Journal of Intelligent Learning Systems and Applications, 2024, 16, 91-106
https://www.scirp.org/journal/jilsa
ISSN Online: 2150-8410
ISSN Print: 2150-8402

Multi-Granularity Neighborhood Fuzzy Rough Set Model with Two Universes

Ju Wang[1*], Xinghu Ai[2], Li Fu[1#]

[1]School of Mathematics and Statistics, Qinghai Nationalities University, Xining, China
[2]Xining Big Date Center, Xining, China
Email: [#]1010949563@qq.com

How to cite this paper: Wang, J., Ai, X.H. and Fu, L. (2024) Multi-Granularity Neighborhood Fuzzy Rough Set Model with Two Universes. *Journal of Intelligent Learning Systems and Applications*, **16**, 91-106.
https://doi.org/10.4236/jilsa.2024.162007

Received: March 30, 2024
Accepted: May 7, 2024
Published: May 10, 2024

Abstract

The two-universes multi-granularity fuzzy rough set model is an effective tool for handling uncertainty problems between two domains with the help of binary fuzzy relations. This article applies the idea of neighborhood rough sets to two-universes multi-granularity fuzzy rough sets, and discusses the two-universes multi-granularity neighborhood fuzzy rough set model. Firstly, the upper and lower approximation operators are defined in the two-universes multi-granularity neighborhood fuzzy rough set model. Secondly, the properties of the upper and lower approximation operators are discussed. Finally, the properties of the two-universes multi-granularity neighborhood fuzzy rough set model are verified through case studies.

Keywords

Fuzzy Set, Two Universes, Multi-Granularity Rough Set, Multi-Granularity Neighborhood Fuzzy Rough Set

1. Introduction

Rough set [1] and fuzzy set [2] are effective methods for processing incomplete data, learning imprecise knowledge, and induction. Rough set emphasizes the roughness of knowledge, whereas fuzzy set emphasizes its fuzziness. The main idea of rough set theory [1] is to approximate uncertain concepts based on existing information or knowledge, using the upper and lower approximation operators in rough set theory, potential knowledge in information systems can be discovered and expressed in the form of decision rules. Fuzzy set [2] theory extends the binary logic in classical set theory to multi-valued logic, solving the knowledge inference problem of "either/or" in classical logic more effec-

*Ju Wang, Female, Research Interest: Uncertain Reasoning.
#Corresponding author: Li Fu, Female, Professor, Research Interest: Uncertain Reasoning.

tively. Due to the differences in handling uncertainty in mathematics between these two theories. Many scholars have combined these two to introduce the concept of fuzzy rough set [3], which can handle continuous data and prevent information loss during the discretization of continuous data. Scholars use distance to define the similarity between objects and have proposed a neighborhood rough set model [4] [5], which can directly handle numerical data. To overcome the low classification accuracy of classical rough sets, a variable precision rough set model [6] was introduced, enhancing the model's fault tolerance. In decision analysis, the relationship between multiple decision-makers may be independent, necessitating the use of multiple binary relations to approximate the target. This led to the proposal of the multi-granularity rough set concept [7]. Building on this, some researchers expanded the domain of discourse from one to two, introducing a two-universes multi-granularity fuzzy rough set model [8]. Based on Bayesian decision-making and binary fuzzy relations, this model considers decision-makers' preferences, thereby reducing decision risk.

Inspired by existing research, this study applies the concept of neighborhood rough sets to two-universes multi-granularity fuzzy rough sets, considers the fuzzy relationship between two domains based on neighborhood relationships, and explores the structure of two-universes multi-granularity neighborhood fuzzy rough set models. The article is organized as follows: First, it introduces basic concepts such as information systems, probabilistic rough sets, variable precision rough sets, neighborhood rough sets, and two-universes multi-granularity fuzzy rough sets. Second, it defines the two-universes multi-granularity neighborhood rough set model, ε-neighborhood, and conditional probability of fuzzy events. Then, it presents the upper and lower approximation operators and their properties from both optimistic and pessimistic perspectives. Finally, the model's relevance is demonstrated through the diagnosis of diseases in nursery rose flowers.

2. Fundamental Principles

Definition 1 [1]: An (U, A, F) is called an information system. Among them, U is a set of objects, $U=\{x_1,x_2,\cdots,x_m\}$, where each element $x_i\,(i\le m)$ is referred to as an object; a set of attributes $A=\{a_1,a_2,\cdots,a_n\}$, with each element $a_l\,(l\le n)$ termed an attribute; and a set of relationships between U and A, denoted as $=\{f_l:U\to V_l\,(l\le n)\}$, where V_l represents the range of a_l.

Definition 2 [9]: Assume U is a non-empty domain, R is an equivalence relation on U, and P is a probability measure defined on the σ-algebra of subsets of U. Then (U,R,P) is called a probability approximation space. For any object set X included in U ($X\subseteq U$), where parameters α and β satisfy $0\le\beta<\alpha\le1$, the lower and upper approximation operators of X with respect to the probability approximation space (U,R,P) are defined respectively.

$$\underline{R}_\alpha(X)=\left\{x\in U \mid P\left(X\mid[x]_R\right)\ge\alpha\right\}, \tag{1}$$

$$\overline{R}_\beta(X)=\left\{x\in U \mid P\left(X\mid[x]_R\right)>\beta\right\}, \tag{2}$$

where $P(X \mid [x]_R) = \frac{|[x]_R \cap X|}{[x]_R}$, and $|\cdot|$ denotes the cardinality of a set. When $\underline{R}_\alpha(X) = \overline{R}_\beta(X)$, X is said to be definable with respect to (U, R, P), otherwise X is said to be rough with respect to (U, R, P).

Definition 3 [6]: In the information system $T = (U, C \cup D, V, f)$, let $P \subseteq C$ and $X \subseteq U$. For the indiscernibility relation $IND(P)$, the upper and lower approximations of X based on variable precision rough sets are defined as follows:

$$R_*^\beta(X) = \cup\{P_i \mid P_i \in U/P, C(P_i, X) \geq \beta\} \quad (3)$$

$$R^{*\beta}(X) = \cup\{P_i \mid P_i \in U/P, C(P_i, X) \geq 1-\beta\} \quad (4)$$

where $\beta \in (0.5, 1]$.

Definition 4 [4]: Consider a neighborhood information system $IS = (U, At, V)$, where $B \subseteq At$. The neighborhood relation induced on the domain U is defined as follows:

$$N_B^\varepsilon = \{(x, y) \in U \times U \mid d_B(x, y) \leq \varepsilon\} \quad (5)$$

where ε is a non-negative constant, called the neighborhood radius, and $d_B(x, y)$ is the distance between objects x and y under attribute set B.

Definition 5 [7]: Consider a neighborhood information system $IS = (U, At, V)$, where $B_1, B_2, \cdots, B_m \subseteq At$ is a family of attribute subsets containing m attribute subsets, and the induced neighborhood relations are $N_{B_1}, N_{B_2}, \cdots, N_{B_m}$. For the target approximation object set $x \subseteq U$, the lower and upper approximations of the optimistic neighborhood multi-granularity rough set based on these m neighborhood relations are defined as follows

$$\underline{\sum_{i=1}^m R_{B_i}^O}(X) = \{x \mid n_{B_1} \subseteq X \vee n_{B_2} \subseteq X \vee \cdots \vee n_m \subseteq X\} \quad (6)$$

$$\overline{\sum_{i=1}^m R_{B_i}^O}(X) = \sim \underline{\sum_{i=1}^m R_{B_i}^O}(\sim X) \quad (7)$$

The $\left(\underline{\sum_{i=1}^m R_{B_i}^O}(X), \overline{\sum_{i=1}^m R_{B_i}^O}(X)\right)$ is called the optimistic neighborhood multi-granularity rough set of X with respect to m neighborhood relations $N_{B_1}, N_{B_2}, \cdots, N_{B_m}$.

