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Mass under the Membrane Theory of Gravitation

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Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

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ABSTRACT

The Cosmic Membrane theory of gravitation (CM) implies Newton's absolute space. We identify the homogeneous vector field used by us since 1994 with the Higgs-field as source of the heavy mass. Following Randall and Sundrum, the introduction of the wafting layer outside the membrane solves the issue of the mobility of particles in a super-strong membrane. Starting with Feynman's radius of excess, we obtain a depth of space of W_{RS} = 1.432×10^6 [m] of the gravitational funnel at the edge of sun. Using Chandrasekhar's gravitational energy, we obtain the tension F_0 of the membrane as F_0=1.820×10^{19} [N/m^2], and the vertical vector field acceleration A_{VFV}, acting perpendicularly from the fourth spatial dimension on the membrane, with A_{VFV}=1.148×10^5 [m/s^2]. The horizontal vector field acceleration A_{VFH}, i.e., inside the wafting layer, is A_{VFH}=1.330×10^5 [m/s^2], and acts as acceleration a=A_{VFH} w' with w' the being slope of the membrane. The mass of the moved membrane in a moving gravitational funnel behaves as an inert mass, but yields a numerical value that is too small to explain the equivalence of heavy and inert mass. Assuming speed of light c for transversal gravitational waves, we obtain a first estimation of the mass distribution ρ_{surf} of the membrane. The clay lump model of the relativistic increase of mass follows the assumption that the energy of the accelerating photons will act again half as mass and half as kinetic energy at the accelerated particle. Our result equals exactly Einstein's SR result.

Keywords: Membrane; absolute space; gravitational energy; relativity; increase of mass.

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1. INTRODUCTION

We have been working since the early 1990s – besides on statistical issues – on the rehabilitation of Newton's absolute space. Important milestones include the works of Einstein, Kaluza and others, according to which our universe has more than three spatial dimensions, and, furthermore, Hubble's discovery of the Nebula Escape [1], as well as the discovery of the cosmic background radiation [2]. Since 1994, we used the model of the cosmic membrane, which posits that the membrane expands as a balloon-shaped 3D-Brane in the 4D-hyperspace. The current boom in cosmological brane world models supports our propositions [3-6]. Other ideas and suggestions come from research areas that are far from our own field of work, for example from Quantum Chromo Dynamics (QCD) or Quantum Electrodynamics (QED).

In this article, we consider the physical property mass from a more phenomenological point of view as it is mapped out by the model of the cosmic membrane. Mass is a property of matter. It exists in the form of heavy mass and inertial mass. The heavy mass causes gravitation. Isaac Newton described the mutual attraction of heavy masses in his universal law of mass attraction, but he gave no explanation of the mechanism. In kinematics, the inert mass appears as the inertia of a mass. It takes a force to change the speed of a mass, i.e., in the cases of acceleration, deceleration or change of direction.

The equivalence principle states that heavy and inert masses are equivalent. Experiments prove this with the high accuracy of 10^{-15}. Albert Einstein also realized the equivalence of mass and energy. Therefore, electromagnetic radiation has mass as well. In his GR, Einstein laid the foundations for all modern theories of gravitation. The mass of particles changes with speed. Therefore, considering impact processes, one calculates with conservation quantity $m(v) = m / \sqrt{1-(v/c)^2}$. The quantity $1/\sqrt{1-(v/c)^2}$ is the Lorentz' factor of the SR and also used by us in [7]. The energy-momentum relation $E^2 - \vec{p}^2c^2 = m^2c^4$ is another universal law.

In 1964, several teams of researchers [8-10] published nearly at the same time and with nearly identical results their insights into the origin of the mass. The best known is Peter Higgs. He postulated the existence of a new particle of the standard model, the Higgs boson, later named after him. Mass is created by the interaction of originally massless particles with the Higgs-field. In this process, the Higgs bosons are created, giving mass to the particles. Since 2012, one has detected particles in the Large Hadron Collider in CERN near Geneva which fit the boson data given by Higgs. This fact has strongly increased the credibility of his and similar theories. A minor fly in the ointment is that Higgs' theory only explains about 1% of the mass.

In 1967, Steven Weinberg applied Higgs' theory to the theory of electroweak interaction [11]. In 1977, George Savvidy posited that the vacuum should contain a not- disappearing real field which is the cause of condensates (particles) [12]. These condensates are an effective description of the vacuum. However, at extremely small dimensions, the vacuum can have a structure [13]. In parallel, bag models were proposed. A hadron (neutron, proton) is an encapsulated piece of distorted vacuum [14-16]. Starting at the millennium, other theories have been developed on the base of Lattice QCD. The space is no longer homogeneous, but consists of a lattice of single points in space and time [17-19]. However, for small distances, one assumes the theory of the continuous QCD.

An important suggestion for our model of the cosmic membrane came from Randall and Sundrum. In 1999, Lisa Randall and Raman Sundrum published a paper [20], in which they proved that the assumption of a 3D double brane, which is embedded in a non-compacted spacetime with 5 dimensions, can generate both, Newtonian gravitation and its refinements according to the GR. In this approach, the distance between the two branes is assumed to be very small, compared with the Planck length. This approach allows for more dimensions, but these had to be compact, and only if they are small enough. In their theory, the authors utilize the generation of gravitons as bosons of the gravitational interaction. The effect of the gravitons is quickly lost with distance from the first brane. This assumption explains the comparatively small gravitational interaction of matter. In a second paper [21], Randall and Sundrum forego the second brane, and instead assume a rapid drop in effectiveness of the gravitons with increasing distance from the brane.

