g) = r^g r^{-1}$. We introduce a topology on $Der_c(G,(A, \mu))$. We show that $Der_c(G,(A, \mu))$ is a topological group, wherever G and R are locally compact. We define the first cohomology, $H^1(G,(A, \mu))$, of G with coefficients in (A, μ) as a quotient space of $Der_c(G,(A, \mu))$. Also, we state conditions under which $H^1(G,(A, \mu))$ is a topological group. Finally, we show that under what conditions $H^1(G,(A, \mu))$ is one of the following: k-space, discrete, locally compact and compact.

Keywords: Non-abelian cohomology of topological groups; Partially crossed topological bimodule; Evaluation map; Compactly generated group

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1 Introduction

The first non-abelian cohomology of groups with coefficients in crossed modules (algebraically) was introduced by Guin [1]. The Guin's approach is extended by Inassaridze to any dimension with coefficients in (partially) crossed bimodules ([2],[3]). Hu [4] defined the cohomology of topological groups with coefficients in abelian topological modules. This paper is a part of an investigation about *non-abelian cohomology of topological groups*. We consider the first non-abelian cohomology in the topological context. The methods used here are motivated by [2] and [3].

All topological groups are assumed to be Hausdorff (not necessarily abelian), unless otherwise specified. Let G and A be topological groups. It is said that A is a topological G-module, whenever G acts continuously on the left of A. For all $g \in G$ and $a \in A$ we denote the action of g on a by ${}^g a$. The centre and the commutator of a topological group G is denoted by $Z(G)$ and $[G, G]$, respectively. If G and H are topological groups and $f : G \to H$ is a continuous homomorphism

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we denote by $\bar{f} : G \to f(G)$ the restricted map of f on its range and by $\mathbf{1} : G \to H$ the trivial homomorphism. The topological isomorphism and isomorphism are denoted respectively by " $\simeq$ " and " $\cong$ ". If the topological groups G and R act continuously on a topological group A, then the notation ${}^{gr}a$ means ${}^{g}({}^{r}a)$, $g \in G$, $r \in R$, $a \in A$. We assume that every topological group acts on itself by conjugation.

In section 2, we define precrossed, partially crossed and crossed topological R-module (A, μ), where A is a topological R-module and $\mu : A \to R$ is a continuous homomorphism. Also, we generalize, these definitions to precrossed, partially crossed and crossed topological $G-R$-bimodule (A, μ), when G and R act continuously on A, and G acts continuously on R. We define the set $Der_c(G, (A, \mu))$, for a partially crossed topological $G-R$-bimodule. We denote the set of all continuous maps from G into A, with compact-open topology, by $\mathcal{C}_k(G, A)$. Since $Der_c(G, (A, \mu)) \subset \mathcal{C}_k(G, A) \times R$, then we may consider $Der_c(G, (A, \mu))$ as a topological subspace of $\mathcal{C}_k(G, A) \times R$. We show that $Der_c(G, (A, \mu))$ is a topological group, whenever G and R are locally compact (Theorem 2.5). In addition, we prove that $Der_c(G, (A, \mu))$ is a topological G-module. Furthermore, we show that under what conditions, $Der_c(G, (A, \mu))$ is a precrossed topological $G-R$- bimodule (Proposition 2.4).

In section 3, we define $H^1(G, (A, \mu))$ as a quotient of $Der_c(G, (A, \mu))$, where (A, μ) is a partially crossed topological $G-R$-bimodule. We state conditions under which $H^1(G, (A, \mu))$ is a topological group (see Theorem 3.1). Moreover, since each partially crossed topological G-modulecan be naturally viewed as a partially crossed topological $G-G$-bimodule, then we may define $H^1(G, (A, \mu))$, when (A, μ) is a partially crossed topological G-module. Finally, we find conditions under which $H^1(G, (A, \mu))$ is one of the following: k-space, discrete, locally compact and compact.

2 Partially Crossed topological $G-R$-bimodule (A, μ)

In this section, we define a partially crossed topological $G-R$-bimodule (A, μ). We give some examples of precrossed, partially crossed and crossed topological $G-R$-bimodules. Also, we define $Der_c(G, (A, \mu))$ and prove that if G and R are locally compact, then $Der_c(G, (A, \mu))$ is a topological group. Moreover, if the topological groups G and R act continuously on each other and on A compatibly, then $(Der_c(G, (A, \mu)), \gamma)$ is a precrossed topological $G-R$-bimodule, where $\gamma : Der_c(G, (A, \mu)) \to R$, $(\alpha, r) \mapsto r$.

Definition 2.1. By a precrossed topological R-module we mean a pair (A, μ) where A is a topological R-module and $\mu : A \to R$ is a continuous homomorphism such that

$$\mu({}^{r}a) = {}^{r}\mu(a), \forall r \in R, a \in A.$$

If in addition we have the *Pieffer identity*

$${}^{\mu(a)}b = {}^{a}b, \forall a, b \in A,$$

then (A, μ) is called a crossed topological R-module.

Definition 2.2. A precrossed topological R-module (A, μ) is said to be a partially crossed topological R-module, whenever it satisfies the following equality

$${}^{\mu(a)}b = {}^{a}b,$$

for all $b \in A$ and for all $a \in A$ such that $\mu(a) \in [R, R]$.

It is clear that every crossed topological R-module is a partially crossed topological R-module.

Example 2.1. *Suppose that A is a non-abelian topological group with nilpotency class of two (i.e., $[A, A] \subseteq Z(A)$). Take $R = A/\overline{[A, A]}$. Let $\pi : A \to R$ be the canonical surjective map and suppose that R acts trivially on A. It is clear that ${}^{\pi(a)}b = {}^{a}b$, for all $b \in A$ if and only if $a \in Z(A)$. Hence, (A, π) is a partially crossed topological R-module which is not a crossed topological R-module.*

Definition 2.3. Let G, R and A be topological groups. A precrossed topological R-module (A, μ) is said to be a precrossed topological $G - R$-bimodule, whenever

(1) G acts continuously on R and A;

(2) $\mu : A \to R$ is a continuous G-homomorphism;

(3) ${}^{({}^{g}r)}a = {}^{grg^{-1}}a$ (i.e., compatibility condition) for all $g \in G$, $r \in R$ and $a \in A$.