Definition 6 [7]: Consider a neighborhood information system $IS = (U, At, V)$, where $B_1, B_2, \cdots, B_m \subseteq At$ is a family of attribute subsets containing m attribute subsets, and the induced neighborhood relations are $N_{B_1}, N_{B_2}, \cdots, N_{B_m}$. For the target approximation object set $x \subseteq U$, the lower and upper approximations of the pessimistic neighborhood multi-granularity rough set based on these m neighborhood relations are defined as follows:

$$\underline{\sum_{i=1}^m R_{B_i}^P}(X) = \{x \mid n_{B_1} \subseteq X \wedge n_{B_2} \subseteq X \wedge \cdots \vee n_m \subseteq X\} \quad (8)$$

$$\overline{\sum_{i=1}^m R_{B_i}^P}(X) = \sim \underline{\sum_{i=1}^m R_{B_i}^P}(\sim X) \quad (9)$$

We call $\left(\underline{\sum_{i=1}^m R_{B_i}^P}(X), \overline{\sum_{i=1}^m R_{B_i}^P}(X)\right)$ the pessimistic neighborhood multi-granularity rough set of X with respect to m neighborhood relations $N_{B_1}, N_{B_2}, \cdots, N_{B_m}$.

Definition 7 [10]: Given a two-domain approximate space (U, W, R), R is a binary relation on $U \times W$, and let $r: U \to 2^W$ be a set-valued mapping, denoted as $r(u) = \{v \in W \mid (u, w) \in R, u \in U\}$. For any non-empty object set $Y \subseteq W$, the lower and upper approximation operators for Y with respect to (U, W, R) are respectively

$$\underline{apr}_R(Y) = \{u \in U \mid r(u) \subseteq Y\}, \tag{10}$$

$$\overline{apr}_R(Y) = \{u \in U \mid r(u) \cap Y \neq \varnothing\} \tag{11}$$

The sequence of $\left(\underline{apr}_R(Y), \overline{apr}_R(Y)\right)$ is called the two-universes rough set of Y with respect to (U, W, R). When $U = W$, $r(u)$ can be considered as the neighborhood of *u*, thus, this model will degenerate into a single-domain rough set model.

Definition 8 [8]: Let *U*, *V* be two non-empty finite domains, and $\mathcal{R}$ a family of binary fuzzy relations between U and *V*, with $R_i \in F(U \times V)$ and $R_i \in \mathcal{R}$, where $i = 1, 2, \cdots, m$. P is a class of fuzzy subsets defined by a probability measure on domain V for the object $x \in U$, then $(U, V, \mathcal{R}, P)$ is called a two-universes multi-granularity fuzzy approximation space.

Definition 9 [8]: $(U, V, \mathcal{R}, P)$ is a bi-domain multi-granularity fuzzy approximation space. For any threshold parameter $0 \le \beta < 0.5 \le \alpha \le 1$, fuzzy set $A \in F(V)$, and precision parameter $0.5 < \delta < 1$, the lower and upper approximations of A with respect to $(U, V, \mathcal{R}, P)$ are defined as follows:

$$\underline{\mathcal{R}}^{\delta,\alpha}_{\sum_{i=1}^{m} R_i}(A) = \left\{ x \in U \mid \frac{\left|\left\{ R_i \mid P\left(A \mid F_{R_i}(x)(y)\right) \ge \alpha, y \in V, i = 1, 2, \cdots, m \right\}\right|}{m} \ge \delta \right\} \tag{12}$$

$$\overline{\mathcal{R}}^{\delta,\beta}_{\sum_{i=1}^{m} R_i}(A) = \left\{ x \in U \mid \frac{\left|\left\{ R_i \mid P\left(A \mid F_{R_i}(x)(y)\right) \le \beta, y \in V, i = 1, 2, \cdots, m \right\}\right|}{m} > 1 - \delta \right\} \tag{13}$$

3. Double-Domain Multi-Granularity Neighborhood Fuzzy Rough Set

The object of study is a two-universes multi-granularity neighborhood fuzzy rough set model. For the sake of symbol unification, the following symbols are used in this article to represent their corresponding meanings: $U = \{x_1, x_2, \cdots, x_m\}$, $V = \{y_1, y_2, \cdots, y_n\}$ are two non-empty finite domains; $\Re$ represents all fuzzy relations on the domain *U*, that is: $\Re = \{\tilde{R} \mid \tilde{R}$ is a fuzzy relation between U and *V*}); $\forall \tilde{R} \in \Re$, $\forall x \in U, y \in V$, $\tilde{R}(x, y)$ represents the degree of correlation between x and y under the fuzzy relation, that is, the membership degree.

Definition 10: Let U and V be two different non-empty domian, $U = \{x_1, x_2, \cdots, x_m\}$, $V = \{y_1, y_2, \cdots, y_n\}$, for all x in *U*, and for all $\tilde{\mathcal{R}}$ in *R*, let $N^{\varepsilon}_{\tilde{R}_i}(x, V)$ denote the ε-neighborhood formed by x and the elements in V under the relation $\tilde{\mathcal{R}}$. It is defined as follows:

$$N_{\tilde{R}_i}^{\varepsilon}(x,V)=\left\{z\in U\mid\frac{\sum_{y\in V}\left|\tilde{R}(x,y)-\tilde{R}(z,y)\right|}{n}\leq\varepsilon,y\in V\right\} \tag{14}$$

Similarly, one can define the ε-neighborhood of elements in V under the relation $\tilde{\mathcal{R}}$ in y as follows:

$$N_{\tilde{R}_i}^{\varepsilon}(U,y)=\left\{z\in V\mid\frac{\sum_{x\in U}\left|\tilde{R}(x,y)-\tilde{R}(x,z)\right|}{m}\leq\varepsilon,x\in U\right\} \tag{15}$$

Next, we can consider the fuzzy relationship between $N_{\tilde{R}_i}^{\varepsilon}(x,V)$ and V, $N_{\tilde{R}_i}^{\varepsilon}(v,y)$ and U, which is denoted as $\tilde{F}$. The membership degree is defined as

$$\tilde{F}\left(N_{\tilde{R}_i}^{\varepsilon}(x,V)\right)=\tilde{R}(z,y),\ z\in N^{\varepsilon}(x) \tag{16}$$

$$\tilde{F}\left(N_{\tilde{R}_i}^{\varepsilon}(U,y)\right)=\tilde{R}(x,z),\ z\in N^{\varepsilon}(y) \tag{17}$$

Definition 11: Let U and V be two different non-empty domian, $\forall U=\{x_1,x_2,\cdots,x_m\}$, $\forall V=\{y_1,y_2,\cdots,y_n\}$, $\Re\in F(U\times V)$ is a binary fuzzy relation between U and V, $\tilde{F}\left(N_{\tilde{R}_i}^{\varepsilon}(x,V)\right)$ is a fuzzy relation under the neighborhood relation $N_{\tilde{R}_i}^{\varepsilon}(x,V)$, and $P\left(\tilde{A}\mid\tilde{F}\left(N_{\tilde{R}_i}^{\varepsilon}(x,V)\right)\right)$ is the conditional probability of a fuzzy event under $\tilde{F}\left(N_{\tilde{R}_i}^{\varepsilon}(x,V)\right)$. Definition:

$$P\left(\tilde{A}\mid\tilde{F}\left(N_{\tilde{R}_i}^{\varepsilon}(x,V)\right)\right)=\frac{\sum_{y\in V}K\left(\tilde{A}(y),\tilde{F}\left(N_{\tilde{R}_i}^{\varepsilon}(x,V)\right)\right)}{\sum_{y\in V}\tilde{F}\left(N_{\tilde{R}_i}^{\varepsilon}(x,V)\right)},x\in U \tag{18}$$

Similarly, we have

$$P\left(\tilde{A}\mid\tilde{F}\left(N_{\tilde{R}_i}^{\varepsilon}(U,y)\right)\right)=\frac{\sum_{x\in U}K\left(\tilde{A}(x),\tilde{F}\left(N_{\tilde{R}_i}^{\varepsilon}(U,y)\right)\right)}{\sum_{x\in U}\tilde{F}\left(N_{\tilde{R}_i}^{\varepsilon}(U,y)\right)},y\in V \tag{19}$$

where $K:[0,1]\times[0,1]\to[0,1]$ is a fuzzy logic operator. Then $\left(U,V,\tilde{R},N_{\tilde{R}_i}^{\varepsilon},P\right)$ is called a two-universes multi-granularity neighborhood fuzzy approximation space.

Note: Let $\left(U,V,\tilde{R},N_{\tilde{R}_i}^{\varepsilon},P\right)$ be a neighborhood multi-granularity fuzzy approximation space under the two universes.