2. THE REFINED MODEL OF THE COSMIC MEMBRANE

The 3D brane expands in a way similar to a balloon that is inflated in a 4D hyperspace. A homogeneous vector field acts parallel to the direction of expansion and perpendicularly to the membrane. The vector field can easily permeate the undisturbed membrane [22]. One can imagine the whole model of the cosmic membrane as a sieve that is blown on. The vector field has the same effect as the Higgs field. Furthermore, we assume that a wafting layer is generated by the influence of the vector field directly above the membrane. The density of this layer decreases exponentially with the distance from the membrane, in a way similar to the decrease of pressure of our atmosphere with increasing altitude. This Randall-Sundrum wafting layer is the actual space in which matter resides and can move freely. Furthermore, we assume that a matter-free area of the wafting layer including the underlying membrane generates only little resistance to the vector field, but each kind of matter greatly increases the resistance. Examples from fluid mechanics of gases and liquids show that, for example, a laminar flow has much less resistance than a turbulent flow. We therefore postulate:

2.1 Each Kind of Matter Increases the Resistance of the Membrane in the Homogeneous Vector Field. This Resistance Generates a Force that Bends the Membrane

Additionally, we also postulate that, without motion, there is no mass. Either we have a wave with phase velocity *c* in undisturbed space, or we have a particle with matter waves of speed *c*, but group speed *v*. Fig. 1 illustrates the refined model of the cosmic membrane. The vector field acts perpendicularly from the fourth dimension, *w*, on the membrane. The jam of the vector flow generates the wafting layer. Inside the wafting layer, waves or particles consisting of matter waves move in x-, y- or z-direction.

The waves and the resulting lateral displacement cause a perturbation of the flow of the vector field through the membrane. An additional jam occurs, which significantly increases the pressure on the membrane and bends the membrane.

In the further course of this section, we quantify the interaction of the mass with the vector field. We consider a mass, e.g., the matter of our sun. The vector field creates a force effect, that is, a load that acts from the fourth dimension. It causes a gravitational funnel with spherical symmetry. The tension of the undisturbed membrane is F_0 with unit [N/m^2]. The decomposition of the radial force F_r and of the line element *ds* are shown in Fig. 2.

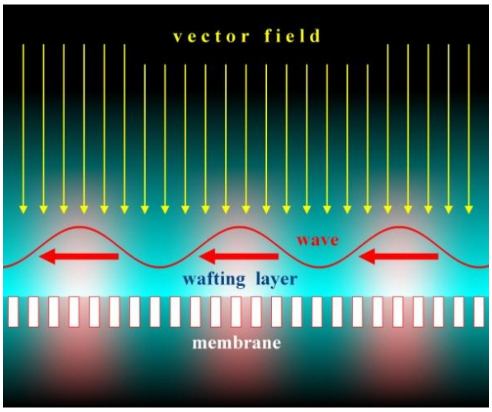

Fig. 1. Membrane with wafting layer, wave and vector field

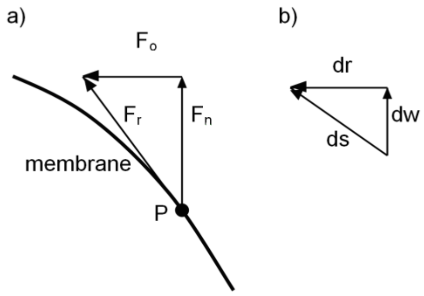

Fig. 2. Decomposition of radial force F_r (Panel a) and of line element *ds* (Panel b)

In the gravitational funnel, the tension of the curved membrane is higher than F_0, but the increase is very small. One proof is the validity of Newton's law of gravitation with an error of 10^{-6}. That means, in addition, that the elasticity coefficient of the membrane has to be lower than 10^{11} [N/m^2]. The radial amount of tension F_0 at point *P* is F_r. By the decomposition of F_r, we obtain the horizontal component F_0 and the vertical component F_n . It is physically obvious that the horizontal component F_0 remains unchanged. We differentiate the line element $ds = \sqrt{dr^2 + dw^2}$ under the assumption of ideal elasticity and obtain

$$\frac{ds}{dr} = \frac{\sqrt{dr^2 + dw^2}}{dr} = \frac{F_r}{F_0}. \tag{2.1}$$

Component F_r is the radial tension at point *P* with radius *r* from the center of the gravitational funnel. The vertical axis is *w*. With *dw/dr = w'*, we obtain $F_r / F_0 = \sqrt{1+w'^2}$.

The decomposition of F_r yields $F_r^2 = F_0^2 + F_n^2$ or $F_n = \sqrt{F_r^2 - F_0^2}$. With $F_r = F_0 \sqrt{1+w'^2}$, we obtain

$$F_n = \sqrt{F_0^2 \left(1+w'^2\right) - F_0^2} = F_0\, w' . \qquad (2.2)$$

The section surrounding the center of the funnel in the constant distance *r* describes a sphere. The central load must equal the sum of the vertical components of all forces which pull at the surface of the sphere. We obtain $4\pi r^2 F_n = L$ or $4\pi r^2 F_0 w' = L$. By differentiation, we obtain $0 = 8\pi r F_0 w' + 4\pi r^2 F_0 w''$, or

$$w'' = -\frac{2\,w'}{r} . \qquad (2.3)$$

This is the ordinary differential equation of a 3D membrane that is curved in a 4D space by a single central load (symmetric case). Each function $w(r)=C_1+C_2/r$ is a solution of the ODE Eq. (2.3). Differentiation of $w(r)= C_1+C_2/r$ yields $w'(r)= -C_2/r^2$, the slope of the membrane in distance *r*. The decomposition of the force acting on a small mass *m* inside the sloped membrane at distance *r* from the w-axis yields the downhill force F_{DH} as $F_{DH}=m\,A_{VFH}\,sin(\alpha)$. In the original cosmic membrane model, before the refinement, we used only the vector field acceleration A_{VF} [23]. Here, A_{VFH} is the horizontal vector field acceleration and *α* is the angle of the slope. Considering only small angles (i.e. small gravity), one can set *sin(α) ~ tan(α) = w'*. Replacing *sin(α)* according to $-C/r^2$ = *tan(α)=w'* by *w'(r)*, one obtains

$$F_{DH} = m\, A_{VFH}\, w'(r). \qquad (2.4)$$

This is equivalent to Newton's Law of Universal Gravitation. In the case of two masses, that is, a great central mass which causes the gravitational funnel and a small mass *m*, F_{DH} is the force of attraction. Now, we apply Eq. (2.4) to the solar system. Here, R_S is the radius of the sun, M_S its mass, W_{RS} the depth *w* of the deformed membrane at the edge of the sun, and W'_{RS} is the slope of the membrane at this position. Dividing Eq. (2.4) by mass *m*, one obtains the gravitational acceleration g_{RS} at the edge of the sun, and finds $g_{RS} = A_{VF}\, W'_{RS}$. By Newton's law, $g_{RS} = -\gamma\, M_S / R_S^2$. If the numerical value of W'_{RS} were known, one could calculate the value of the horizontal vector field acceleration A_{VFH}. Supposing, that the depth of space *w* equals zero for *r→∞*, one can express the function *w(r)* of the gravitational funnel surrounding the sun as