Definition 2.4. A precrossed topological $G - R$-bimodule (A, μ) is said to be a crossed topological $G - R$-bimodule, if (A, μ) is a crossed topological R-module.

Example 2.2. *(1) Let A be an arbitrary topological G-module. Then $Z(A)$ is a topological G-module. Since A is Hausdorff, then $Z(A)$ is a closed subgroup of A. Thus, the quotient group $R = A/Z(A)$ is Hausdorff. Now, we define an action of R on A and an action of G on R by:*

$$ {}^{aZ(A)}b = {}^{a}b, \forall a, b \in A, \qquad {}^{g}(aZ(A)) = {}^{g}a, \forall g \in G, a \in A. \tag{2.1} $$

Let $\pi_A : A \to R$ be the canonical homomorphism. It is easy to see that under (2.1) the pair (A, π_A) is a crossed topological $G - R$-bimodule.

(2) By part (1), for any topological group G the pair (G, π_G) is a crossed topological $G - G/Z(G)$-bimodule.

Definition 2.5. A precrossed topological $G - R$-bimodule (A, μ) is said to be partially crossed topological $G - R$-bimodule, if (A, μ) is a partially crossed topological R-module.

Let G be a locally compact group and $Aut(G)$ the group of all topological group automorphisms (i.e., continuous and open automorphisms) of G with the *Birkhoff topology* (see [5], [6] and [7]). This topology is known as the *generalized compact-open topology*. A neighborhood basis of the identity automorphism consists of sets $N(C, V) = \{\alpha \in Aut(G) : \alpha(x) \in Vx, \alpha^{-1}(x) \in Vx, \forall x \in C\}$, where C is a compact subset of G and V is a neighborhood of the identity of G. It is well-known that $Aut(G)$ is a Hausdorff topological group (see page 40 of [7]). The generalized compact-open topology is finer than the compact-open topology in $Aut(G)$ and if G is compact, then the generalized compact-open topology coincides with compact-open topology in $Aut(G)$ (see page 324 of [6]).

Lemma 2.3. *Let A be a locally compact group and G a topological group. Suppose that A is a topological G-module. Then*

(i) the homomorphism $\imath_A : A \to Aut(A)$, $a \mapsto c_a$, is continuous, where $c_a(b) = aba^{-1}, \forall b \in A$;

(ii) A is a topological $Aut(A)$-module by the action ${}^{\alpha}x = \alpha(x)$, $\forall \alpha \in Aut(A), x \in A$;

(iii) $Aut(A)$ is a topological G-module by the action $({}^{g}\alpha)(x) = {}^{g}\alpha({}^{g^{-1}}x)$, $\forall g \in G, \alpha \in Aut(A), x \in A$.

Proof. For (i) and (ii) see page 324 of [6], and Proposition 3.1 of [7]. (iii): It is enough to prove that the map $\chi : G \times Aut(A) \to Aut(A)$, $(g, \alpha) \mapsto {}^{g}\alpha$ is continuous. By (ii), the maps $\phi : (G \times Aut(A)) \times A \to A$, $((g, \alpha), x) \mapsto {}^{g}\alpha({}^{g^{-1}}x)x^{-1}$ and $\psi : (G \times Aut(A)) \times A \to A$, $((g, \alpha), x) \mapsto {}^{g}\alpha^{-1}({}^{g^{-1}}x)x^{-1}$ are continuous. Let ${}^{g}\alpha \in N(C, V)$. Then, $\phi((g, \alpha), x) \in V$ and $\psi((g, \alpha), x) \in V$, for all $x \in C$. Thus, $\phi(\{(g, \alpha)\} \times C) \subset V$ and $\psi(\{(g, \alpha)\} \times C) \subset V$. Now, $\phi^{-1}(V)$ and $\psi^{-1}(V)$ are open in $(G \times Aut(A)) \times A$ containing $\{(g, \alpha)\} \times C$. Hence, $\phi^{-1}(V) \cap \psi^{-1}(V) \cap (G \times Aut(A)) \times C$ is an open set in $(G \times Aut(A)) \times C$ containing the slice $\{(g, \alpha)\} \times C$ of $(G \times Aut(A)) \times C$. The tube lemma (Lemma 5.8 of [8]) implies that there is an open neighbourhood U of (g, α) in $G \times Aut(A)$ such that the tube $U \times C$ lies in $\phi^{-1}(V) \cap \psi^{-1}(V)$. Then, for every $(h, \beta) \in U$, $x \in C$, we have $\phi((h, \beta), x) \in V$ and $\psi((h, \beta), x) \in V$, i.e., ${}^h\beta({}^{h^{-1}}x) \in Vx$ and ${}^h\beta^{-1}({}^{h^{-1}}x) \in Vx$. Therefore, ${}^h\beta \in N(C, V)$, for all $(h, \beta) \in U$. So χ is continuous. □

Proposition 2.1. *Let A be a topological G-module and A a locally compact group. Then, $(A, \imath_A)$ is a crossed topological $G - Aut(A)$-bimodule, where the homomorphism $\imath_A$ and the actions are defined as in Lemma 2.3.*

Proof. By Lemma 2.3, the homomorphism $\imath_A$ and the actions are continuous. Also,

1. For every $g \in G$ and $a, b \in A$, $\imath_A({}^g a)(b) = c_{{}^g a}(b) = {}^g a b {}^g a^{-1} = {}^g c_a(b)$. Hence, $\imath_A$ is a G-homomorphism.

2. For every $\alpha \in Aut(A)$ and $x, a \in A$, $\imath_A({}^\alpha x)(a) = \imath_A(\alpha(x))(a) = c_{\alpha(x)}(a) = \alpha(x)a\alpha(x)^{-1} = \alpha(x\alpha^{-1}(a)x^{-1}) = \alpha \circ c_x \circ \alpha^{-1}(a) = {}^\alpha c_x(a)$. So $\imath_A$ is a $Aut(A)$-homomorphism.