When $U=V$, it is a single domain, which is the general neighborhood multi-granularity rough set.

When $V=AT$, it is a general information system.

3.1. Two-Universes Optimistic Multi-Granularity Neighborhood Fuzzy Rough Set

Definition 12: Let $\left(U,V,\tilde{R},N_{\tilde{R}_i}^{\varepsilon},P\right)$ be a two-domain multi-granularity neighborhood fuzzy approximation space. For any threshold parameter $0\leq\beta<0.5\leq\alpha\leq1$, the upper and lower approximations of fuzzy set $\tilde{A}\in F(V)$ in the two-domain optimistic multi-granularity neighborhood fuzzy rough set are as follows:

$$\underline{\mathcal{R}}^{O,\alpha}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right)=\left\{x\in U \middle| P\left(\tilde{A}\mid \tilde{F}_{\tilde{R}_1}\left(N^{\varepsilon}_{\tilde{R}_1}\left(x,V\right)\right)\right)\geq\alpha\vee P\left(\tilde{A}\mid \tilde{F}_{\tilde{R}_2}\left(N^{\varepsilon}_{\tilde{R}_2}\left(x,V\right)\right)\right)\geq\alpha \vee\cdots\vee P\left(\tilde{A}\mid \tilde{F}_{\tilde{R}_m}\left(N^{\varepsilon}_{\tilde{R}_m}\left(x,V\right)\right)\right)\geq\alpha\right\} \quad (20)$$

$$\overline{\mathcal{R}}^{O,\beta}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right)=\sim\underline{\mathcal{R}}^{O,\beta}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\sim\tilde{A}\right) \quad (21)$$

Let $\left(U,V,R,N^{\varepsilon}_{\tilde{R}_i},P\right)$ be a two-universes multi-granularity neighborhood fuzzy approximation space. For any threshold parameter $0\leq\beta<0.5\leq\alpha\leq1$, fuzzy set $\tilde{A}\in F\left(V\right)$, we have

$$\overline{\mathcal{R}}^{O,\beta}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right)=U-\left\{x\in U\mid P\left(\tilde{A}\mid \tilde{F}_{\tilde{R}_1}\left(N^{\varepsilon}_{\tilde{R}_1}\left(x,V\right)\right)\right)<\beta\vee P\left(\tilde{A}\mid \tilde{F}_{\tilde{R}_2}\left(N^{\varepsilon}_{\tilde{R}_2}\left(x,V\right)\right)\right)<\beta \vee\cdots\vee P\left(\tilde{A}\mid \tilde{F}_{\tilde{R}_m}\left(N^{\varepsilon}_{\tilde{R}_m}\left(x,V\right)\right)\right)<\beta\right\} \quad (22)$$

$$=\left\{x\in U\mid P\left(\tilde{A}\mid \tilde{F}_{\tilde{R}_1}\left(N^{\varepsilon}_{\tilde{R}_1}\left(x,V\right)\right)\right)>\beta\wedge P\left(\tilde{A}\mid \tilde{F}_{\tilde{R}_2}\left(N^{\varepsilon}_{\tilde{R}_2}\left(x,V\right)\right)\right)>\beta \wedge\cdots\wedge P\left(\tilde{A}\mid \tilde{F}_{\tilde{R}_m}\left(N^{\varepsilon}_{\tilde{R}_m}\left(x,V\right)\right)\right)>\beta\right\} \quad (23)$$

With the help of rough set theory, the positive domain, negative domain, and boundary domain of the two-universes optimistic multi-granularity neighborhood fuzzy rough set $\tilde{A}$ can be obtained.

$$POS^{O,\alpha}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right)=\underline{\mathcal{R}}^{O,\alpha}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right) \quad (24)$$

$$NEG^{O,\alpha}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right)=U-\overline{\mathcal{R}}^{O,\alpha}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right) \quad (25)$$

$$BND^{O,\alpha}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right)=\overline{\mathcal{R}}^{O,\alpha}_{\sum_{i=1}^{m}\tilde{R}_i}\left(A\right)-\underline{\mathcal{R}}^{O,\alpha}_{\sum_{i=1}^{m}V_i}\left(\tilde{A}\right) \quad (26)$$

Property 1: Let U and V be two different non-empty domian, $U=\{x_1,x_2,\cdots,x_m\}$, $V=\{y_1,y_2,\cdots,y_n\}$ $\tilde{R}_i\left(i=1,2,\cdots,m\right)$ be m different binary relations on $U\times V$, for any $0\leq\beta<0.5\leq\alpha\leq1$, any fuzzy sets $\tilde{A}$ and $\tilde{B}$ have

1. $\underline{\mathcal{R}}^{O,\beta}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right)\subseteq\overline{\mathcal{R}}^{O,\beta}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right)$
2. For $\forall\tilde{A}\subseteq\tilde{B}$: $\underline{\mathcal{R}}^{O,\alpha}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right)\subseteq\underline{\mathcal{R}}^{O,\alpha}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{B}\right)$

   $\overline{\mathcal{R}}^{O,\beta}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right)\subseteq\overline{\mathcal{R}}^{O,\beta}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{B}\right)$
3. $\underline{\mathcal{R}}^{O,\alpha}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\cap\tilde{B}\right)\subseteq\underline{\mathcal{R}}^{O,\alpha}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right)\cap\underline{\mathcal{R}}^{O,\alpha}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{B}\right)$

   $\underline{\mathcal{R}}^{O,\alpha}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\cup\tilde{B}\right)\supseteq\underline{\mathcal{R}}^{O,\alpha}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right)\cup\underline{\mathcal{R}}^{O,\alpha}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{B}\right)$
4. $\overline{\mathcal{R}}^{O,\beta}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\cap\tilde{B}\right)\subseteq\overline{\mathcal{R}}^{O,\beta}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right)\cap\overline{\mathcal{R}}^{O,\beta}_{\sum_{i=1}^{m}R_i}\left(\tilde{B}\right)$

   $\overline{\mathcal{R}}^{O,\beta}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\cup\tilde{B}\right)\supseteq\overline{\mathcal{R}}^{O,\beta}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right)\cup\overline{\mathcal{R}}^{O,\beta}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{B}\right)$

Proving:

1. $\forall x\in\underline{\mathcal{R}}^{O,\beta}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right)$ do $x\notin\underline{\mathcal{R}}^{O,\beta}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\sim\tilde{A}\right)$,and then: $x\in\sim\underline{\mathcal{R}}^{O,\beta}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\sim\tilde{A}\right)$,

That is: $x \in \overline{\mathcal{R}}^{O,\beta}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right)$ According to the upper and lower approximation properties of rough sets, we can get: $\underline{\mathcal{R}}^{O,\beta}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right) \subseteq \overline{\mathcal{R}}^{O,\beta}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right)$

2. It can be known from the definition:

$$\underline{\mathcal{R}}^{O,\alpha}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right)=\left\{x\in U \mid P\left(\tilde{A}\mid \tilde{F}_{\tilde{R}_1}\left(N^{\varepsilon}_{\tilde{R}_1}(x,V)\right)\right)\geq \alpha \vee P\left(\tilde{A}\mid \tilde{F}_{\tilde{R}_2}\left(N^{\varepsilon}_{\tilde{R}_2}(x,V)\right)\right)\geq \alpha \vee\cdots\vee P\left(\tilde{A}\mid \tilde{F}_{\tilde{R}_m}\left(N^{\varepsilon}_{\tilde{R}_m}(x,V)\right)\right)\geq \alpha\right\}$$

$\forall \tilde{A}\subseteq \tilde{B}$, do: $\tilde{A}(y)\leq \tilde{A}(y)$

Due to:

$$P\left(\tilde{A}\mid \tilde{F}\left(N^{\varepsilon}(x,V)\right)\right)=\frac{\sum_{y\in V}K\left(\tilde{A}(y),\tilde{F}\left(N^{\varepsilon}_{\tilde{R}_i}(x,V)\right)\right)}{\sum_{y\in V}\tilde{F}\left(N^{\varepsilon}_{\tilde{R}_i}(x,V)\right)}, x\in U:$$

$$P\left(\tilde{A}\mid \tilde{F}_{\tilde{R}_i}\left(N^{\varepsilon}_{\tilde{R}_i}(x,V)\right)\right)\leq P\left(\tilde{B}\mid \tilde{F}_{\tilde{R}_i}\left(N^{\varepsilon}_{\tilde{R}_i}(x,V)\right)\right)$$