$$w(r) = -\frac{W_{RS} R_S}{r} . \qquad (2.5)$$

With $w'(r) = W_{RS}\, R_S / r^2$, one finds

$$W'_{RS} = \frac{W_{RS}}{R_S} . \qquad (2.6)$$

Now, one has to find a value for the depth of space, $W_{RS.}$ We treat Feynman's radius of excess, $r_{Ex}=a/3$=491[m], as formally equivalent to the extension d_{SR} of the geometrical path from the edge of the sun to its center [24]. Here, the magnitude *a* is the Schwarzschild radius of the sun. The radius of excess, r_{Ex}, was calculated by Feynman for a sphere with constant density, but one can show that the extensions of the geometrical path within and outside a sphere of constant density are equal. The depth of space, *w(r),* and, therefore, the extension *dS* of the geometrical path outside a sphere depend only on the total mass of the sphere, but not on its interior density distribution.

The dilation *Δdr* of a piece *dr* of the membrane is $\Delta dr \approx dr\sqrt{1+w'^2(r)} - dr \approx dr\, w'^2(r)/2$

or $\Delta dr \approx \left(W_{RS}^2\, R_S^2\, dr\right)/\left(2r^4\right)$. Now, we calculate the extension of the exterior geometrical path, *dS*, by solving the integral (2.7).

$$dS = \int_{RS}^{\infty} \frac{1}{2} w'^2(r)\, dr = \int_{RS}^{\infty} \frac{W_{RS}^2\, R_S^2}{2r^4}\, dr = \frac{W_{RS}^2}{6R_S} \qquad (2.7)$$

With Feynman's value of r_{EX} = *dS* = 491 [m] and R_S =6.958×10^8 [m], one obtains a value of W_{RS} = 1.432×10^6 [m] or 1432 [km] for the depth of space at the edge of sun in our cosmic membrane model. By $g_{RS} = A_{VFH}\, W'_{RS}$ and

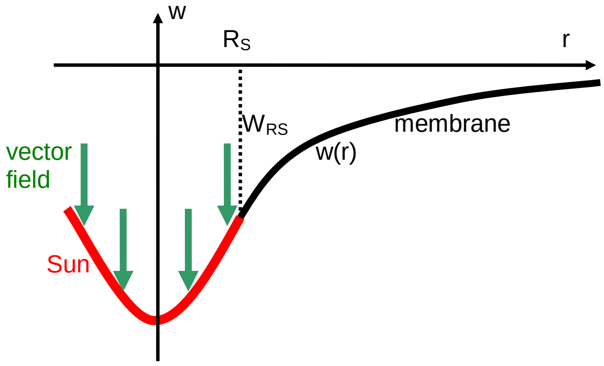

Fig. 3. Depth of space at the edge of sun

Eq. (2.6), the horizontal vector field acceleration A_{VFH} is

$$A_{VFH} = \frac{g_{RS}}{W'_{RS}} = \frac{g_{RS}\, R_S}{W_{RS}} . \qquad (2.8)$$

With the gravitational acceleration $g_{RS} = \gamma\, M_S / R_S^2$ = 273.65 [m/s^2] at the edge of the sun and the above given values of R_S and W_{RS}, one obtains an amount of A_{VFH} = 1.330×10^5 [m/s^2] for the horizontal vector field acceleration.

The membrane holds the sun. The vertical action of the vector field is compensated for by the tension F_0 of the membrane. This implies that the force $F = M_S\, A_{VFV}$ with the vertical vector field acceleration A_{VFV} must be compensated for by the vertical (in *w*-direction) components of the tension F_0 that draws at the surface $4\pi R_S^2$ of the sun. The vertical component of the sloped tension F_0 is $F_{0w} = F_0 \sin(\alpha)$. The slope of the membrane is $w' = \tan(\alpha)$. For small angles α, one obtains for the edge of the sun the equation $M_S\, A_{VFV} = 4\pi\, R_S^2\, F_o\, W'_{RS}$. With Eq. (2.6) and Eq. (2.8), one obtains the relation between the tension F_0 of the membrane and the vertical vector field acceleration A_{VFV} as

$$F_0 = \frac{M_S\, A_{VFV}}{4\pi\, R_S\, W_{RS}} . \qquad (2.9)$$

The numerical values of both constants are derived in the next section.

3. GRAVITATIONAL ENERGY OF THE SUN

Gravitational energy, E_g, is the energy that is released, when many small masses, coming from infinity, agglomerate in a great mass *M*. In 1939, Chandrasekhar [25] has estimated the energy E_g for the case of a sphere with constant density, radius *R* and mass *M*. His estimation is

$$E_{g,Ch} = \frac{3\gamma\, M^2}{5R} . \qquad (3.1)$$

Chandrasekhar's starting point is the integral

$$E_{g,Ch} = \int_0^{RS} \left(\frac{\gamma\, 4\pi\, r^2\, \rho_S\, dr}{r} \right) \left(\frac{4\pi\, r^3\, \rho_S}{3} \right) . \qquad (3.2)$$

The first bracket of the integrand mainly is the mass of the added spherical shell. The second bracket is the mass of the already agglomerated central sphere. Both brackets together give the potential of the added spherical shell in the gravitational field of the already agglomerated central sphere. Here, γ is the gravitational constant. By backward substitution of $4\pi\, R_S^3\, \rho_S / 3 = M_S$ in the solution of the integral, one obtains Chandrasekhar's formula.

$$E_{g,Ch} = \left[\gamma\, \frac{4^2\, \pi^2\, r^5\, \rho_S^2}{5 \cdot 3} \right]_0^{RS} = \frac{3\gamma\, M_S^2}{5\, R_S} . \qquad (3.3)$$

The sun is no exact sphere, and its density is not a constant. However, we work here with Chandrasekhar's assumptions for the sake of simplicity. Substituting the sun data into Chandrasekhar's equation, we obtain the value $E_{g,Ch}$ = 2.268×10^{41} [J].