3. For every $a, b \in A$, ${}^{\imath_A(a)}b = c_a(b) = aba^{-1} = {}^a b$. Thus, the Pieffer identity is satisfied.

4. The compatibility condition is satisfied. Since for every $g \in G, \alpha \in Aut(A), x \in A$, then ${}^{{}^g\alpha}x = ({}^g\alpha)(x) = {}^g\alpha({}^{g^{-1}}x) = {}^{g\alpha g^{-1}}x$.

Therefore, $(A, \imath_A)$ is a crossed topological $G - Aut(A)$-bimodule. □

Remark 2.1. In a natural way any precrossed (crossed) topological R-module is a precrossed (crossed) topological $R - R$-bimodule.

Remark 2.2. Let (A, μ) be a partially crossed (crossed) topological $G - R$-bimodule. Then, $(A, \overline{\mu})$ is a partially crossed (crossed) topological $G - \mu(A)$-bimodule. Thus, by Proposition 2.1, for any topological G-module A in which A is locally compact, we may associate the crossed topological $G - Inn(A)$-bimodule $(A, \overline{\imath_A})$, where $Inn(A)$ is the topological group of all inner automorphisms of A.

Definition 2.6. Let (A, μ) be a partially crossed topological $G - R$-bimodule. The map $\alpha : G \to A$ is called a crossed homomorphism whenever,

$$\alpha(gh) = \alpha(g){}^g\alpha(h), \forall g, h \in G.$$

Denote by $Der(G, (A, \mu))$ the set of all pairs (α, r) where $\alpha : G \to A$ is a crossed homomorphism and r is an element of R such that

$$\mu \circ \alpha(g) = r{}^g r^{-1}, \forall g \in G.$$

Let $Der_c(G, (A, \mu)) = \{(\alpha, r) | (\alpha, r) \in Der(G, (A, \mu))$ and α is continuous$\}$. H. Inassaridze [3] introduced the product $\star$ in $Der(G, (A, \mu))$ by

$$(\alpha, r) \star (\beta, s) = (\alpha * \beta, rs), \text{ where } \alpha * \beta(g) = {}^r\beta(g)\alpha(g), \forall g \in G.$$

Definition 2.7. A family η of subsets of a topological space X is called a network on X if for each point $x \in X$ and each neighbourhood U of x there exists $P \in \eta$ such that $x \in P \subset U$. A network η is said to be compact (closed) if all its elements are compact (closed) subspaces of X. We say that a closed network η is hereditarily closed if for each $P \in \eta$ and any closed set B in P, $B \in \eta$.

Let X and Y be topological spaces. The set of all continuous functions $f : X \to Y$ is denoted by $\mathcal{C}(X, Y)$. Suppose that $U \subset X$ and $V \subset Y$. Take

$$[U, V] = \{f \in \mathcal{C}(X, Y) : f(U) \subset V\}.$$

Let X and Y be topological spaces, and η a network in X. The family $\{[P,V] : P \in \eta$ and V is open in $Y\}$ is a subbase for a topology on $\mathcal{C}(X,Y)$, called the η-topology. We denote the set $\mathcal{C}(X,Y)$ with the η-topology by $\mathcal{C}_\eta(X,Y)$. If η is the family of all singleton subsets of X, then the η-topology is called the point-open topology; in this case $\mathcal{C}_\eta(X,Y)$ is denoted by $\mathcal{C}_p(X,Y)$. If η is the family of all compact subspaces of X, then the η-topology is called the compact-open topology and $\mathcal{C}_\eta(X,Y)$ is denoted by $\mathcal{C}_k(X,Y)$ (see [9]).

Now, suppose that A is a topological group, then $\mathcal{C}(X,A)$ is a group. For $f,g \in \mathcal{C}(X,A)$ the product, $f.g$, is defined by

$$(f.g)(x) = f(x).g(x), \forall x \in X. \tag{2.2}$$

Lemma 2.4. *Let X be a Tychonoff space and A a topological group. If η is a hereditarily closed, compact network on X, then under the product (2.2), $\mathcal{C}_\eta(X,A)$ is a topological group. In particular, $\mathcal{C}_p(X,A)$ and $\mathcal{C}_k(X,A)$ are topological groups.*

Proof. See Theorem 1.1.7 of [9]. In particular, the set of all finite subsets of X and the set of all compact subsets of X are hereditarily closed, compact networks on X. □

Suppose that X is a topological space and A a topological R-module. Then, $\mathcal{C}(X,A)$ is an R-module. If $r \in R, f \in \mathcal{C}(X,A)$, then the action ${}^r f$ is defined by

$$({}^r f)(x) = {}^r(f(x)), \forall x \in X. \tag{2.3}$$

Proposition 2.2. *Let X be a locally compact Hausdorff space, R a locally compact group and A a topological R-module. Then, by (2.3), $\mathcal{C}_k(X,A)$ is a topological R-module.*

Proof. Since X is a locally compact Hausdorff space, then by Lemma 2.4, $\mathcal{C}_k(X,A)$ is a topological group. By Theorem 5.3 of [8], the evaluation map $e : X \times \mathcal{C}_k(X,A) \to A$, $(x,f) \mapsto f(x)$ is continuous. Thus, the map $F : R \times X \times \mathcal{C}_k(X,A) \to A$, $(r,x,f) \mapsto {}^r f(x)$ is continuous. By Corollary 5.4 of [8], the induced map $\hat{F} : \mathcal{C}_k(X,A) \to \mathcal{C}_k(R \times X, A)$ is continuous, where $\hat{F}$ is defined by

$$\hat{F}(f)(r,x) = {}^r f(x).$$

On the other hand the exponential map $\Lambda : \mathcal{C}_k(R \times X, A) \to \mathcal{C}_k(R, C_k(X,A))$, $u \mapsto \Lambda(u)$; $\Lambda(u)(r)(x) = u(r,x)$, is a homeomorphism (see Corollary 2.5.7 of [9]. Therefore, $\Lambda \circ \hat{F} : \mathcal{C}_k(X,A) \to \mathcal{C}_k(R, \mathcal{C}_k(X,A))$ is a continuous map. Since R is locally compact and Hausdorff then by Corollary 5.4 of [8], $\Lambda \circ \hat{F}$ induces the continuous map $\chi : R \times \mathcal{C}_k(X,A) \to \mathcal{C}_k(X,A)$, $\chi(r,f) = (\Lambda \circ \hat{F}(f))(r) = {}^r f$. Therefore, $\mathcal{C}_k(X,A)$ is a topological R-module. □

Note that $Der_c(G,(A,\mu)) \subset Der_c(G,A) \times R \subset \mathcal{C}(G,A) \times R$, where $Der_c(G,A) = \{\alpha | \alpha$ is a continuous crossed homomorphism from G into $A\}$. Thus, $\mathcal{C}_k(G,A) \times R$ induces the subspace topology on $Der_c(G,(A,\mu))$. Here, the induced subspace topology on $Der_c(G,(A,\mu))$ is called the *induced topology by compact-open topology.* From now on, we consider $Der_c(G,(A,\mu))$ with this topology.