So: for $\forall \tilde{A}\subseteq \tilde{B}$, we can get: $\underline{\mathcal{R}}^{O,\alpha}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right)\subseteq \underline{\mathcal{R}}^{O,\alpha}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{B}\right)$

And the same: $\forall \tilde{A}\subseteq \tilde{B}$, do: $\overline{\mathcal{R}}^{O,\beta}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right)\subseteq \overline{\mathcal{R}}^{O,\beta}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{B}\right)$

3. For $\forall \tilde{A}\subseteq \tilde{B}$, we can get: $\tilde{A}\cap\tilde{B}\subseteq \tilde{A}$ and $\tilde{A}\cap\tilde{B}\subseteq \tilde{B}$

Due to (2), for $\forall \tilde{A}\subseteq \tilde{B}$, we have: $\underline{\mathcal{R}}^{O,\alpha}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right)\subseteq \underline{\mathcal{R}}^{O,\alpha}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{B}\right)$

So: $\underline{\mathcal{R}}^{O,\alpha}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\cap\tilde{B}\right)\subseteq \underline{\mathcal{R}}^{O,\alpha}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right)$ and $\underline{\mathcal{R}}^{O,\alpha}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\cap\tilde{B}\right)\subseteq \underline{\mathcal{R}}^{O,\alpha}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{B}\right)$

And then: $\underline{\mathcal{R}}^{O,\alpha}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\cap\tilde{B}\right)\subseteq \underline{\mathcal{R}}^{O,\alpha}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right)\cap\underline{\mathcal{R}}^{O,\alpha}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{B}\right)$

For $\forall \tilde{A}\subseteq B$, we have: $\tilde{A}\cup\tilde{B}\supseteq \tilde{A}$ and $\tilde{A}\cup B\supseteq B$

Due to (2), for $\forall \tilde{A}\subseteq \tilde{B}$, we can get: $\underline{\mathcal{R}}^{O,\alpha}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right)\subseteq \underline{\mathcal{R}}^{O,\alpha}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{B}\right)$

So: $\underline{\mathcal{R}}^{O,\alpha}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\cup\tilde{B}\right)\supseteq \underline{\mathcal{R}}^{O,\alpha}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right)$ and $\underline{\mathcal{R}}^{O,\alpha}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\cup\tilde{B}\right)\supseteq \underline{\mathcal{R}}^{O,\alpha}_{\sum_{i=1}^{m}R_i}\left(\tilde{B}\right)$

And then: $\underline{\mathcal{R}}^{O,\alpha}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\cup\tilde{B}\right)\supseteq \underline{\mathcal{R}}^{O,\alpha}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right)\cup\underline{\mathcal{R}}^{O,\alpha}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{B}\right)$

Note: The inverse inclusion of all inclusion relationships in 3 and 4 in property 1 does not hold.

For example:

Example 1: Let $U=\{x_1,x_2,x_3,x_4\}$, $V=\{y_1,y_2,y_3\}$, neighborhood $\varepsilon=0.31$, $\alpha=0.5$, $\beta=0.47$ (here the fuzzy logic operator adopts the method of taking the minimum), the fuzzy binary relation from U to V is as follows:

$$\tilde{R}_1=\begin{bmatrix}0.3 & 0.5 & 0.6\\ 0.5 & 0.4 & 0.7\\ 0.2 & 0.8 & 0.5\\ 0.7 & 0.5 & 0.3\end{bmatrix}\quad \tilde{R}_2=\begin{bmatrix}0.5 & 0.4 & 0.6\\ 0.7 & 0.8 & 0.5\\ 0.2 & 0.4 & 0.4\\ 0.4 & 0.3 & 0.6\end{bmatrix}$$

From

$$N^{\varepsilon}_{\tilde{R}_i}(x,V)=\left\{z\in U \mid \frac{\sum_{y\in V}\left|\tilde{R}(x,y)-\tilde{R}(z,y)\right|}{n}\leq \varepsilon, y\in V\right\}$$

We have:

$$N_{\tilde{R}_1}^{\varepsilon}(x)=\{x_1,x_2\}$$

$$N_{\tilde{R}_2}^{\varepsilon}(x)=\{x_1,x_4\}$$

And then:

$$\tilde{R}_1'=\begin{bmatrix}0.3 & 0.5 & 0.6\\ 0.5 & 0.4 & 0.7\end{bmatrix}\quad \tilde{R}_2'=\begin{bmatrix}0.5 & 0.4 & 0.6\\ 0.4 & 0.3 & 0.6\end{bmatrix}$$

$$\tilde{A}=\frac{0.15}{y_1}+\frac{0.6}{y_2}+\frac{0.2}{y_3}$$

$$\tilde{B}=\frac{0.4}{y_1}+\frac{0.2}{y_2}+\frac{0.15}{y_3}$$

$$\tilde{A}\cup\tilde{B}=\frac{0.4}{y_1}+\frac{0.6}{y_2}+\frac{0.2}{y_3}$$

$$\tilde{A}\cap\tilde{B}=\frac{0.15}{y_1}+\frac{0.2}{y_2}+\frac{0.15}{y_3}$$

According to **Tables 1-4**, it can be concluded that the upper and lower

Table 1. Conditional probability of fuzzy event $\tilde{A}$.

$\tilde{R}_i$	$P\left(\tilde{A}\mid\tilde{F}_{\tilde{R}_i}\left(N_{\tilde{R}_i}^{\varepsilon}(x_1,V)\right)\right)$	$P\left(\tilde{A}\mid\tilde{F}_{\tilde{R}_i}\left(N_{\tilde{R}_i}^{\varepsilon}(x_2,V)\right)\right)$	$P\left(\tilde{A}\mid\tilde{F}_{\tilde{R}_i}\left(N_{\tilde{R}_i}^{\varepsilon}(x_3,V)\right)\right)$	$P\left(\tilde{A}\mid\tilde{F}_{\tilde{R}_i}\left(N_{\tilde{R}_i}^{\varepsilon}(x_4,V)\right)\right)$
$\tilde{R}_1$	0.607	0.4688		
$\tilde{R}_2$	0.5			0.5

Table 2. Conditional probability of fuzzy event $\tilde{B}$.

$\tilde{R}_i$	$P\left(\tilde{B}\mid\tilde{F}_{\tilde{R}_i}\left(N_{\tilde{R}_i}^{\varepsilon}(x_1,V)\right)\right)$	$P\left(\tilde{B}\mid\tilde{F}_{\tilde{R}_i}\left(N_{\tilde{R}_i}^{\varepsilon}(x_2,V)\right)\right)$	$P\left(\tilde{B}\mid\tilde{F}_{\tilde{R}_i}\left(N_{\tilde{R}_i}^{\varepsilon}(x_3,V)\right)\right)$	$P\left(\tilde{B}\mid\tilde{F}_{\tilde{R}_i}\left(N_{\tilde{R}_i}^{\varepsilon}(x_4,V)\right)\right)$
$\tilde{R}_1$	0.4673	0.4688		
$\tilde{R}_2$	0.5			0.5769

Table 3. Conditional probability of fuzzy event $\tilde{A}\cup\tilde{B}$.

$\tilde{R}_i$	$P\left(\tilde{A}\cup\tilde{B}\mid\tilde{F}_{\tilde{R}_i}\left(N_{\tilde{R}_i}^{\varepsilon}(x_1,V)\right)\right)$	$P\left(\tilde{A}\cup\tilde{B}\mid\tilde{F}_{\tilde{R}_i}\left(N_{\tilde{R}_i}^{\varepsilon}(x_2,V)\right)\right)$	$P\left(\tilde{A}\cup\tilde{B}\mid\tilde{F}_{\tilde{R}_i}\left(N_{\tilde{R}_i}^{\varepsilon}(x_3,V)\right)\right)$	$P\left(\tilde{A}\cup\tilde{B}\mid\tilde{F}_{\tilde{R}_i}\left(N_{\tilde{R}_i}^{\varepsilon}(x_4,V)\right)\right)$
$\tilde{R}_1$	0.7143	0.625		
$\tilde{R}_2$	0.6667			0.6923

Table 4. Conditional probability of fuzzy event $\tilde{A}\cap\tilde{B}$.