Assuming a membrane with tension F_0, the energy $E_{g,M}$, which is released when the sun sinks into the gravitational funnel to a depth of space W_{RS}, is given by the integral Eq. (3.4)

$$E_{g,M} = \int_0^{Wrs} F_0 W'(w) 4\pi R_S^2 \, dw. \qquad (3.4)$$

Here, $4\pi\ R_S^2 F_0\ W'(w)$ is the w-direction force component of the tension force that supports the sun's weight. It is multiplied by the distance *dw*, thus resulting in an energy. With $W'(w) = w/R_S$, the solution of the integral (3.4) is $E_{g,M} = F_0 4\pi R_S W_{RS}^2 / 2$. However, we examine Fig. 4.

Fig. 4 shows the projection of the 4-dimensional gravitational funnel onto a plane. Moreover, the scaling of the w-axis is extremely exaggerated, because the actual ratio of W_{RS} / R_S is about 1/500. The abscissa, here placed in the numerator, gives the distance *r* from the center of the sun in units of radii (R_S) of the sun. The ordinate gives the depth of space *w* in units of W_{RS}. The dashed red line shows a sun that closes the funnel down like a flat lid at the depth W_{RS} of space. This does, physically, make no sense. The pink line draws the more likely funnel shape. The interior of the sun sinks deeper than the rim in the distance R_S from the center of sun, because the sun has, in the center, its greatest substantial extension. Assuming a constant density ρ_S of the sun, the course *w(r)* of the depth of space inside the sun can be computed iteratively by way of the following system of difference equations:

$$\begin{aligned} dw_i &= w_i' \, dr \\ w_i &= w_{i-1} + dw_i \\ r_i &= r_{i-1} - dr \end{aligned} . \qquad (3.5)$$

Here, *dr* is an arbitrarily chosen small quantity (e.g., 1/1000-th of R_S). The quantity $w_i' = (M_i A_{VFV}) / (F_0 4\pi r_i^2)$ is the slope of the membrane at the distance r_i from the center, and the quantity $M_i = 4\pi \rho_S r_i^3$ is the mass of the remaining central sphere with radius r_i. The radius *r* runs backwards from the starting value $r_0 = R_S$ until $r = dr$ ($r = 0$ is impossible). The depth *w* of space starts at $w_0 = W_{RS}$. The increase in energy is, in each step, $dE_i = F_0 w_i' 4\pi r_i^2 dw_i$, and, thus, the additional gravitational energy $E_{g,add}$ the sum of all increases, i. e., $E_{g,add} = \Sigma dE_i$.

However, because the numerical values of the two constants, F_0 and A_{VFV}, are not known yet, we have to perform the numerical solution of the system (3.5) of difference equations with the aim $E_{g,M} + E_{g,add} = E_{g,Ch} = 2.268 \times 10^{41}$ [J] under the additional condition of Eq. (2.9). With the thus found values of $F_0 = 1.820 \times 10^{19}$ [N/m^2] and $A_{VFV} = 1.148 \times 10^5$ [m/s^2], the numerical solution of the system (3.5) of difference equations provides the gravitational energy $E_{g,M} = 1.631 \times 10^{41}$ [J] and the additional energy $E_{g,add} = 0.637 \times 10^{41}$ [J]. This results in the total value $E_g = 2.268 \times 10^{41}$ [J] for the gravitational energy of the sun, that is, the value given by Chandrasekhar.

The advantage of the system (3.5) of difference equations is that one can use, in a simple way, any non-linear, even discontinuous, density curve *ρ(r)* inside the sphere considered. In this case, the programmed algorithm is only slightly more complex. We, therefore, show the results of a parabolic density curve here for comparison. Factor $m_S = M_S / \left(4\pi R_S^3 \left((1/3) - (1/5)\right)\right)$ ensures that, when integrating the masses of the spherical shells negatively, starting with the full sphere of mass M_S, at the end, we reach mass 0.

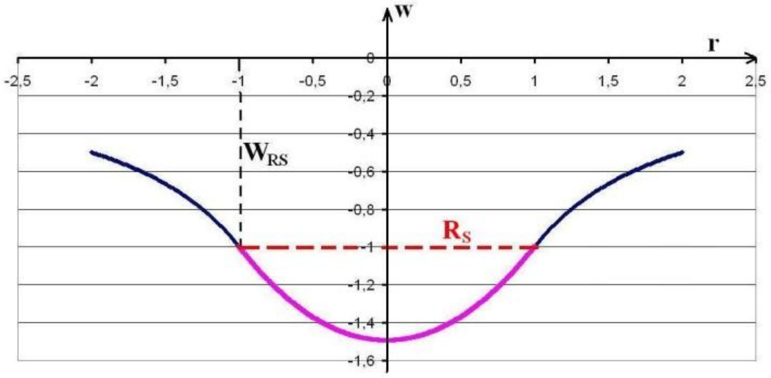

Fig. 4. Shape of the gravitational funnel

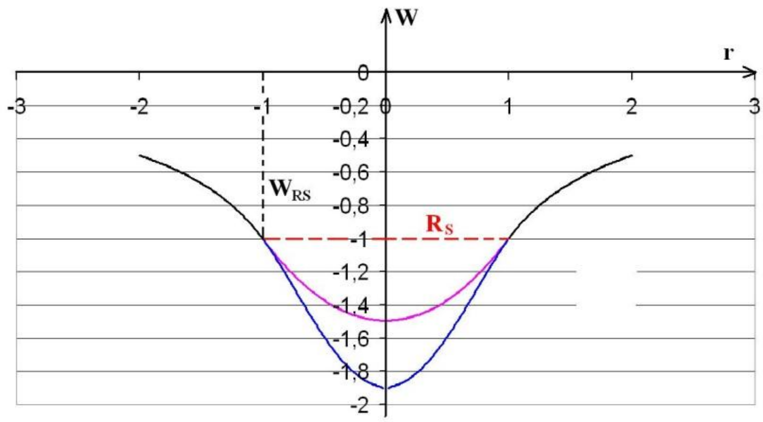

Fig. 5. Shapes of the depth of space for various density distributions

The only needed change in the DE-system (3.5) is that we use instead of equation $M_i = 4\pi \rho_S r_i^3$ the additional difference equation $M_i = M_{i-1} - 4\pi r_{i-1}^2 \rho_{i-1} dr$ with the starting value M_0=M_S. Naturally, we must calculate the density values ρ_i in agreement with the chosen density model. Fig. 5 shows the computed courses of the depth of space for the two cases: (1) constant density; (2) parabolic density curve.