Theorem 2.5. *Let G and R be locally compact groups and (A,μ) a partially crossed topological $G-R$-bimodule. Then, $(Der_c(G,(A,\mu)),\star)$ is a topological group.*

Proof. By Proposition 3 of [3], $Der(G,(A,\mu))$ is a group. If $(\alpha,r),(\beta,s) \in Der_c(G,(A,\mu)) \subset Der(G,(A,\mu))$, then $(\alpha,r) \star (\beta,s) \in Der_c(G,(A,\mu))$ and $(\alpha,r)^{-1} = (\bar{\alpha}, r^{-1}) \in Der_c(G,(A,\mu))$, where $\bar{\alpha}(g) = {}^{r^{-1}}\alpha(g)^{-1}, \forall g \in G$. It is clear that $\alpha * \beta$ and $\bar{\alpha}$ are continuous. Thus, $Der_c(G,(A,\mu))$ is a subgroup of $Der(G,(A,\mu))$.

By Proposition 2.2, $\mathcal{C}_k(G,A)$ is a topological R-module. Thus, it is clear that

$$\phi : (\mathcal{C}_k(G,A) \times R) \times (\mathcal{C}_k(G,A) \times R) \to \mathcal{C}_k(G,A) \times R$$

$$((f,r),(g,s)) \mapsto ({}^{r}gf, rs)$$

and

$$\psi : \mathcal{C}_k(G,A) \times R \to \mathcal{C}_k(G,A) \times R$$
$$(f,r) \mapsto \bar{f} = ({}^{r^{-1}}f^{-1}, r^{-1})$$

are continuous. Obviously, the restrictions of ϕ and ψ to $Der_c(G,(A,\mu)) \times Der_c(G,(A,\mu))$ and $Der_c(G,(A,\mu))$ are continuous, respectively. Consequently, $(Der_c(G,(A,\mu)),\star)$ is a topological group. □

Proposition 2.3. *(i) Let (A,μ) be a partially crossed topological $G-R$-bimodule. Then, $Der_c(G,(A,\mu))$ is a closed subspace of $Der_c(G,A) \times R$;*

(ii) Let A be a topological G-module. Then, $Der_c(G,A)$ is a closed subspace of $\mathcal{C}_k(G,A)$.

Proof. (i). Consider the map

$$\phi_g : \mathcal{C}_k(G,A) \times R \to R, (\alpha,r) \mapsto r^{-1}\mu\alpha(g)^g r,$$

for $g \in G$. By 9.6 Lemma of [10], ϕ_g is continuous, for all $g \in G$. Hence, $\phi_g^{-1}(1)$ is closed in $\mathcal{C}_k(G,A) \times R$, for all $g \in G$. It is easy to see that

$$Der_c(G,(A,\mu)) = \bigcap_{g \in G} \phi_g^{-1}(1) \bigcap (Der_c(G,A) \times R).$$

Therefore, $Der_c(G,(A,\mu))$ is closed in $Der_c(G,A) \times R$.

(ii). By a similar argument as in (i), we consider the continuous map

$$\chi_{(g,h)} : \mathcal{C}_k(G,A) \to A, \alpha \mapsto \alpha(gh)^{-1}\alpha(g)^g\alpha(h),$$

for $(g,h) \in G \times G$. Since

$$Der_c(G,A) = \bigcap_{(g,h) \in G \times G} \chi_{(g,h)}^{-1}(1),$$

then $Der_c(G,A)$ is closed in $\mathcal{C}_k(G,A)$. □

We immediately obtain the following two corollaries.

Corollary 2.6. *Let (A,μ) be a partially crossed topological $G-R$-bimodule. Then, $Der_c(G,(A,\mu))$ is a closed subspace of $\mathcal{C}_k(G,A) \times R$.*

Corollary 2.7. *Let G be a topological group and A an abelian topological group. Then, $Hom_c(G,A)$ is a closed subgroup of $\mathcal{C}_k(G,A)$.*

Suppose that (A,μ) is a partially crossed topological $G-R$-bimodule. There is an action of G on $Der(G,(A,\mu))$ defined by

$$^{g}(\alpha,r) = (\tilde{\alpha}, {}^{g}r), g \in G, r \in R \tag{2.4}$$

with $\tilde{\alpha}(h) = {}^{g}\alpha({}^{g^{-1}}h), h \in G$ [3].

Note that if $(\alpha,r) \in Der_c(G,(A,\mu))$, then ${}^{g}(\alpha,r) \in Der_c(G,(A,\mu)), \forall g \in G$, since $\tilde{\alpha}$ is continuous. This shows that $Der_c(G,(A,\mu))$ is a G-submodule of $Der(G,(A,\mu))$.