$\tilde{R}_i$	$P\left(\tilde{A}\cap\tilde{B}\mid\tilde{F}_{\tilde{R}_i}\left(N_{\tilde{R}_i}^{\varepsilon}(x_1,V)\right)\right)$	$P\left(\tilde{A}\cap\tilde{B}\mid\tilde{F}_{\tilde{R}_i}\left(N_{\tilde{R}_i}^{\varepsilon}(x_2,V)\right)\right)$	$P\left(\tilde{A}\cap\tilde{B}\mid\tilde{F}_{\tilde{R}_i}\left(N_{\tilde{R}_i}^{\varepsilon}(x_3,V)\right)\right)$	$P\left(\tilde{A}\cap\tilde{B}\mid\tilde{F}_{\tilde{R}_i}\left(N_{\tilde{R}_i}^{\varepsilon}(x_4,V)\right)\right)$
$\tilde{R}_1$	0.3571	0.3125		
$\tilde{R}_2$	0.3333			0.3846

approximations of two-universes optimistic neighborhood multi-granularity fuzzy rough sets are as follows:

$$\underline{\mathcal{R}}^{O,0.5}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right)=\left\{x_1,x_4\right\}$$

$$\overline{\mathcal{R}}^{O,0.47}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right)=\left\{x_1,x_4\right\}$$

$$\underline{\mathcal{R}}^{O,0.5}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{B}\right)=\left\{x_1,x_4\right\}$$

$$\overline{\mathcal{R}}^{O,0.47}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{B}\right)=\left\{x_4\right\}$$

$$\underline{\mathcal{R}}^{O,0.5}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\cup\tilde{B}\right)=\left\{x_1,x_2,x_4\right\}$$

$$\overline{\mathcal{R}}^{O,0.47}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\cup\tilde{B}\right)=\left\{x_1,x_2,x_4\right\}$$

$$\underline{\mathcal{R}}^{O,0.5}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\cap\tilde{B}\right)=\left\{\varnothing\right\}$$

$$\overline{\mathcal{R}}^{O,0.47}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\cap\tilde{B}\right)=\left\{\varnothing\right\}$$

It can be obtained from the calculation results:

$$\underline{\mathcal{R}}^{O,0.5}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right)\cap\underline{\mathcal{R}}^{O,0.5}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{B}\right)=\left\{x_1,x_4\right\},\quad \underline{\mathcal{R}}^{O,0.5}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right)\cup\underline{\mathcal{R}}^{O,0.5}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{B}\right)=\left\{x_1,x_4\right\}$$

$$\overline{\mathcal{R}}^{O,0.47}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right)\cap\overline{\mathcal{R}}^{O,0.47}_{\sum_{i=1}^{m}R_i}\left(\tilde{B}\right)=\left\{x_4\right\},\quad \overline{\mathcal{R}}^{O,0.47}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right)\cup\overline{\mathcal{R}}^{O,0.47}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{B}\right)=\left\{x_1,x_4\right\}$$

That is: $\left\{\varnothing\right\}\not\supseteq\left\{x_1,x_4\right\}\Rightarrow\underline{\mathcal{R}}^{O,\alpha}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\cap\tilde{B}\right)\not\supseteq\underline{\mathcal{R}}^{O,\alpha}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right)\cap\underline{\mathcal{R}}^{O,\alpha}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{B}\right)$

$$\left\{x_1,x_2,x_4\right\}\subsetneq\left\{x_1,x_4\right\}\Rightarrow\underline{\mathcal{R}}^{O,\alpha}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\cup\tilde{B}\right)\not\subseteq\underline{\mathcal{R}}^{O,\alpha}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right)\cup\underline{\mathcal{R}}^{O,\alpha}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{B}\right)$$

$$\left\{\varnothing\right\}\not\supseteq\left\{x_4\right\}\Rightarrow\overline{\mathcal{R}}^{O,\beta}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\cap\tilde{B}\right)\not\supseteq\overline{\mathcal{R}}^{O,\beta}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right)\cap\overline{\mathcal{R}}^{O,\beta}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{B}\right)$$

$$\left\{x_1,x_2,x_4\right\}\subsetneq\left\{x_1,x_4\right\}\Rightarrow\overline{\mathcal{R}}^{O,\beta}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\cup\tilde{B}\right)\not\subseteq\overline{\mathcal{R}}^{O,\beta}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right)\cup\overline{\mathcal{R}}^{O,\beta}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{B}\right)$$

3.2. Two-Universes Pessimistic Multi-Granularity Neighborhood Fuzzy Rough Set

Definition 13: $\left(U,V,R,N^{\varepsilon}_{\tilde{R}_i},P\right)$ is called a two-universes multi-granularity neighborhood fuzzy approximation space. For any threshold parameter $0\le\beta<0.5\le\alpha\le1$, fuzzy set $\tilde{A}\in F\left(V\right)$, the upper and lower approximations of the two-universes pessimistic multi-granularity neighborhood fuzzy rough set are as follows

$$\begin{aligned}\underline{\mathcal{R}}^{p,\alpha}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right)=\Big\{x\in U\mid P\left(\tilde{A}\mid\tilde{F}_{R_1}\left(N^{\varepsilon}_{\tilde{R}_1}\left(x,V\right)\right)\right)\ge\alpha\wedge P\left(\tilde{A}\mid\tilde{F}_{R_2}\left(N^{\varepsilon}_{\tilde{R}_2}\left(x,V\right)\right)\right)\ge\alpha\\ \wedge\cdots\wedge P\left(\tilde{A}\mid\tilde{F}_{R_m}\left(N^{\varepsilon}_{\tilde{R}_m}\left(x,V\right)\right)\right)\ge\alpha\Big\}\end{aligned} \tag{27}$$

$$\overline{\mathcal{R}}^{P,\beta}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right)=\sim\underline{\mathcal{R}}^{P,\beta}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\sim\tilde{A}\right) \tag{28}$$

Let $\left(U,V,R,N^{\varepsilon}_{\tilde{R}_i},P\right)$ be a two-universes multi-granularity neighborhood fuzzy approximation space. For any threshold parameter $0\le\beta<0.5\le\alpha\le1$, fuzzy set $\tilde{A}\in F\left(V\right)$, we have

$$\begin{aligned}\overline{\mathcal{R}}^{p,\beta}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right)&=U-\left\{x\in U\mid P\left(\tilde{A}\mid\tilde{F}_{R_1}\left(N^{\varepsilon}_{\tilde{R}_1}\left(x,V\right)\right)\right)<\beta\wedge P\left(\tilde{A}\mid\tilde{F}_{R_2}\left(N^{\varepsilon}_{\tilde{R}_2}\left(x,V\right)\right)\right)<\beta\right.\\&\quad\left.\wedge\cdots\wedge P\left(\tilde{A}\mid\tilde{F}_{R_m}\left(N^{\varepsilon}_{\tilde{R}_m}\left(x,V\right)\right)\right)<\beta\right\}\\&=\left\{x\in U\mid P\left(\tilde{A}\mid\tilde{F}_{R_1}\left(N^{\varepsilon}_{\tilde{R}_1}\left(x,V\right)\right)\right)>\beta\vee P\left(\tilde{A}\mid\tilde{F}_{R_2}\left(N^{\varepsilon}_{\tilde{R}_2}\left(x,V\right)\right)\right)>\beta\right.\\&\quad\left.\vee\cdots\vee P\left(\tilde{A}\mid\tilde{F}_{R_m}\left(N^{\varepsilon}_{\tilde{R}_m}\left(x,V\right)\right)\right)>\beta\right\}\end{aligned}\tag{29}$$

With the help of rough set theory, the positive domain, negative domain, and boundary domain of the two-universes pessimistic multi-granularity neighborhood fuzzy rough set $\tilde{A}$ can be obtained.

$$POS^{p,\alpha}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right)=\underline{\mathcal{R}}^{p,\alpha}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right)\tag{30}$$

$$NEG^{p,\beta}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right)=U-\overline{\mathcal{R}}^{p,\beta}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right)\tag{31}$$

$$BND^{p,\alpha}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right)=\overline{\mathcal{R}}^{p,\alpha}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right)-\underline{\mathcal{R}}^{p,\alpha}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right)\tag{31}$$