The pink curve shows the course of w(r) in the case of the constant density distribution inside the sun, the blue curve for the parabolic density distribution with its maximum at the center of sun. The center of the sun sinks correspondingly to the greater mass in the center deeper into the membrane, i.e., until *1.87* W_{RS} instead of *1.5* W_{RS}, in the case of constant density. Correspondingly, the value of the gravitational energy E_g=2.268×10^{41} [J] increases now to E_g=3.013×10^{41} [J]. As a matter of course, a parabolic density curve, like the constant density curve, is also only a model assumption.

4. INERTIAL MASS

The inertial mass of a particle or a body could, in our cosmic membrane model, be connected with the mass of the membrane. If a body, e.g., the sun, moves with speed *v* in the space (here, e.g., in direction x), the gravitational funnel moves as well. In the direction of this movement, the membrane sinks downwards (in the direction of the negative w-axis, see the red arrows). Behind the moved body, the membrane is lifted in the direction of the positive w-axis (green arrows, see Fig. 6).

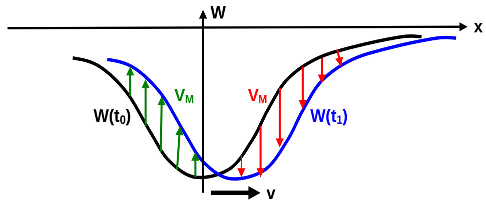

Fig. 6. Moved body with moved gravitational funnel

In this up-and-down movement, kinetic energy is stored. If a body is accelerated, decelerated, or deviated from its direction, then this also implies an intervention in the movement of the membrane. Accelerating the body must also accelerate the up-and-down movement of the diaphragm.

In the case of a spherical body, e.g., the sun, the depth *w* of space outside the body is [26]

$$w(r) = -W_{RS} R_S / r \qquad (4.1)$$

However, Eq. (4.1) holds only in the case of weak gravitation, e.g., in the case of the sun. Quantity *w* is the depth of space in the fourth dimension, W_{RS} is the depth of space at the edge of the sun, and R_S is the radius of the sun. The origin of the coordinate system x, y, z is the center of the sun at time t_0. If the sun moves with speed *v* in direction of the positive x-axis, then the gravitational funnel is commoving. For small velocities, i. e., v<<c, we neglect the finite propagation velocity *c* of gravitation. If we hold a point in space, $\vec{r} = (x, y, z)$, then the gravitational funnel deepens for positive x-values, i.e., we have a negative speed of the membrane, $-v_W(\vec{r})$, in the fourth dimension at position $\vec{r}$. In the case of negative x-values, the sign of the speed $v_W(\vec{r})$ changes. Assuming density ρ_M [kg/m^3] for the membrane, we can calculate the kinetic energy $E_{M,kin}$ of the movement of the membrane in the fourth dimension. With $r=\sqrt{x^2+y^2+z^2}$, $v=dx/dt$, and $v_W(\vec{r}(t))=dw(\vec{r}(t))/dt$, the speed $v_W(\vec{r})$ of the membrane is

$$v_W(\vec{r})=-\frac{dw}{dr}\frac{dr}{dx}\frac{dx}{dt}=-\frac{W_{RS}R_S}{r^2}\frac{x}{r}v. \quad (4.2)$$

Then, the kinetic energy is the volume integral $E_{M,kin}=\iiint(\rho_M/2)v_W^2(\vec{r})dV$ over the entire space of the gravity funnel. Hereby, for this part of the calculation, however, we recess the volume of the sun itself, since 1) the course of the membrane deflection is unknown there because of the unknown density course inside the sun, and 2) the integrand in its form above would be singular because of $r \to 0$. In addition, we neglect a possible density change of the membrane in the gravitational funnel. For the spherical coordinates r, φ, ϑ, we obtain

$$E_{M,kin}=\frac{\rho_M W_{RS}^2 R_S^2 v^2}{2}\iiint\frac{x^2}{r^6}r^2\cos(\vartheta)\,dr\,d\varphi\,d\vartheta \quad (4.3)$$

Here, the radius r runs from R_S to ∞, the angle φ runs from 0 to 2π, and the angle ϑ runs from $-\pi/2$ to $\pi/2$. With $x=r\cos(\varphi)\sin(\vartheta)$, $\int(-1/r^2)dr$ = 1/R$_S$, $\int\cos^2(\varphi)d\varphi$ = π, and $\int\sin^2(\vartheta)\cos(\vartheta)d\vartheta$ = 2/3, the kinetic energy of the membrane is

$$E_{M,kin}=\frac{\pi\,\rho_M\,W_{RS}^2\,R_S\,v^2}{3}. \quad (4.4)$$

However, the recessed volume of the moving sun contributes significantly to the kinetic energy, because there the deviation of the membrane has its maximum. The course of the density, $\rho_S(r)$, inside the sun is not known exactly. Therefore, we work with a density model. We postulate tentatively

$$\rho_S(r)=m_S\left(1-(r/R_S)^k\right). \quad (4.5)$$

In the case of k=2, the density course $\rho_S(r)$ would be a parabola with its maximum in the center of the sun. For the reason of simple integrability, we deliberately did not choose a density curve that is based on temperature curves or nuclear processes. The factor m_S is calculated only once. It ensures that the volume integral of the density over the volume of the sun fits their mass M_S for each value of parameter k. With Integral m_S=M_S/ $\iiint\left(1-(r/R_S)^k\right)dV$, we obtain $m_S=M_S/\left(4\pi R_S^3\left((1/3)-(1/(k+3))\right)\right)$. Fig. 7 shows the course of density, $\rho_S(r)$, over the diameter of the sun for different values of the parameter k. For $k\to\infty$, factor m_S fits the constant density, i.e., the arithmetic mean value, $\bar{\rho}_S$ =1407 [kg/m3], of the density of the sun.