Lemma 2.8. *Let G and R be locally compact groups and (A,μ) a partially crossed topological module. Then by (2.4), $Der_c(G,(A,\mu))$ is a topological G-module.*

Proof. Since G is locally compact and Hausdorff, then the evaluation map $e : G \times \mathcal{C}_k(G, A) \to A$, $(g, \alpha) \mapsto \alpha(g)$ is continuous. Thus, the map

$$\Phi : G \times G \times \mathcal{C}_k(G, A) \to A, (g, h, \alpha) \mapsto {}^{g}\alpha({}^{g^{-1}}h)$$

is continuous. By a similar argument as in the proof of Proposition 2.2, the map $G \times \mathcal{C}_k(G, A) \to \mathcal{C}_k(G, A)$, $(g, \alpha) \mapsto \tilde{\alpha}$ is continuous, where $\tilde{\alpha}(h) = {}^{g}\alpha({}^{g^{-1}}h)$, $h \in G$. Hence,

$$(G \times \mathcal{C}_k(G, A)) \times R \to \mathcal{C}_k(G, A) \times R$$

$$((g, \alpha), r) \mapsto (\tilde{\alpha}, {}^{g}r)$$

is continuous. Therefore, by restriction of this map to $G \times Der_c(G, (A, \mu))$ we get the continuous map

$$G \times Der_c(G, (A, \mu)) \to Der_c(G, (A, \mu))$$

$$((g, \alpha), r) \mapsto (\tilde{\alpha}, {}^{g}r)$$

and this completes the proof. □

Let (A, μ) be a partially crossed topological $G - R$-bimodule. If G is a topological R-module, and the compatibility condition

$${}^{({}^{r}g)}a = {}^{rgr^{-1}}a \text{ and } {}^{({}^{r}g)}s = {}^{rgr^{-1}}s;\ \forall r, s \in R, g \in G, a \in A,$$

holds, then $Der(G, (A, \mu))$ is an R-module via

$${}^{r}(\alpha, s) = (\tilde{\alpha}, {}^{r}s) \tag{2.5}$$

where $\tilde{\alpha}(g) = {}^{r}\alpha({}^{r^{-1}}g), g \in G$ [3].

It is easy to see that $Der_c(G, (A, \mu))$ is an R-submodule of $Der(G, (A, \mu))$.

Lemma 2.9. *Let G and R be locally compact groups and (A, μ) a partially crossed topological $G - R$-bimodule. Then by (2.5), $Der_c(G, (A, \mu))$ is a topological R-module.*

Proof. This can be proved by a similar argument as in Lemma 2.8. □

Definition 2.8. Let G and R be topological groups acting continuously on each other. These actions are said to be compatible if

$${}^{({}^{r}g)}s = {}^{rgr^{-1}}s \text{ and } {}^{({}^{g}r)}h = {}^{grg^{-1}}h;\ \forall r, s \in R, g, h \in G.$$

Also, it is said that the topological groups G and R act (continuously) on a topological group A compatibly if

$${}^{({}^{r}g)}a = {}^{rgr^{-1}}a \text{ and } {}^{({}^{g}r)}a = {}^{grg^{-1}}a;\ \forall r \in R, g \in G, a \in A.$$

Proposition 2.4. *Let G and R be locally compact groups and (A, μ) a partially crossed topological $G - R$-bimodule. Let the topological groups G and R act continuously on each other and on A compatibly. Then, $(Der_c(G, (A, \mu)), \gamma)$ is a precrossed topological $G - R$-bimodule, where $\gamma : Der_c(G, (A, \mu)) \to R$, $(\alpha, r) \mapsto r$.*

Proof. Since G and R are locally compact groups, then by Lemma 2.8 and Lemma 2.9, G and R act continuously on $Der_c(G, (A, \mu))$. The map γ is continuous, since $\pi_2 : \mathcal{C}_k(G, A) \times R \to R, (\alpha, r) \mapsto r$ is continuous. Also, γ is a G-homomorphism and an R-homomorphism. Since ${}^{{}^{g}r}(\alpha, s) = {}^{grg^{-1}}(\alpha, s)$ for all $g \in G, r \in R, (\alpha, s) \in Der_c(G, (A, \mu))$ (Proposition 5 of [3]), we conclude that $(Der_c(G, (A, \mu)), \gamma)$ is a precrossed topological $G - R$-bimodule. □

3 The first non-abelian cohomology of a topological group as a topological space

In this section we define the first non-abelian cohomology $H^1(G,(A,\mu))$ of G with coefficients in a partially crossed topological $G-R$-bimodule (A,μ). We will introduce a topological structure on $H^1(G,(A,\mu))$. It will be shown that under what conditions $H^1(G,(A,\mu))$ is a topological group. As a result, $H^1(G,(A,\mu))$ is a topological group for every partially crossed topological G-module. In addition, we verify some topological properties of $H^1(G,(A,\mu))$.

Let R ba a topological G-module, then we define

$$H^0(G,R) = \{r|^g r = r, \forall g \in G\}.$$

Let (A,μ) be a partially crossed topological $G-R$-bimodule. H. Inassaridze [2] introduced an equivalence relation on the group $Der(G,(A,\mu))$ as follows:

$$(\alpha,r) \sim (\beta,s) \Leftrightarrow (\exists\ a \in A \wedge (\forall g \in G \Rightarrow \beta(g) = a^{-1}\alpha(g)^g a)) \\ \wedge\ (s = \mu(a)^{-1} r \text{ mod } H^0(G,R))$$

Let $\sim'$ be the restriction of $\sim$ to $Der_c(G,(A,\mu))$. Therefore, $\sim'$ is an equivalence relation. In other word, $(\alpha,r) \sim' (\beta,s)$ if and only if $(\alpha,r) \sim (\beta,s)$, whenever $(\alpha,r),(\beta,s) \in Der_c(G,(A,\mu))$.

Definition 3.1. Let (A,μ) be a partially crossed topological $G-R$-bimodule. The quotient set $Der_c(G,(A,\mu))/\sim'$ will be called the first cohomology of G with the coefficients in (A,μ) and is denoted by $H^1(G,(A,\mu))$. (In this definition, the groups G, R and A are not necessarily Hausdorff.)

Theorem 3.1. *Let G and R be locally compact groups and (A,μ) a partially crossed topological $G-R$-bimodule satisfying the following conditions*

(i) $H^0(G,R)$ is a normal subgroup of R;

(ii) for every $c \in H^0(G,R)$ and $(\alpha,r) \in Der_c(G,(A,\mu))$, there exists $a \in A$ such that $\mu(a) = 1$ and ${}^c\alpha(g) = a^{-1}\alpha(g)^g a$, $\forall g \in G$.

Then, $Der_c(G,(A,\mu))$ induces a topological group structure on $H^1(G,(A,\mu))$.