Property 2: Let U and V be two different non-empty domian, $U=\{x_1,x_2,\cdots,x_m\}$, $V=\{y_1,y_2,\cdots,y_n\}$, and $\tilde{R}_i\left(i=1,2,\cdots,m\right)$ be m different binary relations on $U\times V$. For any $0\le\beta<0.5\le\alpha\le1$, and any fuzzy sets $\tilde{A}$ and $\tilde{B}$ have

1. $\underline{\mathcal{R}}^{P,\beta}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right)\subseteq\overline{\mathcal{R}}^{P,\beta}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right)$
2. $\forall A\subseteq B$, we have: $\underline{\mathcal{R}}^{P,\alpha}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right)\subseteq\underline{\mathcal{R}}^{P,\alpha}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{B}\right)$
$\overline{\mathcal{R}}^{P,\beta}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right)\subseteq\overline{\mathcal{R}}^{P,\beta}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{B}\right)$
3. $\underline{\mathcal{R}}^{P,\alpha}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\cap\tilde{B}\right)\subseteq\underline{\mathcal{R}}^{P,\alpha}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right)\cap\underline{\mathcal{R}}^{P,\alpha}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{B}\right)$
$\underline{\mathcal{R}}^{P,\alpha}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\cup\tilde{B}\right)\supseteq\underline{\mathcal{R}}^{P,\alpha}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right)\cup\underline{\mathcal{R}}^{P,\alpha}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{B}\right)$
4. $\overline{\mathcal{R}}^{P,\beta}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\cap\tilde{B}\right)\subseteq\overline{\mathcal{R}}^{P,\beta}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right)\cap\overline{\mathcal{R}}^{P,\beta}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{B}\right)$
$\overline{\mathcal{R}}^{P,\beta}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\cup\tilde{B}\right)\supseteq\overline{\mathcal{R}}^{P,\beta}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right)\cup\overline{\mathcal{R}}^{P,\beta}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{B}\right)$

Proof: same property 1

Note: The inverse inclusion of all the inclusion relationships in (3) and (4) in property 2 does not hold. See the case study for counterexamples.

Example 2: see example 1

The upper and lower approximations of the two-universes pessimistic multi-granularity neighborhood fuzzy rough set are

$$\underline{\mathcal{R}}^{P,0.5}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right)=\{x_1,x_4\}$$

$$\overline{\mathcal{R}}^{P,0.47}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right)=\{x_1,x_4\}$$

$$\underline{\mathcal{R}}^{P,0.5}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{B}\right)=\{x_4\}$$

$$\overline{\mathcal{R}}^{P,0.47}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right)=\{x_1,x_4\}$$

$$\underline{\mathcal{R}}^{P,0.5}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\cup\tilde{B}\right)=\{x_1,x_2,x_4\}$$

$$\overline{\mathcal{R}}^{P,0.47}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\cup\tilde{B}\right)=\{x_1,x_2,x_4\}$$

$$\underline{\mathcal{R}}^{O,0.5}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\cap\tilde{B}\right)=\{\varnothing\}$$

$$\overline{\mathcal{R}}^{P,0.47}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\cap\tilde{B}\right)=\{\varnothing\}$$

From the calculation results:

$$\underline{\mathcal{R}}^{P,0.5}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right)\cap\underline{\mathcal{R}}^{P,0.5}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{B}\right)=\{x_4\},\quad \underline{\mathcal{R}}^{P,0.5}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right)\cup\underline{\mathcal{R}}^{P,0.5}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{B}\right)=\{x_1,x_4\}$$

$$\overline{\mathcal{R}}^{P,0.47}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right)\cap\overline{\mathcal{R}}^{P,0.47}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{B}\right)=\{x_1,x_4\},\quad \overline{\mathcal{R}}^{P,0.47}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right)\cup\overline{\mathcal{R}}^{P,0.47}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{B}\right)=\{x_1,x_4\}$$

That is: $\{\varnothing\}\not\supseteq\{x_4\}\Rightarrow\underline{\mathcal{R}}^{P,\alpha}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\cap\tilde{B}\right)\not\supseteq\underline{\mathcal{R}}^{P,\alpha}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right)\cap\underline{\mathcal{R}}^{P,\alpha}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{B}\right)$

$$\{x_1,x_2,x_4\}\subsetneq\{x_1,x_4\}\Rightarrow\underline{\mathcal{R}}^{P,\alpha}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\cup\tilde{B}\right)\not\subseteq\underline{\mathcal{R}}^{P,\alpha}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right)\cup\underline{\mathcal{R}}^{P,\alpha}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{B}\right)$$

$$\{\varnothing\}\not\supseteq\{x_1,x_4\}\Rightarrow\overline{\mathcal{R}}^{P,\beta}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\cap\tilde{B}\right)\not\supseteq\overline{\mathcal{R}}^{P,\beta}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right)\cap\overline{\mathcal{R}}^{P,\beta}_{\sum_{i=1}^{m}R_i}\left(\tilde{B}\right)$$

$$\{x_1,x_2,x_4\}\subsetneq\{x_1,x_4\}\Rightarrow\overline{\mathcal{R}}^{P,\beta}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\cup\tilde{B}\right)\not\subseteq\overline{\mathcal{R}}^{P,\beta}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right)\cup\overline{\mathcal{R}}^{P,\beta}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{B}\right)$$

4. Case Analysis

A nursery has a batch of sick roses, which can be divided into eight varieties. It is necessary to diagnose which disease the roses have and provide corresponding treatment. The domain $U=\{x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8\}$ is the eight varieties of roses that are sick in the nursery. The domain $V=\{y_1,y_2,y_3,y_4\}$ is the pathological characteristics represented by the roses. y_1 to y_4 are respectively represented as: leaves with reddish-brown small patches, leaves with small white spots, leaves with yellowing, and leaves with small black spots. $\tilde{R}_1,\tilde{R}_2,\tilde{R}_3$ are three horticulturists who judge whether the roses have certain symptoms based on their own experience and professional knowledge. The fuzzy set $\tilde{A}$ represents rust disease, the fuzzy set $\tilde{B}$ represents downy mildew, the fuzzy set $\tilde{A}\cup\tilde{B}$ represents rust disease or downy mildew, and the fuzzy set $\tilde{A}\cap\tilde{B}$ represents both rust disease and downy mildew. The neighborhood $\varepsilon = 0.24$ (Here the fuzzy logic operator adopts the method of taking the minimum).

$\tilde{R}_1,\tilde{R}_2,\tilde{R}_3$ are fuzzy relations between sets U and V, where

$$\tilde{R}_1=\begin{bmatrix}0.5&0.6&0.6&0.7\\0.6&0.8&0.6&0.5\\0.8&0.7&0.9&0.6\\0.7&0.8&0.6&0.7\\0.8&0.8&0.7&0.9\\0.5&0.7&0.5&0.6\\0.6&0.8&0.6&0.5\\0.5&0.6&0.5&0.6\end{bmatrix}\quad \tilde{R}_2=\begin{bmatrix}0.7&0.8&0.7&0.5\\0.7&0.6&0.5&0.5\\0.6&0.4&0.5&0.6\\0.8&0.8&0.7&0.6\\0.5&0.7&0.6&0.7\\0.8&0.9&0.8&0.7\\0.5&0.6&0.5&0.7\\0.4&0.5&0.4&0.5\end{bmatrix}\quad \tilde{R}_3=\begin{bmatrix}0.8&0.6&0.7&0.6\\0.6&0.5&0.4&0.6\\0.6&0.5&0.4&0.6\\0.5&0.6&0.6&0.7\\0.7&0.8&0.8&0.9\\0.8&0.9&0.6&0.7\\0.4&0.5&0.4&0.5\\0.6&0.7&0.4&0.6\end{bmatrix}$$