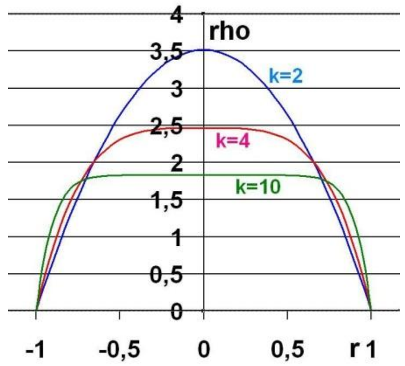

Fig. 7. Courses of density ρ in 1000 kg/m^3 for various values of *k*

In the case of a sphere with radius r and mass M, one obtains by Eq. (2.8) the slope $w'(r)=g/A_{VFH}$ of the membrane at the edge of the sphere. Hereby, g is the gravitational acceleration, $g=\gamma M/r^2$, at the edge of the sphere, and A_{VFH} is the horizontal vector field acceleration, and γ is the gravitational constant. We replace in Eq. (3.2) the derivation $w'(r)=dw/dr$ by $w'(r)=\gamma M(r)/\left(r^2 A_{VFH}\right)$, and obtain, for the vertical speed $v_W(\vec{r})$ of the membrane, at the surface of the sphere with radius r

$$v_W(\vec{r})=-\frac{\gamma M(r)}{r^2 A_{VF}}\frac{x}{r}v. \quad (4.6)$$

M(r) is here the mass of the sphere until the radius r, i.e., $M(r)=\iiint m_S\left(1-(r/R_S)^k\right)dV$, or $M(r)=4\pi m_S\left((r^3/3)-\left(r^{k+3}/\left(R_S^k(k+3)\right)\right)\right)$.

Now, one obtains, instead of Eq. (4.3), the kinetic energy of the moved membrane inside the sun as

$$E_{M,kin,\text{int}} = \frac{\rho_M \gamma^2 v^2}{2 A_{VEH}^2} \iiint \frac{M^2(r) x^2}{r^6} r^2 \cos(\vartheta)\, dr\, d\varphi\, d\vartheta \quad (4.7)$$

Because of the relation $M^2(r) = 16\pi^2 m_S^2 \left(r^6/9 - 2r^{k+6}/\left(3R_S^k(k+3)\right) + r^{2k+6}/\left(R_S^{2k}(k+3)^2\right)\right)$, one can subdivide integral (4.7) into three parts. In the case of the sun, r runs from 0 to R_S, angle φ from 0 to 2π, and angle ϑ from $-\pi/2$ to $\pi/2$. With $x = r\cos(\varphi)\sin(\vartheta)$, $\int \cos^2(\varphi)\, d\varphi$ = π, and $\int \sin^2(\vartheta)\cos(\vartheta)\, d\vartheta$ = 2/3 (as above), one obtains the solution

$$E_{M,kin,\text{int}} = E_0 \left(\frac{1}{45} - \frac{2}{3(k+5)(k+3)} + \frac{1}{(2k+5)(k+3)^2} \right) \quad (4.8)$$

With

$$E_0 = \frac{16\pi^3 m_S^2 \rho_M \gamma^2 v^2 R_S^5}{3 A_{VFH}^2} . \quad (4.9)$$

Eq. (4.8) shows that the kinetic energy of the membrane depends only on the square v^2 of the speed v of the sun in our three spatial dimensions. This is the same behavior as that of the moved sun.

We seek an estimation of the density ρ_M of the membrane. The cosmic membrane is a flat object under tension. Therefore, transversal waves, e.g., gravitational waves, can occur. We use the relation Eq. (4.10), following from the wave equation, e.g., given for a taut guitar string [27],

$$v^2 = F_0/\rho. \quad (4.10)$$

Here, F_0 is the tension, ρ the mass distribution, and v the speed of the wave. The membrane is presumably very thin in the fourth dimension. Therefore, ρ is not a density, but a mass distribution ρ_{surf} , i.e., the mass of one cubic meter of the tree-dimensional surface of the balloon, which is our cosmos. However, the dimension is that of a common density. With a taut guitar string, the difference becomes more apparent: The density of steel is about 8000 Kg/m^3, but the density distribution of the guitar string is only about 0.01 Kg/m.). By solving the formula $v^2=F_0/\rho_{surf}$ with $v=c$, we obtain $\rho_{surf} = F_0/c^2$ = 2.025×10^2 [Kg/m^3].

Inserting the mass distribution ρ_{surf} into Eqs. (4.4) and (4.9) instead of the density ρ_M of the membrane, one obtains for the total amount of the kinetic energy, $E_{M,kin}$, of the moved membrane inside and outside the sun for speed $v=1$ [m/s] and density parameter $k=2$, the value of $E_{M,kin}$ = 4.26×10^{23} [J]. Unfortunately, this amount is only a small fraction of the true kinetic energy of $E_{S,kin}=(M_S/2)v^2$ =0.997×10^{30} [J] of the sun for speed $v=1$ [m/s].

The trial, to use another relation for the longitudinal gravitational waves in the membrane, has had no success. We suppose that longitudinal waves can occur, but, because of the extreme thinness of the membrane compared with its cosmic extension, they have no relevance for the transport of energy.

5. THE CLAY LUMP MODEL OF THE RELATIVISTIC INCREASE OF MASS

Mass increases with the speed in the absolute space. This applies equally to the inert and heavy masses. Since mass and energy are equivalent to each other, a particle with higher velocity (and therefore with higher mass) produces more electron-positron pairs, than a particle with lower speed. In our membrane theory, a particle has its rest mass m, when it is resting in the absolute space. Energy $E = mc^2/\sqrt{1-(v/c)^2}$ is a conservation quantity, where m and v are to be taken in the dimensions of the rest frame. The Lorentz factor, above used, is an implication of the addition theorem of velocities of the special relativity (SR). It describes the behavior of the mass of a particle under acceleration. Here, however, we try to find another foundation of the factor, because the addition theorem of velocities of the SR does not hold in the same way under the paradigm of the absolute space assumed by us with a relativistic length and cross contraction [28].