Proof. By Theorem 2.1 of [2], the group $Der(G,(A,\mu))$ induces the following action on $Der(G,(A,\mu))/\sim$

$$[(\alpha,r)][(\beta,s)] = [({}^r\beta\alpha, rs)].$$

Thus, $N = \{(\alpha,r)|(\alpha,r) \in Der(G,(A,\mu)), (\alpha,r) \sim (\mathbf{1},1)\}$ is a normal subgroup of $Der(G,(A,\mu))$. Therefore, $N' = \{(\alpha,r)|(\alpha,r) \in Der_c(G,(A,\mu)), (\alpha,r) \sim (\mathbf{1},1)\}$ is a normal subgroup of $Der_c(G,(A,\mu))$. By Theorem 2.5, $Der_c(G,(A,\mu))$ is a topological group. Obviously, $H^1(G,(A,\mu)) = Der_c(G,(A,\mu))/N'$. Therefore, $H^1(G,(A,\mu))$ is a topological group. □

Notice 3.2. *(i) Note that Hausdorffness of A is not needed in Theorem 3.1.*

(ii) Let A be a topological G-module. The first cohomology, $H^1(G,A)$, of G with coefficients in A is defined as in [11]. Thus, the compact-open topology on $Der_c(G,A)$ induces a quotient topology on $H^1(G,A)$. From now on, we consider $H^1(G,A)$ with this topology. Define $Inn(G,A) = \{Inn(a)|a \in A\}$, where for all $a \in A, g \in G$, $Inn(a)(g) = a^g a^{-1}$. If A is abelian, then by Remark 2.4. (i) of [11], $Inn(G,A)$ is a normal subgroup of $Der_c(G,A)$ and $H^1(G,A) = Der_c(G,A)/Inn(G,A)$; moreover, $H^1(G,A)$ is a topological group, and it is Hausdorff if and only if $Inn(G,A)$ is closed in $Der_c(G,A)$.

(iii) Define $Inn(G,(A,\mu)) = \{(Inn(a), \mu(a)z) | a \in A, z \in H^0(G,R)\}$. *Note that* $H^1(G,(A,\mu))$ *is a topological group if and only if* $Inn(G,(A,\mu))$ *is a normal subgroup of* $Der_c(G,(A,\mu))$. *Thus, by hypotheses of Theorem 3.1,* $Inn(G,(A,\mu))$ *is a normal subgroup of* $Der_c(G,(A,\mu))$ *and* $H^1(G,(A,\mu)) = Der_c(G,(A,\mu))/Inn(G,(A,\mu))$.

In the following, we give an example for this fact that: in general, $H^1(G,A)$ and $H^1(G,(A,\mu))$ are not necessarily Hausdorff.

Example 3.3. *Let G be an abelian discrete group; let* $(\mathbb{Z},+)$ *be the integer numbers group with the indiscrete topology* τ, *(i.e.,* $\tau = \{\mathbb{Z},\emptyset\}$*) such that* $\chi : G \to Aut(\mathbb{Z}) \cong \mathbb{Z}/2\mathbb{Z}$ *is a nontrivial homomorphism. Equip* $Aut(\mathbb{Z})$ *with the compact-open topology. Then,* χ *induces a nontrivial continuous action of G on* $\mathbb{Z}$ *given by* ${}^{g}z = \chi(g)(z)$, $\forall g \in G, z \in \mathbb{Z}$. *For all* $g \in G$, *we have* $[\{g\},\mathbb{Z}] \cap Der_c(G,\mathbb{Z}) = Der_c(G,\mathbb{Z})$. *Hence, the compact-open topology on* $Der_c(G,\mathbb{Z})$ *is the indiscrete topology. Thus,* $H^1(G,\mathbb{Z}) = Der_c(G,\mathbb{Z})/Inn(G,\mathbb{Z})$ *has the indiscrete topology. On the other hand, discreteness of G implies that* $Der_c(G,\mathbb{Z}) = Der(G,\mathbb{Z})$. *Hence by Theorem 3.2 of [12],* $H^1(G,\mathbb{Z}) \cong \mathbb{Z}/2\mathbb{Z} \neq 1$. *Hence,* $H^1(G,\mathbb{Z})$ *is not Hausdorff. Consequently,* $Inn(G,\mathbb{Z})$ *is not closed in* $Der_c(G,\mathbb{Z})$. *Now, note that* $(\mathbb{Z}, \mathbf{1} : \mathbb{Z} \to G)$ *is a crossed* $G-G$*-bimodule. It is easy to see that* $Inn(G,(\mathbb{Z},\mathbf{1})) = Inn(G,\mathbb{Z}) \times G$. *Hence* $Inn(G,(\mathbb{Z},\mathbf{1}))$ *is not closed in* $Der_c(G,(\mathbb{Z},\mathbf{1}))$ *and so* $H^1(G,(\mathbb{Z},\mathbf{1}))$ *is not Hausdorff.*

Remark 3.1. Let A be an abelian topological G-module and A be compact Hausdorff. Then, $H^1(G,A)$ is a Hausdorff topological group.

Let (A,μ) be a partially crossed G-module. Naturally (A,μ) is a crossed $G-G$-bimodule. Thus, we define the first cohomology of G with coefficients in (A,μ) as the set $H^1(G,(A,\mu))$.

Theorem 3.4. *Let G be a locally compact group and* (A,μ) *a partially crossed topological G-module. Then,* $H^1(G,(A,\mu))$ *is a topological group. In addition, if any of the following conditions is satisfied, then* $H^1(G,(A,\mu))$ *is Hausdorff.*

(i) A is compact and G has trivial center;

(ii) A is a trivial G-module;

(iii) A and Z(G) are compact, in particular if both topological groups A and G are compact.

Proof. Note that $H^0(G,G) = Z(G)$. For any $c \in Z(G)$ and $(\alpha, g) \in Der_c(G,(A,\mu))$, $\alpha(cx) = \alpha(xc)$ for all $x \in G$. Thus, ${}^{c}\alpha(x) = \alpha(c)^{-1}\alpha(x)\alpha(c)$, $\forall x \in G$ and $\mu(\alpha(c)) = {}^{g}cc^{-1} = 1$. Since G is locally compact, then by Theorem 3.1, $H^1(G,(A,\mu))$ is a topological group.