From $N^{\varepsilon}_{\tilde{R}_i}(x,V)=\left\{z\in U \mid \frac{\sum_{y\in V}\left|\tilde{R}(x,y)-\tilde{R}(z,y)\right|}{n}<\varepsilon, y\in V\right\}$, We have:

$$N^{\varepsilon}_{\tilde{R}_1}(x,V)=\{x_1,x_2,x_3,x_4,x_7\}$$

$$N^{\varepsilon}_{\tilde{R}_2}(x,V)=\{x_1,x_2,x_3,x_5,x_7\}$$

$$N^{\varepsilon}_{\tilde{R}_3}(x,V)=\{x_1,x_4,x_8\}$$

And then:

$$\tilde{R}_1'=\begin{bmatrix}0.5&0.6&0.6&0.7\\0.6&0.8&0.6&0.5\\0.8&0.7&0.9&0.6\\0.7&0.8&0.6&0.7\\0.6&0.8&0.6&0.5\end{bmatrix}\quad \tilde{R}_2'=\begin{bmatrix}0.7&0.8&0.7&0.5\\0.7&0.6&0.5&0.5\\0.6&0.4&0.5&0.6\\0.5&0.7&0.6&0.7\\0.5&0.6&0.5&0.7\end{bmatrix}\quad \tilde{R}_3'=\begin{bmatrix}0.8&0.6&0.7&0.6\\0.5&0.6&0.6&0.7\\0.6&0.7&0.4&0.6\end{bmatrix}$$

$$\tilde{A}=\frac{0.2}{y_1}+\frac{0.65}{y_2}+\frac{0.15}{y_3}+\frac{0.35}{y_4}$$

$$\tilde{B}=\frac{0.7}{y_1}+\frac{0.2}{y_2}+\frac{0.45}{y_3}+\frac{0.3}{y_4}$$

$$\tilde{A}\cup\tilde{B}=\frac{0.7}{y_1}+\frac{0.65}{y_2}+\frac{0.45}{y_3}+\frac{0.35}{y_4}$$

$$\tilde{A}\cap\tilde{B}=\frac{0.2}{y_1}+\frac{0.2}{y_2}+\frac{0.15}{y_3}+\frac{0.3}{y_4}$$

According to **Table 5** and **Table 6**, it can be concluded that the comprehensive parameters α and β are:

$$\alpha=0.6\times0.5424+0.1\times0.5455+0.3\times0.5714=0.5517$$
$$\beta=0.6\times0.3056+0.1\times0.4222+0.3\times0.3095=0.3184$$

According to **Tables 7-10**, it can be concluded that then the upper and lower approximations of the two-universes optimistic neighborhood multi-granularity fuzzy rough set are

Table 5. Loss function for three scoring criteria.

	λ^k_{11}	λ^k_{21}	λ^k_{31}	λ^k_{12}	λ^k_{22}	λ^k_{32}
$\tilde{R}_1$	0.33	0.85	0.60	0.79	0.36	0.47
$\tilde{R}_2$	0.58	0.89	0.63	0.55	0.30	0.49
$\tilde{R}_3$	0.43	0.84	0.55	0.81	0.52	0.65

Table 6. The value of the threshold parameter.

$\tilde{R}_i$	$\tilde{R}_1$	$\tilde{R}_2$	$\tilde{R}_3$
α^k	0.5424	0.5455	0.5714
β^k	0.3056	0.4222	0.3095

Table 7. Conditional probability of fuzzy event $\tilde{A}$.

$\tilde{R}_i$	$\tilde{R}_1$	$\tilde{R}_2$	$\tilde{R}_3$
$P\left(\tilde{A} \mid \tilde{F}_{\tilde{R}_i}\left(N^{\varepsilon}\left(x_1,V\right)\right)\right)$	0.5416	0.5	0.4815
$P\left(\tilde{A} \mid \tilde{F}_{\tilde{R}_i}\left(N^{\varepsilon}\left(x_2,V\right)\right)\right)$	0.54	0.5652	
$P\left(\tilde{A} \mid \tilde{F}_{\tilde{R}_i}\left(N^{\varepsilon}\left(x_3,V\right)\right)\right)$	0.45	0.5238	
$P\left(\tilde{A} \mid \tilde{F}_{\tilde{R}_i}\left(N^{\varepsilon}\left(x_4,V\right)\right)\right)$	0.4822		0.5417
$P\left(\tilde{A} \mid \tilde{F}_{\tilde{R}_i}\left(N^{\varepsilon}\left(x_5,V\right)\right)\right)$		0.54	
$P\left(\tilde{A} \mid \tilde{F}_{\tilde{R}_i}\left(N^{\varepsilon}\left(x_6,V\right)\right)\right)$			
$P\left(\tilde{A} \mid \tilde{F}_{\tilde{R}_i}\left(N^{\varepsilon}\left(x_7,V\right)\right)\right)$	0.54	0.5652	
$P\left(\tilde{A} \mid \tilde{F}_{\tilde{R}_i}\left(N^{\varepsilon}\left(x_8,V\right)\right)\right)$			0.5869

Table 8. Conditional probability of fuzzy event $\tilde{B}$.

$\tilde{R}_i$	$\tilde{R}_1$	$\tilde{R}_2$	$\tilde{R}_3$
$P\left(\tilde{B} \mid \tilde{F}_{\tilde{R}_i}\left(N^{\varepsilon}\left(x_1,V\right)\right)\right)$	0.6042	0.6111	0.6111
$P\left(\tilde{B} \mid \tilde{F}_{\tilde{R}_i}\left(N^{\varepsilon}\left(x_2,V\right)\right)\right)$	0.62	0.7174	
$P\left(\tilde{B} \mid \tilde{F}_{\tilde{R}_i}\left(N^{\varepsilon}\left(x_3,V\right)\right)\right)$	0.55	0.7381	
$P\left(\tilde{B} \mid \tilde{F}_{\tilde{R}_i}\left(N^{\varepsilon}\left(x_4,V\right)\right)\right)$	0.5893		0.537
$P\left(\tilde{B} \mid \tilde{F}_{\tilde{R}_i}\left(N^{\varepsilon}\left(x_5,V\right)\right)\right)$		0.85	
$P\left(\tilde{B} \mid \tilde{F}_{\tilde{R}_i}\left(N^{\varepsilon}\left(x_6,V\right)\right)\right)$			
$P\left(\tilde{B} \mid \tilde{F}_{\tilde{R}_i}\left(N^{\varepsilon}\left(x_7,V\right)\right)\right)$	0.5893	0.6304	
$P\left(\tilde{B} \mid \tilde{F}_{\tilde{R}_i}\left(N^{\varepsilon}\left(x_8,V\right)\right)\right)$			0.6522

Table 9. Conditional probability of fuzzy event $\tilde{A} \cup \tilde{B}$.

$\tilde{R}_i$	$\tilde{R}_1$	$\tilde{R}_2$	$\tilde{R}_3$
$P\left(\tilde{A} \cup \tilde{B} \mid \tilde{F}_{\tilde{R}_i}\left(N^{\varepsilon}\left(x_1,V\right)\right)\right)$	0.7916	0.7963	0.77
$P\left(\tilde{A} \cup \tilde{B} \mid \tilde{F}_{\tilde{R}_i}\left(N^{\varepsilon}\left(x_2,V\right)\right)\right)$	0.62	0.7174	
$P\left(\tilde{A} \cup \tilde{B} \mid \tilde{F}_{\tilde{R}_i}\left(N^{\varepsilon}\left(x_3,V\right)\right)\right)$	0.55	0.7381	
$P\left(\tilde{A} \cup \tilde{B} \mid \tilde{F}_{\tilde{R}_i}\left(N^{\varepsilon}\left(x_4,V\right)\right)\right)$	0.5893		0.537

Continued

$P\left(\tilde{A}\cup\tilde{B}\mid\tilde{F}_{\tilde{R}_i}\left(N^{\varepsilon}\left(x_5,V\right)\right)\right)$	0.85		
$P\left(\tilde{A}\cup\tilde{B}\mid\tilde{F}_{\tilde{R}_i}\left(N^{\varepsilon}\left(x_6,V\right)\right)\right)$			
$P\left(\tilde{A}\cup\tilde{B}\mid\tilde{F}_{\tilde{R}_i}\left(N^{\varepsilon}\left(x_7,V\right)\right)\right)$	0.5893	0.6304	
$P\left(\tilde{A}\cup\tilde{B}\mid\tilde{F}_{\tilde{R}_i}\left(N^{\varepsilon}\left(x_8,V\right)\right)\right)$			0.6522

Table 10. Conditional probability of fuzzy event $\tilde{A}\cap\tilde{B}$.