In this section, we present a model of the increase of mass by repeated impact processes. A particle with rest mass m_p is in rest in the membrane, i.e., its speed is v = 0. The particle is bombarded with photons of mass $dm_{Ph} = h\nu/c^2$. We suppose that the impact is inelastic, i.e., after impact, the photon transfers its whole energy to the particle, and, at the same time, the mass of the particle increases just as an object increases in mass when sticky clay is thrown at it.

A mass *m* in motion with speed *v* has, in the case of low speed *v* (small compared with the speed of light, c), the kinetic energy

$$E_{kin} = \frac{m_0}{2} v^2 . \tag{5.1}$$

Quantity m_0 is the rest mass. We find the same amount of energy hidden in the relativistic increase *dm* of mass. The relativistic increase of mass is not only a theoretical assumption of the SR, but also an experimentally very exactly measured effect [29, 30]. The relativistic increase of mass is

$$dm = \frac{m_0}{\sqrt{1 - v^2 / c^2}} - m_0 . \tag{5.2}$$

By the serial development of the square root, one obtains, in the cases of low speeds *v*,

$$dm \approx \frac{m_0}{2} \frac{v^2}{c^2} . \tag{5.3}$$

From this, we deduce that the energy $dm_{Ph} = dm c^2$ bears on the increase of the kinetic energy of the particle under consideration as an increase of mass, both in equal parts. If we put this thought into equations, we obtain for the energy Eq. (5.4), i.e.,

$$\frac{m}{2} v^2 + \frac{dm_{Ph}}{2} c^2 = \left(\frac{m + dm_{Ph}}{2} \right) v'^2 , \tag{5.4}$$

Or

$$v'^2 = \frac{m v^2 + dm_{Ph} c^2}{m + dm_{Ph}} . \tag{5.5}$$

Neglecting the small quantity $(dv)^2$, we obtain from $v'^2 = (v + dv)^2 = v^2 + 2v\,dv + (dv)^2$ and Eq. (5.5) the Eq. (5.6).

$$dv = \frac{1}{2v} \left(\frac{m v^2 + dm_{Ph} c^2}{m + dm_{Ph}} - v^2 \right) . \tag{5.6}$$

Now, we have two coupled integrable difference equations:

$$m_{n+1} = m_n + (dm_{Ph} / 2) \ , \tag{5.7}$$

$$v_{n+1} = v_n + dv . \tag{5.8}$$

With the initial values, $v_0 = 0$ and $m_0 = 1$, and the arbitrarily chosen increment $dm_{PH} = 0.00001$, one obtains, by numerical integration, the results in Table 1.

Table 1. Results of integration of the DE-system Eqs. (5.7) and (5.8)

step	*m(v)*	*v [m/s]*	*dv [m/s]*	*v' [m/s]*	$m_{rel}(v)$
0	1	0	948,022	948,022	1
1	1.000005	948,022	392,678	1,340,700	1.000005
2	1.000010	1,340,700	301,309	1,642,010	1.000010
10	1.000050	2,997,808	146,307	3,144,116	1.000050
100	1.000500	9,476,704	47,230	9,523,934	1.000500
1000	1.005000	29,867,322	14,818	29,882,140	1.005000
10000	1.050000	91,409,814	4,246	91,414,061	1.050000
20000	1.100000	124,892,739	2,703	124,895,442	1.100000
30000	1.150000	148,042,796	1,995	148,044,792	1.150000
40000	1.199840	165,666,123	1,570	165,667,693	1.199840
50000	1.249840	179,834,369	1,279	179,835,649	1.249839
60000	1.299840	191,524,367	1,068	191,525,435	1.299839
70000	1.349840	201,368,689	907	201,369,596	1.349839
80000	1.399840	209,785,873	780	209,786,654	1.399839
90000	1.449840	217,069,203	679	217,069,882	1.449839
100000	1.499840	223,432,882	596	223,433,478	1.499839

Column *step* contains the step number of the numerical integration. Column $m_{rel}(v)$ contains the prediction $m_{rel}(v) = m_0 / \sqrt{1 - v^2/c^2}$ of the mass accordingly to Einstein's SR. Column $m(v)$ contains the calculated mass values of the clay lump model. The differences between the values of $m(v)$ and $m_{rel}(v)$ are about 2×10^{-6}. They are caused mostly by the error of the numerical integration which depends on the finite dm_{PH} – value of dm_{PH} = 0.00001. Here, the last value of the particle speed, v=223,432,882 [m/s], is quite close to the speed of light of c=299,792,456 [m/s].

6. RESULTS AND DISCUSSION

The assumption of Newton's absolute space together with the model of a brane world leads to minor changes in the existing worldview. Peter Higgs introduced the Higgs field, and Lisa Randall and Raman Sundrum a world-spanning brane with the property that the influence of gravitons drops exponentially with the distance from the membrane of the position of their production. We have taken up these important suggestions and improved the cosmic membrane model that we proposed in 1994.

In Section 2, we identify the vector field, used by us from the beginning, with the Higgs field. The Higgs field is the cause of the mass of waves and particles. Furthermore, we assume, conform with Randall and Sundrum, a wafting layer directly before the membrane. The membrane has a strength and toughness which is much greater as that of steel. All physical processes do not take place in the membrane, but in the wafting layer. The density of the wafting layer drops exponentially with the distance from the membrane, in accordance with the decrease of atmospheric pressure with increasing altitude.

The membrane was always assumed by us to be permeable for the vector field. But disturbances in the wafting layer as caused by waves or particles increase the resistance of the membrane in the flow of the vector field. This causes an additional load and, consequently, a curvature of the membrane. In the case of spherical masses, we obtain gravitational funnels of spherical symmetry which one can treat mathematically in particularly simple ways. So, one can calculate, from the sun data, the depth of the space at the edge of the sun, and, from this, the differential equation of the curvature of the membrane in the gravitational funnel. However, in all cases of non-spherical masses, the computation of the curvature of the space needs the solution of partial differential equations in 4 dimensions [26].