(i). If A is compact and G has trivial center then by the assumption $Z(G) = 1$. So $Inn(G,(A,\mu)) = \{(Inn(a), \mu(a)) | a \in A\}$. It is easy to see that the map $Inn : A \to Der_c(G,A), a \mapsto Inn(a)$ is continuous. Thus, compactness of A implies that $Inn(G,(A,\mu))$ is a compact subset of $Der_c(G,(A,\mu))$. Hence, $Inn(G,(A,\mu))$ is closed in $Der_c(G,(A,\mu))$. So $H^1(G,(A,\mu))$ is Hausdorff.

(ii). If G acts trivially on A, then ${}^{g}\mu(a) = \mu(a)$, for every $g \in G$ and $a \in A$. Thus, $Inn(G,(A,\mu)) = \{\mathbf{1}\} \times Z(G)$. Hence, $Inn(G,(A,\mu))$ is closed in $Der_c(G,(A,\mu))$.

(iii). Consider the continuous map $A \times Z(G) \to Der_c(G,(A,\mu))$, $(a,z) \mapsto (Inn(a), \mu(a)z)$. Consequently, the part (iii) is proved. □

Lemma 3.5. *Let G be a locally compact group and A an abelian topological group. Then, there is a natural topological isomorphism*

$$Hom_c(G,A) \simeq Hom_c(G/\overline{[G,G]}, A).$$

Proof. Since G is locally compact, then $G/\overline{[G,G]}$ is a locally compact group. Let $\pi : G \to G/\overline{[G,G]}$ be the natural epimorphism. Then, obviously $\chi : Hom_c(G/\overline{[G,G]}, A) \to Hom_c(G, A)$, $f \mapsto \pi f$ is a one to one and onto continuous homomorphism. We show that χ is an open map. It suffices to show that for every neighborhood Γ of $\mathbf{1}$ in $Hom_c(G, A)$, $\chi(\Gamma)$ is a neighborhood of $\mathbf{1}$ in $Hom_c(G/\overline{[G,G]}, A)$. Since $Hom_c(G, A)$ is a topological group, so it is a homogeneous space. It is clear that the network of all compact subset of G is closed under finite unions. Now, by a similar argument as in page 7 of [9], there is an open neighborhood of $\mathbf{1}$ of the form $S(C, U)$ in Γ. Note that $S(C, U) = \{f | f \in Hom_c(G/\overline{[G,G]}, A), f(C) \subset U\}$, where C is compact in $G/\overline{[G,G]}$ and U is open in A. Since G is locally compact, then by 5.24.b of [13], there is a compact subset D of G such that $\pi(D) = C$. It is easy to see that $\chi(S(C, U)) = S(D, U) \subset \chi(\Gamma)$. Therefore, χ is a topological isomorphism. □

Recall that a topological group G *has no small subgroups* (or *is without small subgroups*) if there is a neighborhood of the identity that contains no nontrivial subgroup of G. For example if n is a positive integer number, then the n-dimensional vector group, the n-dimensional tours, and general linear groups over the complex numbers are without small subgroups. It is well-known that the property of having no small subgroups is an extension property (see 6.15 Theorem of [10]). A topological group G is called compactly generated if there exists a compact subset K so that it generates G, that is $G =< K >$.

Proposition 3.1. *(1) If G is a locally compact group and A is a compact abelian group without small subgroups, then $Hom_c(G, A)$ is a locally compact group.*

(2) If G is a locally compact compactly generated group and A is a locally compact abelian group without small subgroups, then $Hom_c(G, A)$ is a locally compact group.

(3) If G is a compact group and A is an abelian group without small subgroups, then $Hom_c(G, A)$ is a discrete group.

(4) If G is a discrete group and A is a compact group, then $Hom_c(G, A)$ is a compact group.

(5) If G is a finite discrete group and A is a compact abelian group without small subgroups, then $Hom_c(G, A)$ is a finite discrete group.

(6) Let A be a topological G-module. If G is discrete and A is compact, then $Der_c(G, A)$ is a compact group.

Proof. Since A is abelian, by Lemma 3.5, $Hom_c(G, A) \simeq Hom_c(G/\overline{[G,G]}, A)$. Therefore, (1) and (2) follow from two corollaries in page 377 of [14]. Also (3) is obtained by Theorem 4.1 of [14].

(4) Since G is discrete, then $\mathcal{C}_k(G, A) = \mathcal{C}_p(G, A)$. By Corollary 2.7, $Hom_c(G, A)$ is closed in $\mathcal{C}_k(G, A)$. Let $B = \prod_{g \in G} A_g$, where $A_g = A, \forall g \in G$. It is clear that the map $\Phi : \mathcal{C}_p(G, A) \to B$, $f \mapsto \{f(g)\}_{g \in G}$ is continuous. In addition, since G is discrete, then the map $G \times B \to A$,$(h, \{a_g\}_{g \in G}) \mapsto a_h$ is continuous. Hence, this map induces the continuous map $\Psi : B \to \mathcal{C}_p(G, A)$, $\{a_g\}_{g \in G} \mapsto f$, where $f(g) = a_g$. Obviously, $\Phi\Psi = Id$ and $\Psi\Phi = Id$. Consequently, $\mathcal{C}_p(G, A)$ is homeomorphic to B. Thus, $\mathcal{C}_p(G, A)$ is compact. So $Hom_c(G, A)$ is compact.

(5) This is an immediate result from (3) and (4).

(6) By Proposition 2.3, $Der_c(G, A)$ is closed in $\mathcal{C}_k(G, A)$. We have seen in the proof of (4) that $\mathcal{C}_k(G, A)$ is compact. Consequently, $Der_c(G, A)$ is compact. □

Recall that a topological space X is called a k-space if every subset of X, whose intersection with every compact $K \subset X$ is relatively open in K, is open in X. A topological space X is a k-space if and only if X is the quotient image of a locally compact space (see Characterization (1) of [15]). For example, locally compact spaces and first-countable spaces are k-spaces. It is well-known that the k-space property is preserved by the closed subsets and the quotients. Also, the product of a locally compact space with a k-space is a k-space (see Result (1) of [15]). We call a topological group to be a k-group if it is a k-space as a topological space.