$\tilde{R}_i$	$\tilde{R}_1$	$\tilde{R}_2$	$\tilde{R}_3$
$P\left(\tilde{A}\cap\tilde{B}\mid\tilde{F}_{\tilde{R}_i}\left(N^{\varepsilon}\left(x_1,V\right)\right)\right)$	0.3542	0.3148	0.3148
$P\left(\tilde{A}\cap\tilde{B}\mid\tilde{F}_{\tilde{R}_i}\left(N^{\varepsilon}\left(x_2,V\right)\right)\right)$	0.34	0.3696	
$P\left(\tilde{A}\cap\tilde{B}\mid\tilde{F}_{\tilde{R}_i}\left(N^{\varepsilon}\left(x_3,V\right)\right)\right)$	0.2833	0.4048	
$P\left(\tilde{A}\cap\tilde{B}\mid\tilde{F}_{\tilde{R}_i}\left(N^{\varepsilon}\left(x_4,V\right)\right)\right)$	0.3036		0.3542
$P\left(\tilde{A}\cap\tilde{B}\mid\tilde{F}_{\tilde{R}_i}\left(N^{\varepsilon}\left(x_5,V\right)\right)\right)$		0.34	
$P\left(\tilde{A}\cap\tilde{B}\mid\tilde{F}_{\tilde{R}_i}\left(N^{\varepsilon}\left(x_6,V\right)\right)\right)$			
$P\left(\tilde{A}\cap\tilde{B}\mid\tilde{F}_{\tilde{R}_i}\left(N^{\varepsilon}\left(x_7,V\right)\right)\right)$	0.34	0.3696	
$P\left(\tilde{A}\cap\tilde{B}\mid\tilde{F}_{\tilde{R}_i}\left(N^{\varepsilon}\left(x_8,V\right)\right)\right)$			0.3696

$$\underline{\mathcal{R}}^{O,0.5517}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right)=\left\{x_2,x_7,x_8\right\}$$

$$\overline{\mathcal{R}}^{O,0.3184}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right)=\left\{x_1,x_2,x_3,x_4,x_5,x_7,x_8\right\}$$

$$\underline{\mathcal{R}}^{O,0.5517}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{B}\right)=\left\{x_1,x_2,x_3,x_4,x_5,x_7,x_8\right\}$$

$$\overline{\mathcal{R}}^{O,0.3184}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{B}\right)=\left\{x_1,x_2,x_3,x_4,x_5,x_7,x_8\right\}$$

$$\underline{\mathcal{R}}^{O,0.5517}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\cup\tilde{B}\right)=\left\{x_1,x_2,x_3,x_4,x_5,x_7,x_8\right\}$$

$$\overline{\mathcal{R}}^{O,0.3184}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\cup\tilde{B}\right)=\left\{x_1,x_2,x_3,x_4,x_5,x_7,x_8\right\}$$

$$\underline{\mathcal{R}}^{O,0.5517}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\cap\tilde{B}\right)=\left\{\varnothing\right\}$$

$$\overline{\mathcal{R}}^{O,0.3184}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\cap\tilde{B}\right)=\left\{x_2,x_5,x_7,x_8\right\}$$

Therefore, in the optimistic case, x_2,x_7,x_8 have rust disease, $x_1,x_2,x_3,x_4,x_5,x_7,x_8$ have downy mildew disease, and $x_1,x_2,x_3,x_4,x_5,x_7,x_8$ have either rust or downy mildew disease

The upper and lower approximations of the two-universes pessimistic neighborhood multi-granularity fuzzy rough set are

$$\underline{\mathcal{R}}^{P,0.5517}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right)=\{x_8\}$$

$$\overline{\mathcal{R}}^{P,0.3184}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\right)=\{x_1,x_2,x_3,x_4,x_5,x_7,x_8\}$$

$$\underline{\mathcal{R}}^{P,0.5517}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{B}\right)=\{x_1,x_2,x_4,x_5,x_7,x_8\}$$

$$\overline{\mathcal{R}}^{P,0.3184}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{B}\right)=\{x_1,x_2,x_3,x_4,x_5,x_7,x_8\}$$

$$\underline{\mathcal{R}}^{P,0.5517}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\cup\tilde{B}\right)=\{x_1,x_2,x_3,x_4,x_5,x_7,x_8\}$$

$$\overline{\mathcal{R}}^{P,0.3184}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\cup\tilde{B}\right)=\{x_1,x_2,x_3,x_4,x_5,x_7,x_8\}$$

$$\underline{\mathcal{R}}^{P,0.5517}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\cap\tilde{B}\right)=\{\varnothing\}$$

$$\overline{\mathcal{R}}^{P,0.3184}_{\sum_{i=1}^{m}\tilde{R}_i}\left(\tilde{A}\cap\tilde{B}\right)=\{x_1,x_2,x_3,x_4,x_5,x_7,x_8\}$$

In the pessimistic scenario, x_8 has rust disease, x_1,x_2,x_4,x_5,x_7,x_8 have downy mildew, and $x_1,x_2,x_3,x_4,x_5,x_7,x_8$ have either rust or downy mildew.

5. Conclusion

This paper combines neighborhood rough sets with double-domain multi-granularity fuzzy rough sets. By considering the fuzzy relationship between two domains using ε-neighborhoods, and defining the conditional probability of fuzzy events based on this fuzzy relationship, a double-domain multi-granularity neighborhood fuzzy rough set model is proposed. From both optimistic and pessimistic perspectives, the definitions of double-domain optimistic multi-granularity neighborhood fuzzy rough sets and double-domain pessimistic multi-granularity neighborhood fuzzy rough sets are given along with their respective properties. For any two fuzzy sets, the upper approximation satisfies the inclusion relationship with intersection, and the lower approximation satisfies the inclusion relationship with union, but the upper approximation intersection does not satisfy reverse inclusion, and the lower approximation union does not satisfy reverse inclusion. Finally, a case study is used to verify the properties of the double-domain multi-granularity fuzzy rough set model.

Funding

This work is supported by National Science Foundation of China (Grant No.12371459) and 2023 Graduate Innovation Project of Qinghai University for Nationalities (No.07M2023007).

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

[1] Pawlak, Z. (1982) Rough Sets. *International Journal of Computer and Information Sciences*, **11**, 341–356. https://doi.org/10.1007/BF01001956

[2] Zadeh L.A. (1965) Fuzzy Sets. *Information and Control*, **8**, 338-353. https://doi.org/10.1016/S0019-9958(65)90241-X

[3] Dubois, D. and Prade, H. (1990) Rough Fuzzy Sets and Fuzzy Rough Sets. *International Journal of General Systems*, **17**, 191-208. https://doi.org/10.1080/03081079008935107

[4] Hou, L.J., Wang, G.Y., Nie, N., *et al*. (2000) Discretization Problems in Rough Set Theory. *Computer Science*, **12**, 89-94.

[5] Teng, S.H., Lu, M., Yang, A.F., *et al*. (2016) Efficient Attribute Reduction from the Viewpoint of Discernibility. *Information Sciences*, **326**, 297-314. https://doi.org/10.1016/j.ins.2015.07.052

[6] Ziarko, W. (1993) Variable Precision Rough Set Model. *Journal of Computer and System Sciences*, **46**, 39-59. https://doi.org/10.1016/0022-0000(93)90048-2

[7] Qian, Y.H., Li, S., Liang, J., Shi, Z. and Wang, F. (2014) Pessimistic Rough Set Based Decision: A Multigranulation Fusion Strategy. *Information Sciences*, **264**, 196-210. https://doi.org/10.1016/j.ins.2013.12.014

[8] Li, X.N., Qi, J.J., Sun, B.Z., *et al*. (2019) Three-Branch Decision-making Theory and Methodolog. Science Press, Beijing.

[9] Yao, Y. (2010) Three-Way Decisions with Probabilistic Rough Sets. *Information Science*, **180**, 341-353. https://doi.org/10.1016/j.ins.2009.09.021

[10] Qiu, Y.Z. and Fu, R. (2005) Rough Set Models on Two Domains and Their Applications. *Journal of Sichuan Normal University* (*Natural Science Edition*), **28**, 15-18.