In Section 3, we deal with the gravitational energy in the classical perspective given by Chandrasekhar and in the perspective of the cosmic membrane model. By use of the cosmic membrane model, we obtain Chandrasekhar's value of the gravitational energy using the value $F_0 = 1.820 \times 10^{19}$ [N/m^2] for the membrane tension and the value $A_{VFV} = 1.148 \times 10^5$ [m/s^2] for the vertical vector field acceleration.

The equivalence of heavy and inert mass is an experimentally well-based finding, and one of the foundations of Einstein's GR. In Section 4, we tried to explain the inert mass with the up and down motion of the curved membrane, and we were able to achieve a partial success. We were able to prove that the kinetic energy of the up and down motion of the membrane during the migration of the gravitational funnel behaves formally as the kinetic energy of a moved mass, i.e., it is proportional to the square of the speed, v^2. Numerically, however, the value is too small. The origin of the equivalence of the heavy and inert mass is, therefore, not yet clear. However, we suspect that the main flow of the vector field permeates the membrane, but, in the case of a sloped membrane, a side stream forms parallel to the membrane in direction of the greatest inclination. So, both phenomena - *warping of space due to a heavy mass* and *acceleration of a mass in a gravitational field* - have nearly identical causes.

In Section 5, we present the *clay lump model* of the relativistic increase of mass. It describes the increase in mass of a particle that is accelerated by repeated impact processes in good agreement with the SR and the experimental findings [29, 30]. We have established our assumption that the energy of the pushing photon, or its mass equivalent *dm*, respectively, i.e., $dm_{Ph} = dmc^2$, affects, in equal parts, the increase of the kinetic energy and the increase of the mass of the particle. With this assumption, the integration of the equation of motion succeeds. The clay lump model suggests the increase in energy of an accelerated particle, and that the increase in mass and energy is real. In the case of an impact, this mass and energy is released in the form of electron-positron pairs or photons. Unfortunately, the clay lump model is only one step in the attempt to explain the relativistic increase of mass from the view of the membrane. There is no direct connection to the cosmic membrane model. After all, the calculated values of the clay lump model are invariant to any constant expansion rate V_E of the membrane in the fourth spatial dimension, because we have used the energy equation (5.4). Replacing v^2 by $v^2 + V_E{}^2$, v'^2 by $v'^2 + V_E{}^2$, and c^2 by $c^2 + V_E{}^2$, one obtains, in Eq. (5.5), in each side of the equation only the additive terms + $V_E{}^2$, which cancel each other out. In this respect, the clay lump model is 4D-suitable in our cosmic membrane model.

7. CONCLUSIONS

We know that gravity in the context of a taut elastically membrane is a wide field. One can exactly solve the simple cases, as that of a single load. However, already in the case of two heavy masses, the equations become involved. The more masses, the more effort is needed. Based on our experience in computing the force of attraction in a many-particle system, we know the importance of efficient software. The difficulty is that all solutions are found only iteratively, and, for the iterations, one needs a criterion to stop the iteration. Another serious problem is that the quantity of improvement of the solution decreases with each step of the iteration. Here, we have to find a way to come out of this vicious circle.

In this paper, we have shown that the kinetic energy of the up and down motion of the membrane during the migration of the gravitational funnel behaves as the kinetic energy of a moved mass. However, numerically, the value found is too small. Now, our aim is to find another approach to the explanation of the equivalence of the heavy and inert masses from the perspective of the membrane theory.

The clay lump model gives an explanation of the relativistic increase of mass of a particle under acceleration. In a next step, we should consider the issue in the context of the membrane theory, specifically under the paradigm of Newton's absolute space.

8. SUMMARY

Newton's absolute space is one of the foundations of the Cosmic Membrane theory (CM). *Membrane* and *absolute space* are synonyms. We had introduced the homogeneous vector field that acts on the membrane as source of the heavy mass, from the beginning in 1994. Now, we identify it with the Higgs-field. In 1999, Randall and Sundrum introduced their model of a world-spanning brane. With this idea, we improve our cosmic brane model by the introduction of the wafting layer outside the membrane. This layer solves the issue of the mobility of particles in a superstrong and superhard membrane.

Feynman's radius of excess is our starting point to explore the curvature of the membrane near heavy masses. Using its value of $r_{Ex}=a/3$=491[m] (with Schwarzschild's radius *a*), we obtain, for the gravitational funnel of sun, a depth of space of W_{RS} = 1.432×10^6 [m] at the edge of sun.

In 1939, Chandrasekhar calculated the gravitational energy of stars. We followed his tracks and obtained, in the case of sun, the same value. As a side result, we obtained the value of the tension F_0 of the cosmic membrane as F_0=1.820×10^{19} [N/m^2], and that of the vertical vector field acceleration A_{VFV}, acting perpendicularly from the fourth spatial dimension on the membrane, as A_{VFV}=1.148×10^5 [m/s^2]. The horizontal vector field acceleration A_{VFH}, i.e., inside the wafting layer, is connected with the acceleration of a probe mass in the gravitational field of a heavy mass, e.g., the sun or the earth. Its value is A_{VFH}=1.330×10^5 [m/s^2]. A_{VFH} acts as acceleration $a=A_{VFH}\ w'$, where w' is the slope of the membrane.

The mass of the moved membrane in a propagating gravitational funnel behaves as an inert mass. Its kinetic energy follows the relation E_{kin}~mv^2. However, the energy of the moving membrane yields a numerical value that is too small to explain the equivalence of heavy and inert mass.

Gravitational waves in connection with the membrane can appear in two kinds of waves, longitudinal and transversal. Assuming speed of light *c* for the speed of transversal gravitational waves, we obtain first estimations of the mass distribution ρ_{surf} of the membrane.

The *clay lump model* describes the increase in mass of a particle that is accelerated by repeated impact processes. The energy of the pushing photons affects, in equal parts, the increase in kinetic energy and the increase in the mass of the particle. The integration of the equation of motion yields results that are exactly the same as Einstein's prediction in his SR. The model suggests the increase in energy in an accelerated particle, and it is invariant to any constant expansion rate V_E of the membrane in the fourth spatial dimension.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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