Theorem 3.6. *Let G be a locally compact group; let (A, μ) be a partially crossed topological $G-R$-bimodule such that G acts trivially on A and R.*

(1) *If R is a k-group and A is compact without small subgroups, then $H^1(G, (A, \mu))$ is a k-space.*

(2) *If G is compactly generated, R is a k-group and A is locally compact without small subgroups, then $H^1(G, (A, \mu))$ is a k-space.*

(3) *If G is compact, A has no small subgroups and R is discrete, then $H^1(G, (A, \mu))$ is discrete.*

(4) *If G and R are finite discrete and A is compact without small subgroups, then $H^1(G, (A, \mu))$ is a finite discrete space.*

Proof. Since G acts trivially on A and R, then it is easy to see that $Der_c(G, (A, \mu))$ is homeomorphic to $Hom_c(G, Ker\mu) \times R$. Note that $Ker\mu$ is closed in $Z(A)$. Now by Proposition 3.1, the assertions (1) to (4) hold. □

Theorem 3.7. *Let G be a locally compact abelian topological group; let (A, μ) be a partially crossed topological G-module and A a trivial G-module.*

(1) *If A is compact without small subgroups, then $H^1(G, (A, \mu))$ is a locally compact abelian group.*

(2) *If G is compactly generated and A is locally compact without small subgroups, then $H^1(G, (A, \mu))$ is a locally compact abelian group.*

(3) *If G is finite discrete and A is compact without small subgroups, then $H^1(G, (A, \mu))$ is a finite discrete abelian group.*

Proof. Since G is a locally compact abelian group and acts trivially on A, one can see $Der_c(G, (A, \mu)) \simeq Hom_c(G, Ker\mu) \times G$. Therefore, by Proposition 3.1, the proof is completed. □

Let G and A be topological groups; let K be an abelian subgroup of A. We denote the set of all continuous homomorphisms $f : G \to A$ with $f(G) \subset K$ by $Hom_c(G, A|K)$. Obviously, if G is locally compact, then $Hom_c(G, A|K)$ with compact-open topology is an abelian topological group.

Remark 3.2. (1) Let (A, μ) be a partially crossed topological G-module. Suppose that G is a locally compact abelian group which acts trivially on A. Then, $H^1(G, (A, \mu)) \simeq Hom_c(G, A|Ker\mu)$.

(2) Let A be an abelian topological G-module. Then, $(A, \mathbf{1})$ is a crossed topological $G-R$-bimodule for every topological group R, and $H^1(G, (A, \mathbf{1}))$ is homeomorphic to $H^1(G, A)$.

(3) Let G be a locally compact group and A an abelian topological G-module. Then, $(A, \mathbf{1})$ is a crossed topological G-module, and $H^1(G, (A, \mathbf{1})) \simeq H^1(G, A)$. In particular if G acts trivially on A, then $H^1(G, (A, \mathbf{1})) \simeq Hom_c(G/\overline{[G, G]}, A)$.

(4) Let G be a locally compact group and A an abelian topological G-module. Then, $H^1(G, (A, \pi_A)) = H^1(G, (A, \mathbf{1})) \simeq H^1(G, A)$.

Theorem 3.8. *Let (A, μ) be a partially crossed topological $G-R$-bimodule. Suppose that G is a discrete group, A and R are compact. Then, $H^1(G, (A, \mu))$ is compact.*

Proof. By Proposition 2.3, $Der_c(G, (A, \mu))$ is closed in $Der_c(G, A) \times R$. Obviously, if R is compact, then $H^1(G, (A, \mu))$ is compact. □

As an immediate result of Theorem 3.8, we have the following corollary:

Corollary 3.9. *Let (A, μ) be a partially crossed topological G-module, G be finite discrete and A be compact. Then $H^1(G, (A, \mu))$ is a compact group.*

Definition 3.2. A topological group A is radical-based, if it has a countable base $\{U_n\}_{n\in\mathbb{N}}$ at 1, such that each U_n is symmetric and for all $n \in \mathbb{N}$:

(1) $(U_n)^n \subset U_1$;

(2) $a, a^2, ..., a^n \in U_1$ implies $a \in U_n$.

For example, if n is a positive integer, then the n-dimensional vector group, the n-dimensional torus and the rational numbers are radical-based groups. For another example see [16].

Theorem 3.10. *Let (A, μ) be a partially crossed topological $G-R$-bimodule, and G a first countable group. Let R be locally compact and A a compact radical-based group with $H^0(G, A) = A$. Then, $H^1(G, (A, \mu))$ is a k-space.*

Proof. Since $H^0(G, A) = A$, then it follows from Proposition 2.3 that $Der_c(G, (A, \mu))$ is closed in $Hom_c(G, A) \times R$. By Theorem 1 of [16], $Hom_c(G, A)$ is a k-space. Thus, $Hom_c(G, A) \times R$ is a k-space. Consequently, $H^1(G, (A, \mu))$ is a k-space. □

By Theorem 3.10, the next corollary is immediate.

Corollary 3.11. *Let (A, μ) be a partially crossed topological G-module, let G be locally compact first countable and A a compact radical-based group with $H^0(G, A) = A$. Then, $H^1(G, (A, \mu))$ is a k-group.*

4 Conclusions

a We have proved that if G and R are locally compact groups and (A, μ) a partially crossed topological $G - R$-bimodule, then $Der_c(G, (A, \mu))$ is a topological group (Theorem 2.5). Moreover, if the locally compact groups G and R act continuously on each other and on A compatibly then $(Der_c(G, (A, \mu)), \gamma)$ is a precrossed topological $G - R$-bimodule, where $\gamma : Der_c(G, (A, \mu)) \to R$, $(\alpha, r) \mapsto r$ (Proposition 2.4).

b We have showed that under what conditions $H^1(G, (A, \mu))$ is a topological group (Theorem 3.1). In particular, if G is a locally compact group, then $H^1(G, (A, \mu))$ is a topological group for every partially crossed topological G-module (A, μ). Furthermore, we have found conditions under which $H^1(G, (A, \mu))$ is one of the following: k-space, discrete, locally compact and compact.

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Competing Interests

The authors declare that they have no competing interests.

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